CS 473 ✧ Spring 2017

Homework 9

Due Wednesday, April 19, 2017 at 8pm

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1. Suppose you are given an arbitrary directed graph $G = (V, E)$ with arbitrary edge weights $\ell : E \to \mathbb{R}$. Each edge in G is colored either red, white, or blue to indicate how you are permitted to modify its weight:

   - You may increase, but not decrease, the length of any red edge.
   - You may decrease, but not increase, the length of any blue edge.
   - You may not change the length of any black edge.

   The ***cycle nullification*** problem asks whether it is possible to modify the edge weights—subject to these color constraints—so that *every cycle in G has length* 0. Both the given weights and the new weights of the individual edges can be positive, negative, or zero. To keep the following problems simple, assume that G is strongly connected.

   (a) Describe a linear program that is feasible if and only if it is possible to make every cycle in G have length 0. *[Hint: Pick an arbitrary vertex s, and let* $\text{dist}(v)$ *denote the length of every walk from s to v.]*

   (b) Construct the dual of the linear program from part (a). *[Hint: Choose a convenient objective function for your primal LP.]*

   (c) Give a self-contained description of the combinatorial problem encoded by the dual linear program from part (b), and prove *directly* that it is equivalent to the original cycle nullification problem. Do not use the words "linear", "program", or "dual". Yes, you have seen this problem before.

   (d) Describe and analyze an algorithm to determine *in $O(EV)$ time* whether it is possible to make every cycle in G have length 0, using your dual formulation from part (c). Do not use the words "linear", "program", or "dual".

2. *There is no problem 2.*