CS 473 ✧ Spring 2017

Homework 6

Due Wednesday, March 16, 2017 at 8pm

1. Suppose you are given a directed graph $G = (V, E)$, two vertices s and t, a capacity function $c : E \to \mathbb{R}^+$, and a second function $f : E \to \mathbb{R}$. Describe and analyze an algorithm to determine whether f is a maximum (s, t)-flow in G. *[Hint: Don't make any "obvious" assumptions!]*

2. Suppose you are given a flow network G with ***integer*** edge capacities and an ***integer*** maximum flow f^* in G. Describe algorithms for the following operations:

   (a) INCREMENT(e): Increase the capacity of edge e by 1 and update the maximum flow.

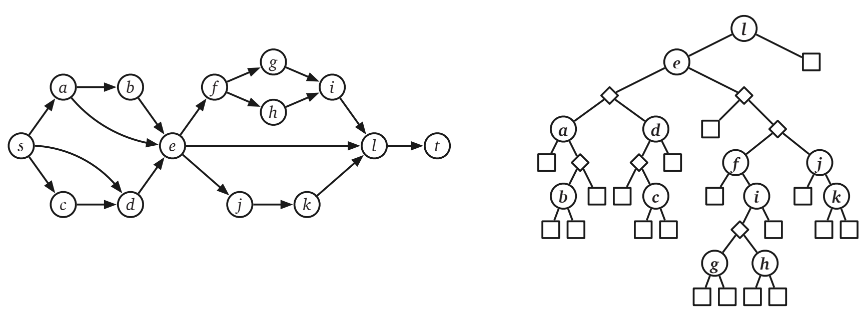
   (b) DECREMENT(e): Decrease the capacity of edge e by 1 and update the maximum flow.

   Both algorithms should modify f^* so that it is still a maximum flow, but more quickly than recomputing a maximum flow from scratch.

3. An ***(s, t)-series-parallel*** graph is a directed acyclic graph with two designated vertices s (the *source*) and t (the *target* or *sink*) and with one of the following structures:

   - **Base case:** A single directed edge from s to t.
   - **Series:** The union of an (s, u)-series-parallel graph and a (u, t)-series-parallel graph that share a common vertex u but no other vertices or edges.
   - **Parallel:** The union of two smaller (s, t)-series-parallel graphs with the same source s and target t, but with no other vertices or edges in common.

   Every (s, t)-series-parallel graph G can be represented by a ***decomposition tree***, which is a binary tree with three types of nodes: leaves corresponding to single edges in G, series nodes (each labeled by some vertex), and parallel nodes (unlabeled).

   An series-parallel graph and its decomposition tree.

   (a) Suppose you are given a directed graph G with two special vertices s and t. Describe and analyze an algorithm that either builds a decomposition tree for G or correctly reports that G is not (s, t)-series-parallel. *[Hint: Build the tree from the bottom up.]*

   (b) Describe and analyze an algorithm to compute a maximum (s, t)-flow in a given (s, t)-series-parallel flow network with arbitrary edge capacities. *[Hint: In light of part (a), you can assume that you are actually given the decomposition tree.]*