Max Flow/min cut

- Augmenting path algo (Ford-Fulkerson)
  - Max Flow = $\Sigma$ paths
  - Any Flow = $\Sigma$ paths + $\Sigma$ cycles
    $\# \leq |E|$
  - Integer cap => integer flow

- Which path?
  - Arbitrary - guarantees weak
  - Fat pipes - $O(E^2 \log E \log F)$
  - Short pipes - $O(E^2 V)$
  - Orlin - $O(EV)$

$\Omega(\text{EF})$
when all $c=1$
$F \leq V-1$

So what? Applications of flows + cuts

- Edge-disjoint paths problem
  - Given directed graph $G=(V,E)$
  - Vertices $s, t$
  - Find max # paths from $s$ to $t$
    that contain each edge at most once.

Solution: Set $c(e)=1$
Run FF $O(VE)$
Compute Flow decomposition

Claim: \(\#\) vertex-disjoint paths in \(G\)
\[= \#\) edge-disjoint paths in \(H\)

Proof: Two things to prove.

1. Suppose there are \(k\) vertex-disjoint paths in \(G\), then there are \(\geq k\) edge-disjoint paths in \(H\).

2. Suppose \(k\) edge-disjoint paths in \(H\):

   \[S \rightarrow v_{in} \rightarrow v_{out} \rightarrow w_{in} \rightarrow w_{out} \rightarrow \ldots \rightarrow z_{out} \rightarrow T\]
   \[S \rightarrow v \rightarrow w \rightarrow \ldots \rightarrow z \rightarrow T\]
So \( k \) vertex disjoint paths in \( G \)

Vertex capacities \( c: V \rightarrow \mathbb{R}^+ \)

Require \( \sum \frac{F(u \rightarrow v)}{c(v)} \leq c(v) \)

Add \( s, t \) with \( c(e) < 1 \)
Compute integral max flow \( f(u \rightarrow v) = 1 \iff uv \in M \) \( O(VE) \)
Each PC member can review \( \leq 15 \) papers
Each paper needs \( \geq 3 \) reviews.

Assignment Problem

Is max flow = \#papers?