A Treap with \( n \) vertices

- \( E[\#\text{leaves}] = \sum_{i=1}^{n} \Pr[\text{node } i \text{ is a leaf}] \)
- \( = \sum_{i=1}^{n} \Pr[\text{node } i \text{ is a local max}] \)
- \( = \sum_{i=2}^{n} \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{n-2}{3} + 1 = \frac{n+1}{3} \)

\( [i \uparrow k] = \text{[node } i \text{ is a proper ancestor of node } k] \)

node \( i \) is a leaf \( \iff [i \uparrow k] = 0 \) for all \( k \)

\( [i \uparrow k] \iff \text{node } i \text{ has smallest priority in } [i,...,k] \)

A Treap Diagram:

\[ \text{node } i \text{ is not smallest} \]

\( i \) is a leaf \( \iff \text{priority}(i) > \text{priority}(i-1) \) and \( \text{priority}(i+1) \)

\( \iff \text{priority}(i) \) is a local max
\[ E[\text{length of left spine}] = \sum_i \Pr[\text{Li on left spine}] = \sum \frac{1}{i^k} \leq \ln + 1 \]

\[ \Pr[i \uparrow k] \leq \ln n \]

\[ \Pr[i \uparrow 1] = \frac{1}{i^c} \]

\[ 1 - \frac{1}{n^c} \]

\[ \# \text{nodes on left spine} = O(\log n) \text{ whp} \]

\[ Pr[X = (1+\delta)\mu] \leq \left( \frac{e^\delta}{(1+\delta)^{1+\delta}} \right)^\mu \]

\[ \mu = E[X] = H_n = \ln n \]

\[ Pr[X \geq 10 \ln n] \leq \left( \frac{e^9}{10^{10}} \right)^n = n^{\ln(e^9/10^{10})} = n^{-\text{const}} \]

\[ a^{\log_b c} = c^{\log_b a} \]
Each day
every marked node
marks one unmarked child

\[
\text{MinDays}(v) = \begin{cases} 
0 & \text{if } v \text{ is a leaf} \\
1 + \text{MinDays}(w) & \text{if } v \text{ has one child } w \\
\min \left\{ \max\left\{ 2 + \text{MinDays}(r), 1 + \text{MinDays}(l) \right\}, \max\left\{ 2 + \text{MinDays}(l), 1 + \text{MinDays}(r) \right\} \right\} & \text{if } v \text{ has two kids } l \text{ and } r
\end{cases}
\]

Memoize in \( T \)
Eval in postorder/DFS
\( \mathcal{O}(n) \) time
\[ (\binom{1}{k}) = (\binom{n-1}{k}) + (\binom{n-1}{k-1}) \]
\[ E[\text{# Pareto opt pts}] = H_n \]

\[ \Pr[x_i \text{ is Pareto opt}] = \frac{1}{n-2 + 1} \]

Chernoff: \[ \Pr[X \geq 10 \ln n] \leq \left( \frac{e^9}{10^{10}} \right)^{\ln n} \]

\[ (1+8) \mu = 9, \quad \mu = h_n = \frac{1}{n} \]

\[ H_n = \frac{1}{i} \sum_{i=1}^{n} i = H_n \]

\[ \sum_{i=0}^{n} \alpha^i = \frac{1}{1-\alpha} \]

\[ \text{if } \alpha < 1 \]

\[ \sum_{i=1}^{n} i^c = \Theta(n^{c+1}) \]

\[ \text{if } c \neq -1 \]