Midterm 2 is graded (and sorted)

Distribution on the web site soon

Mean (stdev) = 26.7 (6.2) ($\frac{1}{2}$pt > MT1)

MT1: 26.2 (8.5)

Hw10 out — due Tue

Hw11 out next Mon/Tue "due" one week later
= last day of class!

Mon	Tue	Wed	Thu	Fri
	19		21 Today	
	26 HW10 due		28 ICES	
	2 Last day Any ?s		4 Review	5 Final 7pm

Hard problems:

NP-hard — no poly time algo unless miracle occurs
(P = NP)

ETH, SETH } no subexp. time algo

↳ no subquadratic-time algorithm.
or worse

We have to solve these anyway!

- Exploit structure in input

k-means Clustering

- Instead of optimal
compute provably good solutions

↑ close to optimal

Approximation algorithm

Scheduling n jobs $T[1..n]$ – running times

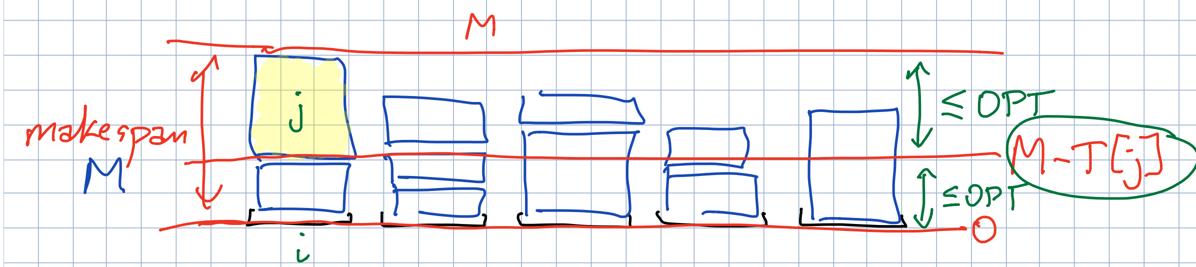
m machines – identical

compute assignment $M[1..n]$

run job j on machine $M[j]$

minimize $\max_i \sum_{j:M[j]=i} T[j]$

NP-hard from 3PARTITION



~~① Sort $T[1..n]$ in decreasing order~~

② for $j \leftarrow 1$ to n
 Assign job j to machine finishing first so far

Guaranteed to compute makespan $\leq \frac{4}{3} \cdot OPT$

$\leq \frac{3}{2} OPT$

Claim: Makespan $\leq 2\,OPT$

Proof: Machine i ends last
 Job j ends last

$\boxed{T[j] \leq OPT}$ $\qquad OPT \geq \sum_{j=1}^{n} T[j] / m$

because algorithm

$M - T[j] \leq$ Total time for any single machine $\qquad OPT \geq T[j]$ for all j

$\boxed{M - T[j] \leq \min_{i'} \text{Total}[i'] \leq \frac{1}{m} \sum_{i'} \text{Total}[i'] \leq OPT}$

$\implies M \leq 2 \cdot OPT$

Makespan $\leq X + Y$

$X \leq OPT$

$Y \leq OPT$

☆ ① Sort $T[1..n]$ in decreasing order ☆

② for $j \leftarrow 1$ to n

Assign job j to machine finishing first so far

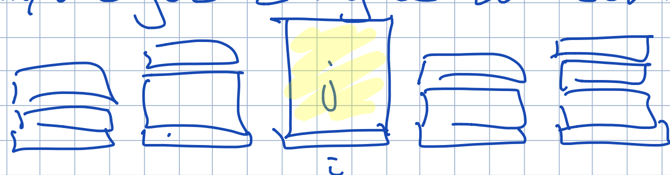
Guaranteed to compute makespan $\leq \frac{4}{3} \cdot OPT$

Claim: Makespan $\leq \frac{3}{2}$ OPT

Proof: Machine i finishes last

Job j finishes last

- IF only one job assigned to machine i

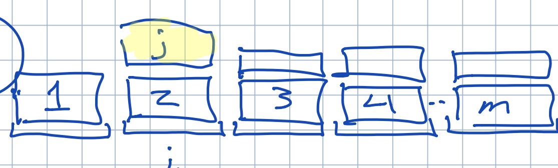


$\Rightarrow M = OPT$

- Otherwise:

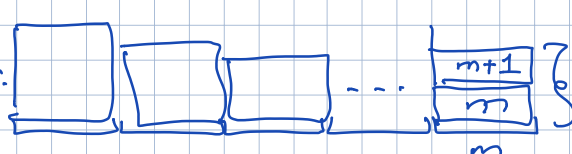
$j \geq m+1$

because algo.



$T[j] \leq T[1] \dots T[m]$ because ①

first ~~m+1~~ jobs:



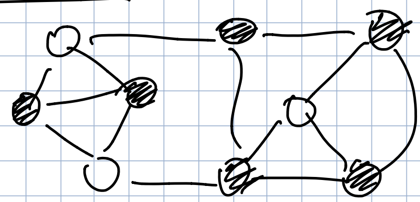
$$T[m] + T[m+1] \leq OPT$$

$$T[j] \leq T[m+1] \leq \frac{1}{2} OPT$$

$$M - T[j] \leq OPT$$

$$\Rightarrow M \leq \frac{3}{2} OPT \qquad \square$$

Vertex Cover



min # vertices
cover every edge

Do reductions preserve approximation? NO!

MaxIndset is NP-hard to approx
within a factor of $O(n^{1-\epsilon})$ for all ϵ.

Sort nodes by decr degree

Mark max deg node v
discard v and neighbors
recurse

H_n-approx