It is possible to win this level \( \iff \) The input formula is satisfiable.

To prove \( X \) is NP-hard

1. Choose known NP-hard problem \( Y \)
   - Circuit SAT
   - 3Color
   - SAT
   - SuperMarioBros
   - 3SAT
   - Ham. Cycle
   - Max. Ind Set
   - Partition
   - Max. Clique
   - 3Partition
   - Min Vertex Cover

2. Reduce \( Y \) to \( X \) in poly time

3. Prove it works
   \[ (\Rightarrow) \quad (\Leftarrow) \]
Hamiltonian Cycle (directed)

Input: Directed graph $G$

Output: Cycle visiting each vertex exactly once in $G$

Prove NP-hard by reduction from 3SAT

Given 3CNF Formula $\overline{\Phi} \rightarrow$ Graph $G$

- Variable gadget:

$$x \rightarrow \begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}$$

$$
\begin{array}{c}
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\bullet \\
\end{array}
$$
Every Ham cycle in $G$ corresponds to a satisfying assignment for $\phi$. 
<table>
<thead>
<tr>
<th>Binary choices</th>
<th>SAT / Partition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choices from a small set</td>
<td>Coloring</td>
</tr>
<tr>
<td>Order/permutation</td>
<td>Ham Cycle/Path</td>
</tr>
<tr>
<td>Small subset</td>
<td>Vertex Cover</td>
</tr>
<tr>
<td>Large subset</td>
<td>Max Ind. Set</td>
</tr>
<tr>
<td>Lots of subsets</td>
<td>3 Partition</td>
</tr>
<tr>
<td>3</td>
<td>3SAT</td>
</tr>
<tr>
<td></td>
<td>3 Color</td>
</tr>
<tr>
<td></td>
<td>3 Partition</td>
</tr>
<tr>
<td>WTF</td>
<td>3SAT / CircuitSAT</td>
</tr>
</tbody>
</table>
International Draughts

Flying Kings
Captures come off at end
Must capture max # pieces

A high level view of the reduction from Hamiltonian cycle to international draughts.
Left: A vertex gadget. Right: A white king emptying the vault.
Gray circles are black pieces that cannot be captured.

A corner gadget.

A crossing gadget.
Classic Nintendo Games are (Computationally) Hard

Greg Aloupis, Erik D. Demaine, Alan Guo, Giovanni Viglietta

(Submitted on 8 Mar 2012 (v1), last revised 8 Feb 2015 (this version, v3))

We prove NP-hardness results for five of Nintendo's largest video game franchises: Mario, Donkey Kong, Legend of Zelda, Metroid, and Pokemon. Our results apply to generalized versions of Super Mario Bros. 1–3, The Lost Levels, and Super Mario World; Donkey Kong Country 1–3; all Legend of Zelda games; all Metroid games; and all Pokemon role-playing games. In addition, we prove PSPACE-completeness of the Donkey Kong Country games and several Legend of Zelda games.
Figure 8: Left: Start gadget for Super Mario Bros. Right: The item block contains a Super Mushroom

Figure 9: Finish gadget for Super Mario Bros.

Figure 10: Variable gadget for Super Mario Bros.
Figure 11: Clause gadget for Super Mario Bros.

Figure 12: Crossover gadget for Super Mario Bros.
Threes!, Fives, 1024!, and 2048 are Hard

Stefan Langerman, Yushi Uno

(Submitted on 16 May 2015)

We analyze the computational complexity of the popular computer games Threes!, 1024!, 2048 and many of their variants. For most known versions expanded to an m x n board, we show that it is NP-hard to decide whether a given starting position can be played to reach a specific (constant) tile value.

Figure 9 An example of NP-hardness reduction from 3SAT to MAKE-T.