It is possible to win this level

$\Updownarrow$

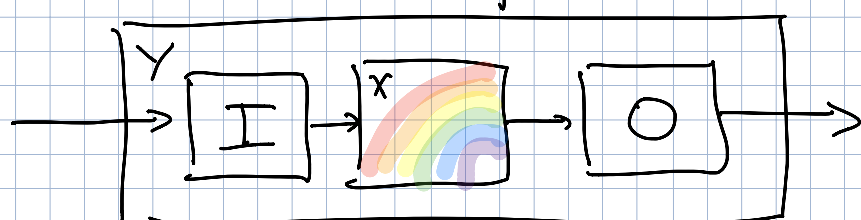
The input formula is satisfiable.

To prove X is NP-hard

1. Choose known NP-hard problem Y

   → Circuit SAT
   SAT
   → 3SAT
   Max Ind Set
   Max Clique
   Min Vertex Cover
   3Color
   Super Mario Bros
   Ham. Cycle
   Partition
   3Partition

2. Reduce Y to X in poly time



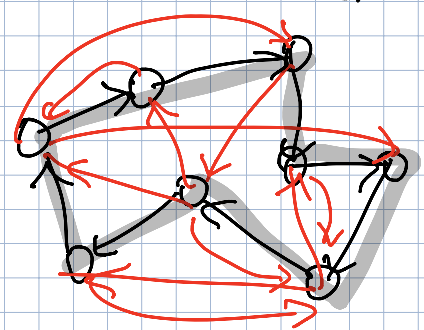
3. Prove it works

   (⇒) (⇐)

Hamiltonian Cycle (directed)

Input: Directed graph G

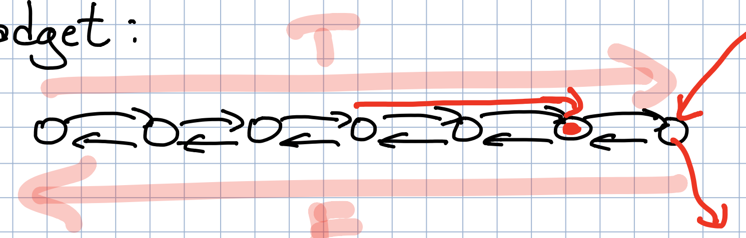
Output: Cycle in G visiting each vertex in G exactly once

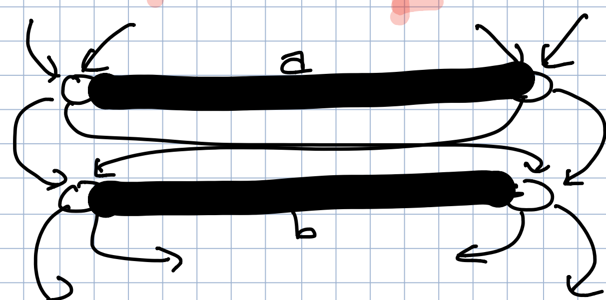


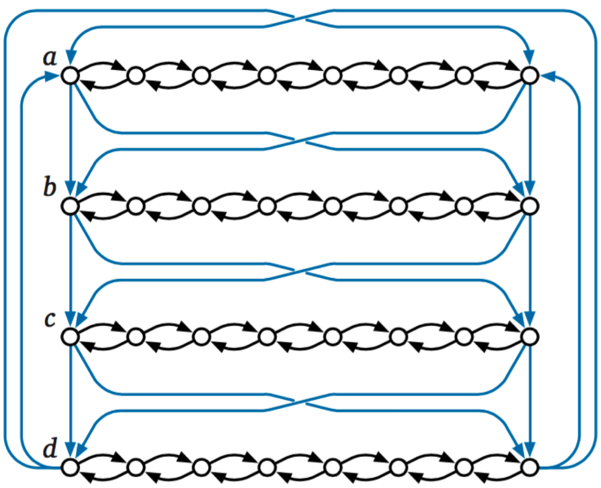
Prove NP-hard by reduction from 3SAT

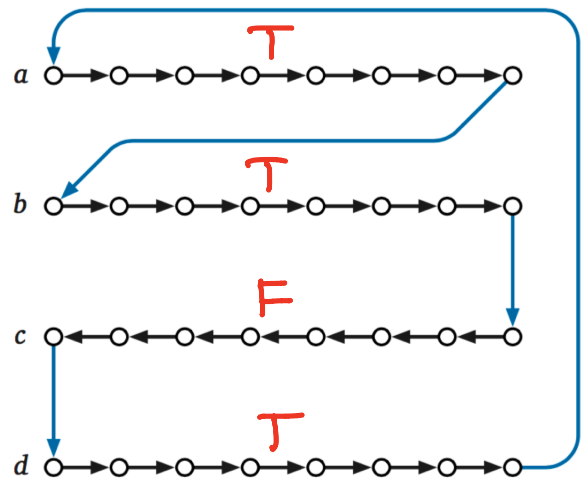
Given 3CNF formula $\Phi \mapsto$ Graph G

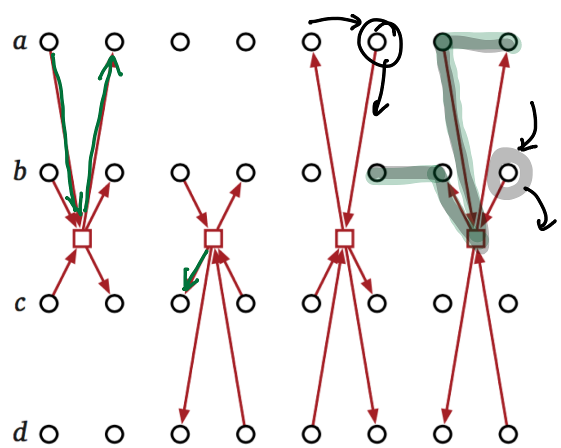
- Variable gadget:

$x \mapsto$ 

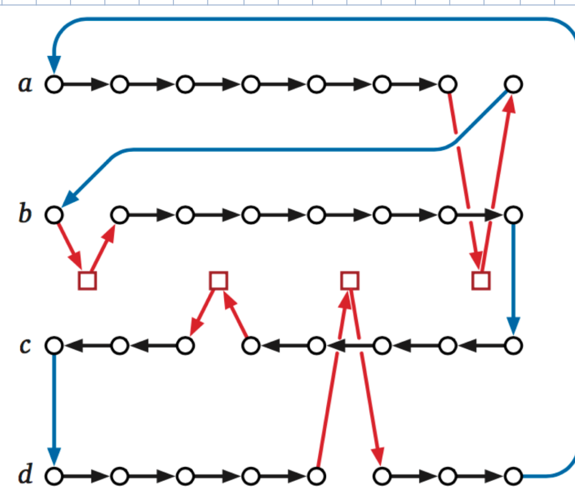








$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$



$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$

HARD PART Every Ham cycle in G corresponds to a satisfying assignment for Φ

Binary choices — SAT / Partition

Choices from a small set — Coloring

Order/permutation — Ham Cycle/Path

Small subset — Vertex Cover

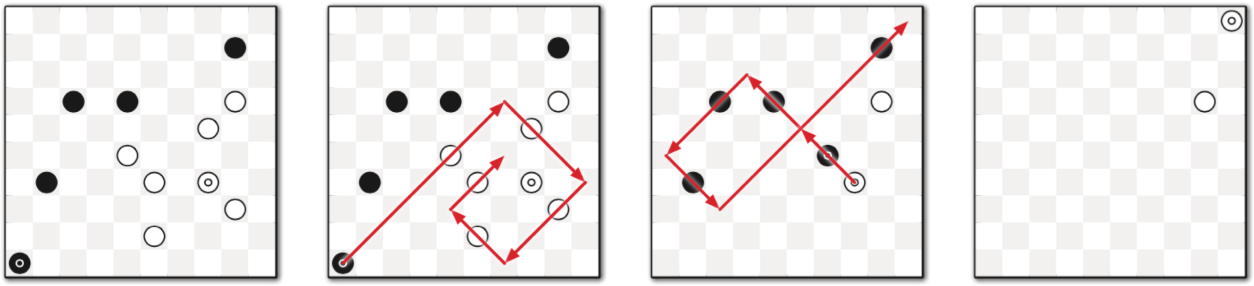
Large subset — Max Ind Set

Lots of subsets — 3Partition

3 — 3SAT
3Color
3Partition

WTF — 3SAT / CircuitSAT

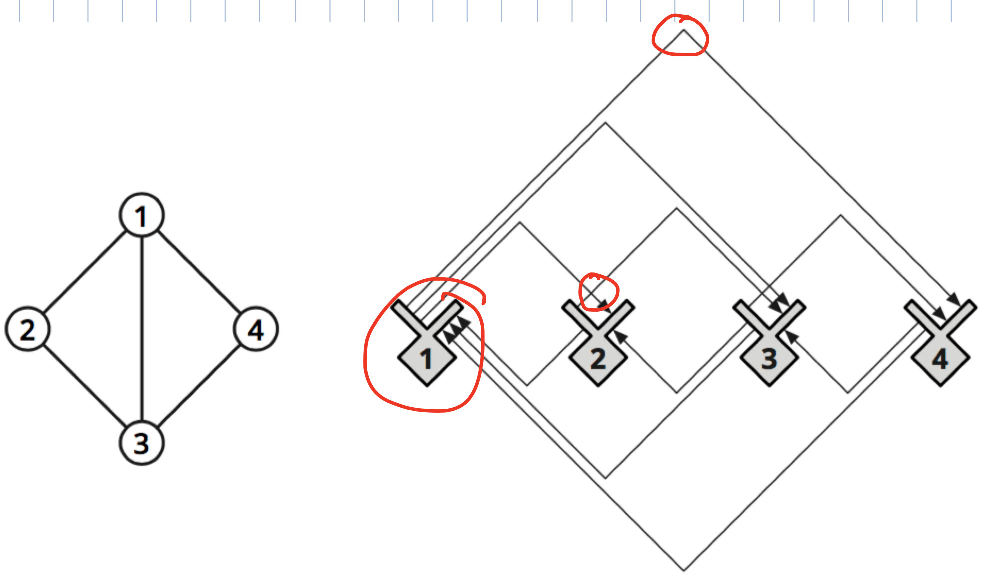
International Draughts



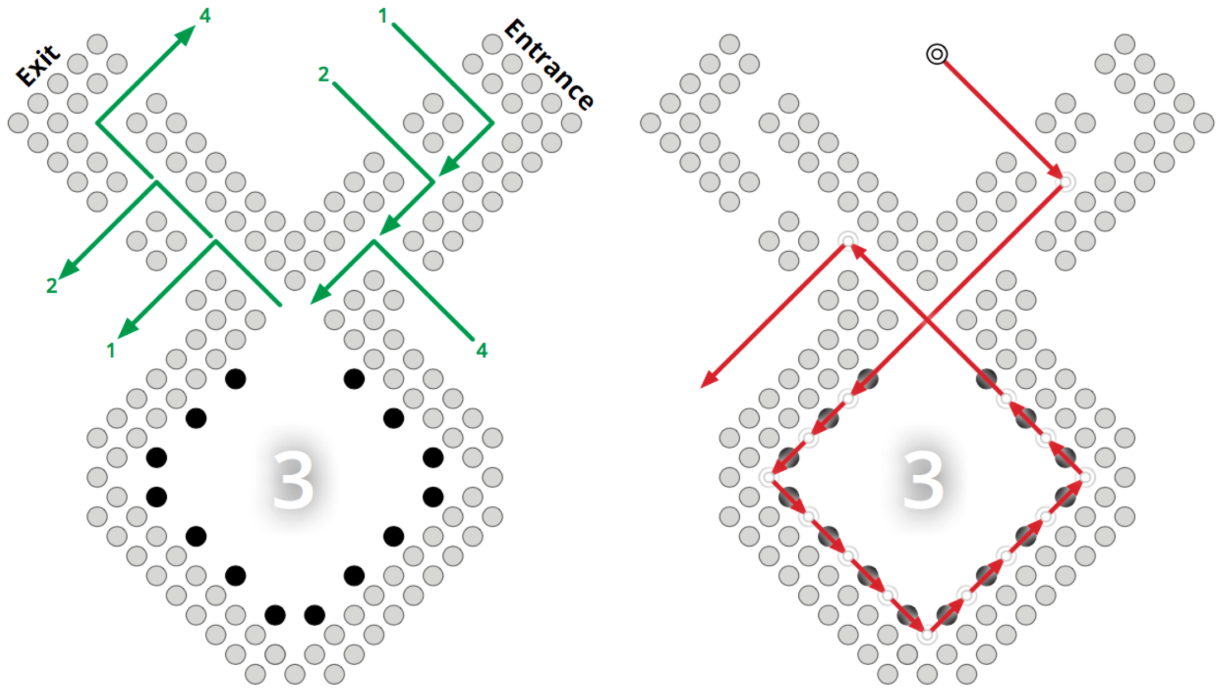
Flying Kings

Captures come off at end

Must capture max # pieces

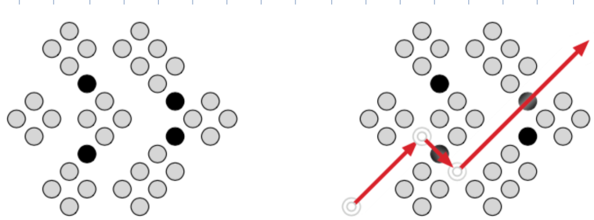


A high level view of the reduction from Hamiltonian cycle to international draughts.

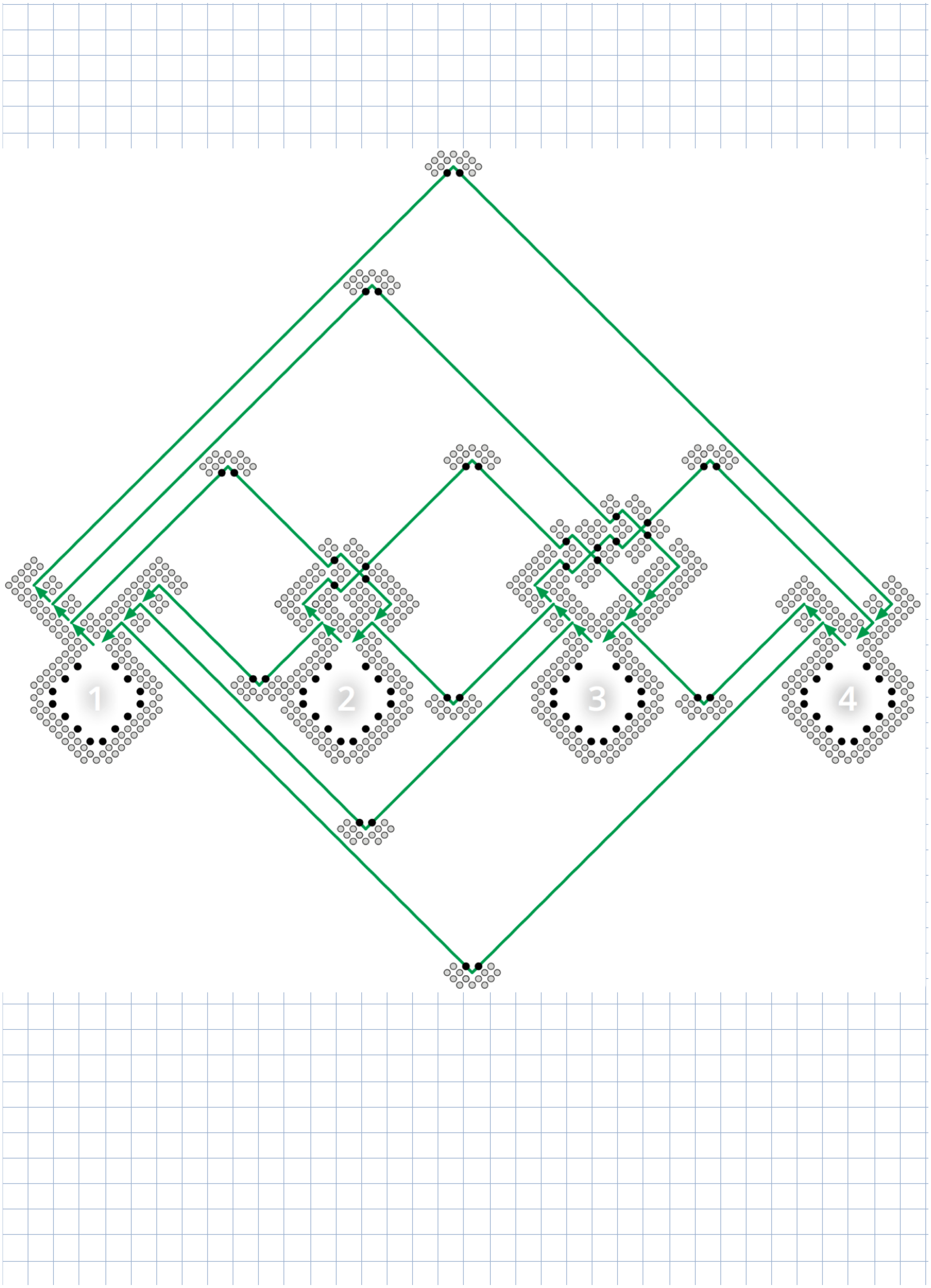


Left: A vertex gadget. Right: A white king emptying the vault.
Gray circles are black pieces that cannot be captured.

A corner gadget.



A crossing gadget.


1
2
3
4

arXiv.org > cs > arXiv:1203.1895

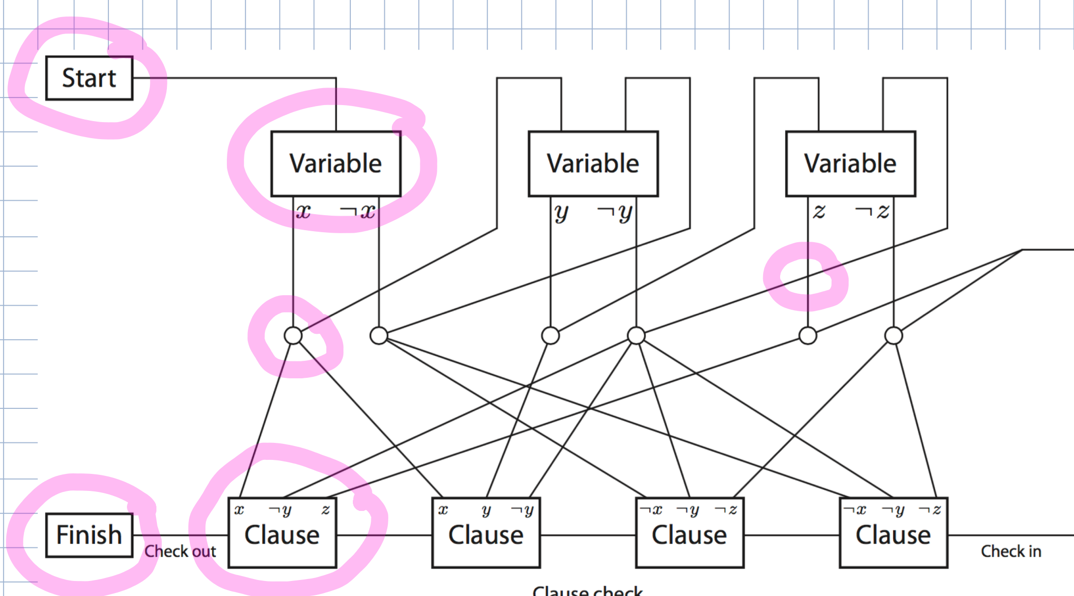
Computer Science > Computational Complexity

Classic Nintendo Games are (Computationally) Hard

Greg Aloupis, Erik D. Demaine, Alan Guo, Giovanni Viglietta

(Submitted on 8 Mar 2012 (v1), last revised 8 Feb 2015 (this version, v3))

We prove NP-hardness results for five of Nintendo's largest video game franchises: Mario, Donkey Kong, Legend of Zelda, Metroid, and Pokemon. Our results apply to generalized versions of Super Mario Bros. 1-3, The Lost Levels, and Super Mario World; Donkey Kong Country 1-3; all Legend of Zelda games; all Metroid games; and all Pokemon role-playing games. In addition, we prove PSPACE-completeness of the Donkey Kong Country games and several Legend of Zelda games.



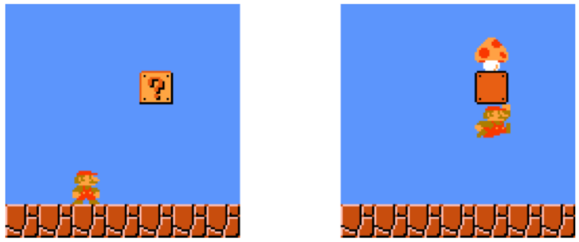


Figure 8: Left: Start gadget for Super Mario Bros. Right: The item block contains a Super Mushroom

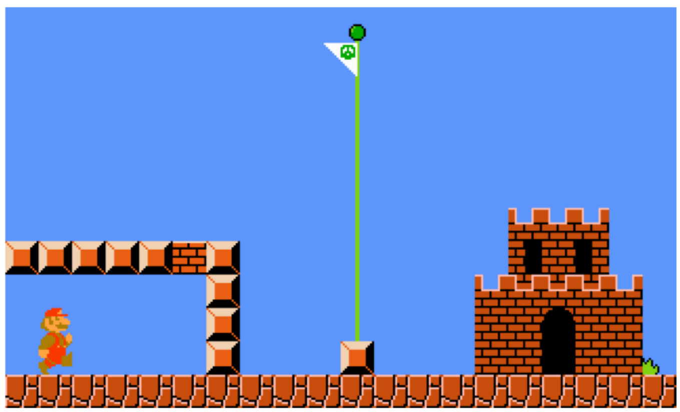


Figure 9: Finish gadget for Super Mario Bros.

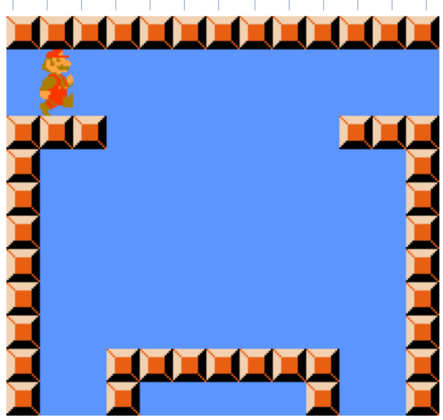


Figure 10: Variable gadget for Super Mario Bros.

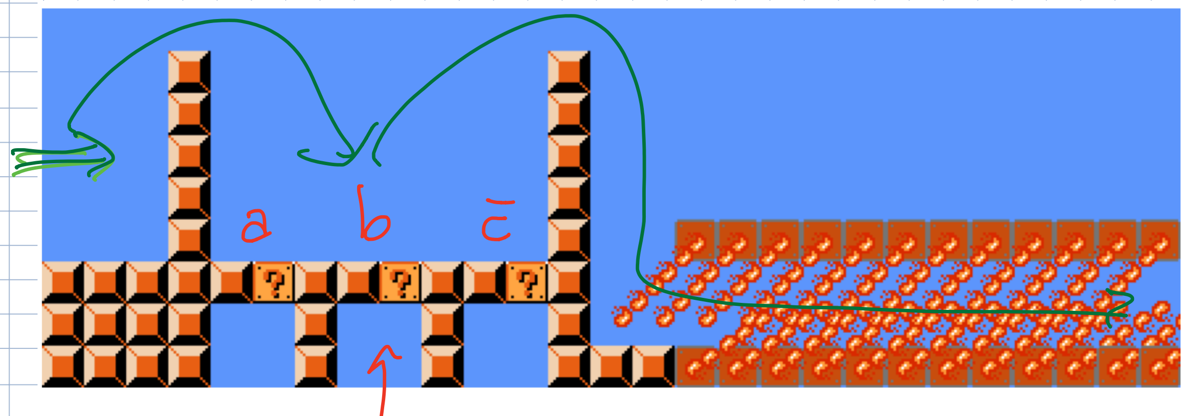


Figure 11: Clause gadget for Super Mario Bros.

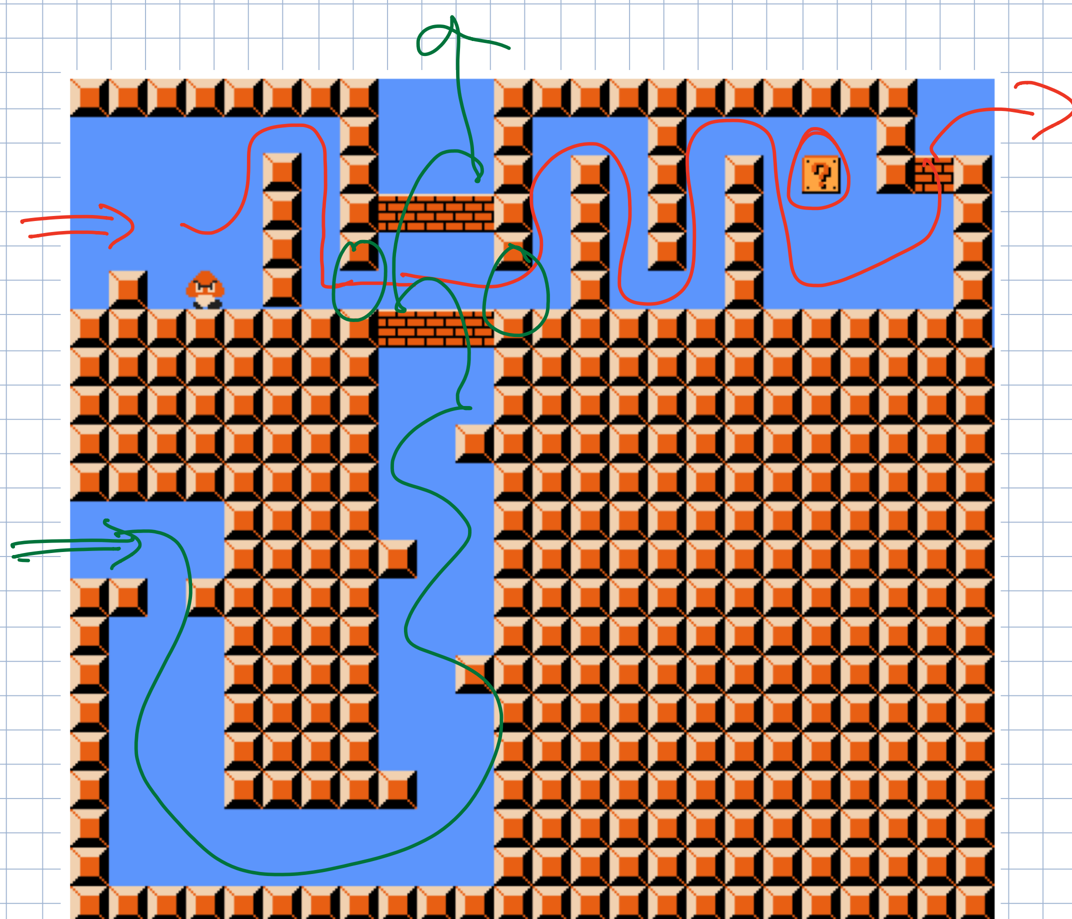


Figure 12: Crossover gadget for Super Mario Bros.

arXiv.org > cs > arXiv:1505.04274

Computer Science > Computational Complexity

Threes!, Fives, 1024!, and 2048 are Hard

Stefan Langerman, Yushi Uno

(Submitted on 16 May 2015)

We analyze the computational complexity of the popular computer games Threes!, 1024!, 2048 and many of their variants. For most known versions expanded to an m x n board, we show that it is NP-hard to decide whether a given starting position can be played to reach a specific (constant) tile value.

$f = (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee x_3 \vee \overline{x_4}) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_4)$

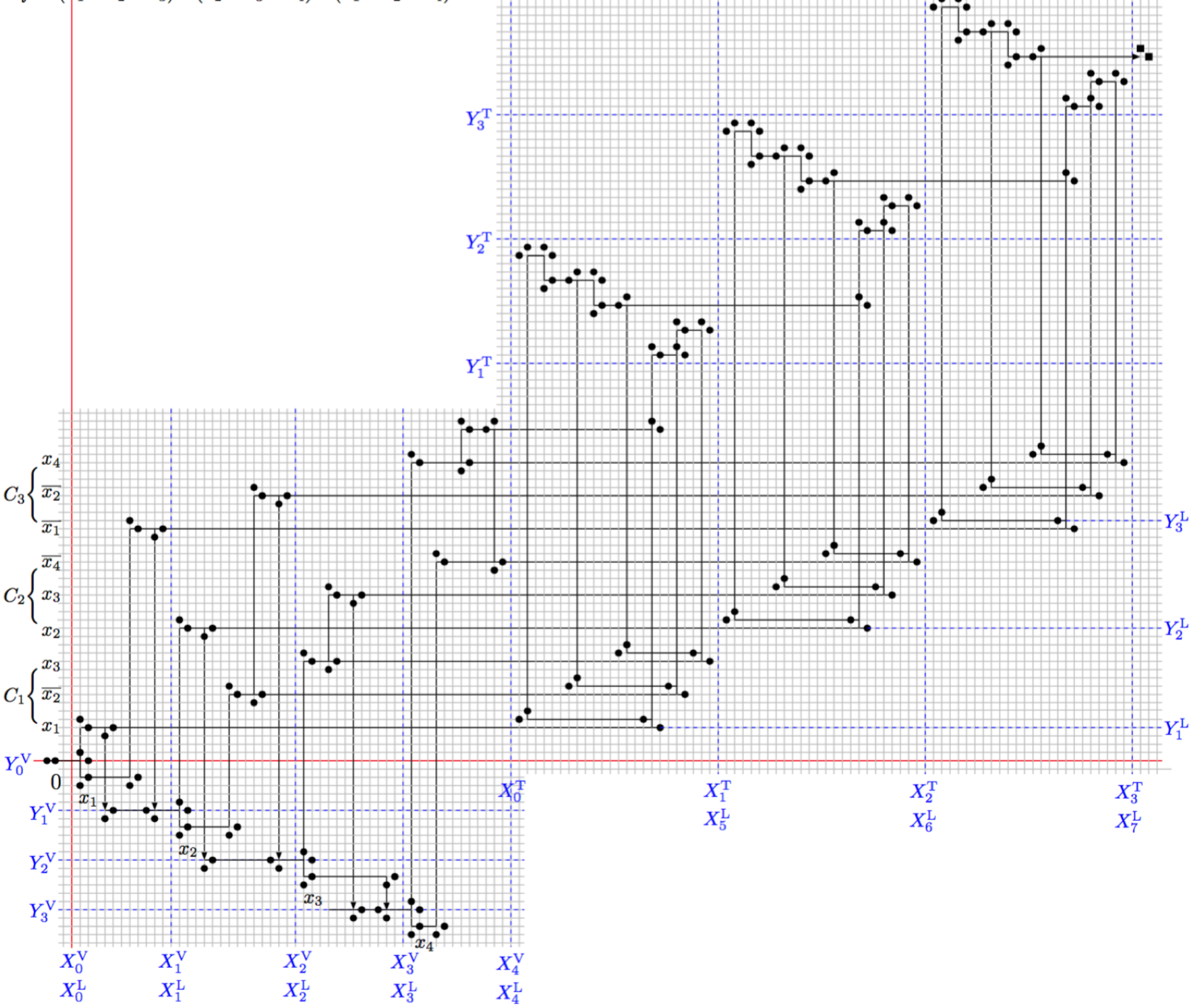


Figure 9 An example of NP-hardness reduction from 3SAT to MAKE-T .