\[
\sum_{i} h - \text{len}(p_i) + 1
\]

Find max collection of edge-disjoint paths with min total length.

Discard any path \( \text{len} > h \).
**NP-hardness**

P: answer in poly time

NP: verify in poly time

X is NP-hard ⇔ if X∈P, then P=NP

**Cook-Levin Theorem:**

\[ \text{CircuitSAT is NP-hard} \]

\[ \text{SAT is NP-hard} \]

\[ \text{[Karp]} \]

\[ \text{3SAT is NP-hard} \]

\[ \text{MIS is NP-hard} \]

\[ (a \lor b \lor c) \land (b \lor c \lor \overline{d}) \land (a \lor c \lor d) \land (a \lor b \lor d) \]

**Reduction:** To prove X is NP-hard

1. Choose known NP-hard Y
2. Build a poly-time algo for Y using a fictional poly-time algo for X as a subroutine
Maximum Independent Set

Input: Graph $G$
Output: max size of ind subset of vertices.

Reduction from 3SAT

Arbitrary 3CNF Formula $\Phi$ \rightarrow 3SAT \rightarrow G \rightarrow MIS_k \rightarrow \#clauses? \rightarrow T/F

(a \lor b \lor c) \land (b \lor c \lor \overline{d}) \land (a \lor c \lor \overline{d}) \land (a \lor \overline{b} \lor \overline{d})

Ind set in $G \iff$ (partial) sat. assignment for $\Phi$

Clause gadget
G has an ind set of size k \iff There is an assignment that satisfies k clauses in \Phi

\(\Rightarrow\) Suppose G has an ind set of size k

\Rightarrow Assign values to variables of \Phi to make those k literals True

Consistent because of \(\times \Rightarrow \times\)

\Rightarrow k clauses, each with \geq 1 True literal

\(\Leftarrow\) Suppose some assqt makes k clauses in \Phi True

Choose one True literal from each of those clauses

Mark corr. verts in G
diff \(\Delta s\)
Ind \(\times \Rightarrow \times\)
Ind set of k verts

---

Max Clique

\[\text{clique} \iff \text{ind set}\]

\[\]
Min Vertex Cover = V - Max Ind Set
3 Color

Given G

Color verts with 3 colors
s.t. every edge touches 2 colors

Reduction from 3SAT
because 3.

Given ARBITRARY 3CNF \( \Phi \)
Build graph G

1. Truth gadget

2. Variable gadget

3. Clause gadget (\( \overline{a} \vee b \vee c \))
A 3-colorable graph derived from the satisfiable 3CNF formula

\[(a \lor b \lor c) \land (b \lor c \lor \tilde{d}) \land (\tilde{a} \lor c \lor d) \land (a \lor \tilde{b} \lor \tilde{d})\]