For problems that ask to prove that a given problem $X$ is NP-hard, a full-credit solution requires the following components:

- Specify a known NP-hard problem $Y$, taken from the problems listed in the notes.
- Describe a polynomial-time algorithm for $Y$, using a black-box polynomial-time algorithm for $X$ as a subroutine. Most NP-hardness reductions have the following form: Given an arbitrary instance of $Y$, describe how to transform it into an instance of $X$, pass this instance to a black-box algorithm for $X$, and finally, describe how to transform the output of the black-box subroutine to the final output. A cartoon with boxes may be helpful.
- Prove that your reduction is correct. As usual, correctness proofs for NP-hardness reductions usually have two components (“one for each $f$”).

1. Consider the following solitaire game. The puzzle consists of an $n \times m$ grid of squares, where each square may be empty, occupied by a red stone, or occupied by a blue stone. The goal of the puzzle is to remove some of the given stones so that the remaining stones satisfy two conditions: (1) every row contains at least one stone, and (2) no column contains stones of both colors. For some initial configurations of stones, reaching this goal is impossible.

Prove that it is NP-hard to determine, given an initial configuration of red and blue stones, whether the puzzle can be solved.

2. Everyone’s having a wonderful time at the party you’re throwing, but now it’s time to line up for *The Algorithm March* (アルゴリズムこうしん)! This dance was originally developed by the Japanese comedy duo Itsumo Kokokara (いつもここから) for the children’s television show *PythagoraSwitch* (ピタゴラスイッチ). The Algorithm March is performed by a line of people; each person in line starts a specific sequence of movements one measure later than the person directly in front of them. Thus, the march is the dance equivalent of a musical round or canon, like “Row Row Row Your Boat”.¹ Proper etiquette dictates that each marcher must know the person directly in front of them in line, lest a minor mistake during lead to horrible embarrassment between strangers.

Suppose you are given a complete list of which people at your party know each other. Prove that it is NP-hard to determine the largest number of party-goers that can participate in the Algorithm March. You may assume without loss of generality that there are no ninjas at your party.

¹そろそろおわりかな。そろそろおわりかな。そろそろおわりかな。