OLD CS 473: Fundamental Algorithms, Spring 2015

Discussion 14

April 30, 2015

14.1. NP Completeness.
Show that the following problems are NP-Complete.

Max Degree Spanning Tree

> **Instance**: Graph $G = (V, E)$ and integer k
> **Question:** Does G contains a spanning tree T where every node in T is of degree at most k?

TILING

> **Instance**: Finite set $\mathcal{RECTS}$ of rectangles and a rectangle R in the plane.
> **Question:** Is there a way of placing the rectangles of $\mathcal{RECTS}$ inside R, so that no pair of the rectangles intersect, and all the rectangles have their edges parallel of the edges of R?

HITTING SET

> **Instance**: A collection C of subsets of a set S, a positive integer K.
> **Question:** Does S contain a *hitting set* for C of size K or less, that is, a subset $S' \subseteq S$ with $|S'| \leq K$ and such that S' contains at least one element from each subset in C.

LARGEST COMMON SUBGRAPH

> **Instance**: Graphs $G = (V_1, E_1), H = (V_2, E_2)$, positive integer K.
> **Question:** Do there exists subsets $E'_1 \subseteq E_1$ and $E'_2 \subseteq E_2$ with $|E'_1| = |E'_2| \geq K$ such that the two subgraphs $G' = (V_1, E'_1)$ and $H' = (V_2, E'_2)$ are isomorphic?

BIN PACKING

> **Instance**: Finite set U of items, a size $s(u) \in \mathbb{Z}^+$ for each $u \in U$, an integer bin capacity B, and a positive integer K.
> **Question:** Is there a partition of U int disjoint sets $U_1, \ldots, U_K$ such that the sum of the sizes of the items inside each U_i is B or less?

14.2. Self reducibility!
For each of the following problems, assume you are given a black box that can solve the decision problem in polynomial time. Show how to solve the optimization version of this problem in polynomial time using this black box.

Shortest Path

Instance: A weighted undirected graph G, vertices s and t and a threshold w.
Question: Is there a path between s and t in G of length at most w?

Independent Set

Instance: A graph G, integer k.
Question: Is there an independent set in G of size k?

3Colorable

Instance: A graph G.
Question: Is there a coloring of G using three colors?

TSP

Instance: A weighted undirected graph G, and a threshold w.
Question: Is there a TSP tour of G of weight at most w?

Vertex Cover

Instance: A graph G, integer k.
Question: Is there a vertex cover in G of size k?

Subset Sum

Instance: S - set of positive integers,t: - an integer number (target).
Question: Is there a subset $X \subseteq S$ such that $\sum_{x \in X} x = t$?

3DM

Instance: X, Y, Z sets of n elements, and T a set of triples, such that $(a, b, c) \in T \subseteq X \times Y \times Z$.
Question: Is there a subset $S \subseteq T$ of n disjoint triples, s.t. every element of $X \cup Y \cup Z$ is covered exactly once.?

Partition

Instance: A set S of n numbers.
Question: Is there a subset $T \subseteq S$ s.t. $\sum_{t \in T} t = \sum_{s \in S \setminus T} s$?

SET COVER

Instance: $(X, \mathcal{F}, k)$:
X: A set of n elements
$\mathcal{F}$: A family of subsets of S, s.t. $\bigcup_{X \in \mathcal{F}} X = X$.
k: A positive integer.
Question: Are there k sets $S_1, \ldots, S_k \in \mathcal{F}$ that cover S. Formally, $\bigcup_i S_i = X$?

CYCLE HATER.

Instance: An undirected graph $\mathsf{G} = (\mathsf{V}, \mathsf{E})$, and an integer $k > 0$.
Question: Is there a subset $X \subseteq \mathsf{V}$ of at most k vertices, such that all cycles in G contain at least one vertices of X.

CYCLE LOVER.

Instance: An undirected graph $\mathsf{G} = (\mathsf{V}, \mathsf{E})$, and an integer $k > 0$.
Question: Is there a subset $X \subseteq \mathsf{V}$ of at most k vertices, such that all cycles in G contain at least two vertices of X.

14.3. Independence

Let $G = (V, E)$ be an undirected graph over n vertices. Assume that you are given a numbering $\pi : V \to \{1, \ldots, n\}$ (i.e., every vertex have a unique number), such that for any edge $ij \in E$, we have $|\pi(i) - \pi(j)| \leq 20$.
Either prove that it is NP-Hard to find the largest independent set in G, or provide a polynomial time algorithm.