

HW 8 (due Tuesday, at noon, April 8, 2014)

CS 473: Fundamental Algorithms, Spring 2014

Version: 1.01

Make sure that you write the solutions for the problems on separate sheets of paper. Write your name and netid on each sheet.

Collaboration Policy: The homework can be worked in groups of up to 3 students each.

1. (35 PTS.) Recomputing correct flow.

Given a flow network G with integer capacities you have computed a maximum flow f between s and t . However you have made a mistake in noting the capacity of an edge e .

- (15 pts) Suppose you *under* estimated the capacity of e by $k > 0$ units. Show that you can compute the correct maximum flow in $O(km)$ time using the current flow f .
- (20 pts) Do the same as above if you *over* estimated the capacity of e by $k > 0$ units. *Hint:* First assume that f is acyclic. How do you reduce flow on e ?

No proof of correctness necessary but justify the running time.

2. (35 PTS.) Special minimum cuts.

Let $G = (V, E)$ be a flow network with integer edge capacities. We have seen algorithms that compute a minimum s - t cut. For both problems below assume that you only have black box access to an algorithm that given G and nodes s, t outputs a minimum cut between s and t .

- (15 pts) Given G and s, t describe an algorithm that computes a minimum cut with a *minimum* number of edges.
- (20 pts) Given G and s, t describe an algorithm that decides whether G has a *unique* minimum s - t cut.

No proof of correctness necessary but we recommend a brief justification.

3. (30 PTS.) Committee Assignment.

The Computer Science Department at UIUC has n professors. They handle department duties by taking part in various committees. There are m committees and the j 'th committee requires k_j professors. The head of the department asked each professor to volunteer for a set of committees. Let $S_i \subseteq \{1, 2, \dots, m\}$ be the set of committees that professor i has volunteered for. A committee assignment consists of sets S'_1, S'_2, \dots, S'_n where $S'_i \subseteq \{1, 2, \dots, m\}$ is the set of committees that professor i will participate in. A *valid* committee assignment has to satisfy two constraints: (i) for each professor i , $S'_i \subseteq S_i$, that is each professor is only given committees that he/she has volunteered for, and (ii) each committee j has k_j professors assigned to it, or in other words j occurs in at least k_j of the sets S'_1, S'_2, \dots, S'_n .

- (a) Describe a polynomial time algorithm that the head of the department can employ to check if there is a valid committee assignment given the lists S_1, S_2, \dots, S_n . The algorithm should output a valid assignment if there is one.
- (b) The head of the department notices that often there is no valid committee assignment because professors naturally are inclined to volunteer for as few committees as possible. To overcome this, the definition of a valid assignment is relaxed as follows. Let ℓ be some integer. An assignment S'_1, S'_2, \dots, S'_n is now said to be valid if (i) $|S'_i - S_i| \leq \ell$ and (ii) each committee j has k_j professors assigned to it. The new condition (i) means that a

professor i may be assigned up to ℓ committees not on the list S_i that he/she volunteered for. Describe an algorithm to check if there is a valid committee assignment with the relaxed definition.

(b) is a more general problem than (a) so you can do (b) for full points or only (a) for 15 pts.