1. Let G be a directed graph with non-negative edge lengths. Describe an algorithm that given G and a number $\lambda > 0$ decides whether G has a cycle of average length strictly less than λ. The average length of a cycle is defined as the total length of the cycle (w/r/t the given edge lengths) divided by the number of edges in the cycle.

   Hint: Use negative cycle detection.

2. Let G be a graph and suppose two coins are placed on two of its vertices. In each step, both coins move to an adjacent node. A coin cannot remain in place on a step.

   a. Describe an algorithm that finds the shortest sequence of steps that puts the coins on the same node.

   b. Suppose both coins start facing heads, and that on every odd step we flip over the first coin and on every even step we flip over the second coin. Describe an algorithm that finds the shortest sequence of steps that puts the coins on the same node with both coins showing heads or both showing tails.

3. Let G be a directed graph and A a subset of its vertices. The distance from A to a vertex v is the shortest distance from any vertex $a \in A$ to v.

   Find the furthest vertex v from A along with the corresponding distance. (Alternatively, if G is a city map and A the set of fire stations, what is the furthest house from any fire station (and how far away)?)

4. Let G be a directed graph with weighted edges and H a subgraph of G with $V(H) = V(G)$. Let s and t be two vertices in G.

   Describe an algorithm that finds the shortest path from s to t using at most one edge not in H, and reports the extra edge not in H if one is used.

5. Let G be a directed graph. Each edge $e \in E$ has a probability $p(e) \in [0, 1]$ of failing. The edges fail independently of each other. The survival-rate of a path is the probability that no edge on it fails.

   Given two nodes s, t the goal is to find an s-t path P with the highest probability of surviving.