CS 473: Fundamental Algorithms, Spring 2011

Discussion 5

February 15, 2011

1. k PARTITION.

   Given a set of $S = \{\alpha_1, \ldots, \alpha_n\}$ of n numbers positive integers in the range 1 to m, decide, given a parameter k, if the numbers in S can be partitioned into k sets $S_1, \ldots, S_k$, such that, for all i, $w(S_i) = w(S)/k$, where $w(X) = \sum_{x \in X} x$. What is the running time of your algorithm as a function of n, m and k.

2. LARGEST COMPLETE SUBTREE.

   For this problem, a *subtree* of a binary tree means any connected subgraph. A binary tree is *complete* if every internal node has two children, and every leaf has exactly the same depth. Describe and analyze a recursive algorithm to compute the *largest complete subtree* of a given binary tree. Your algorithm should return the root and the depth of this subtree.

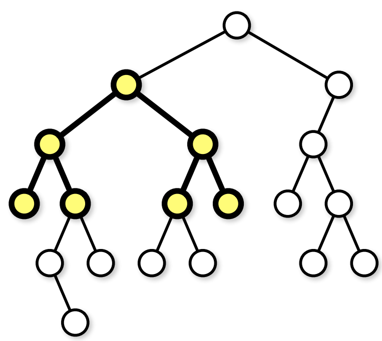

Figure 1: The largest complete subtree of this binary tree has depth 2.

3. ROD CUTTING.

   Suppose we are given a steel rod of length n. Also, assume we are given an array $p[1 \ldots n]$, where $p[i]$ is the price a rod of length i sells for. Given that we can make cuts for free (and that we only cut integer lengths), provide an algorithm that efficiently computes the maximum amount we can make by cutting up and selling our rod of length n.

4. BILLBOARDS.

   Consider a stretch of Interstate-57 that is m miles long. We are given an ordered list of mile markers, $x_1, x_2, \ldots, x_k$ in the range 0 to m, at each of which we are allowed to construct billboards (suppose they are given as an array $X[1 \ldots k]$). Suppose we can construct billboards for free, and that we are given an array $R[1 \dots k]$, where $R[i]$ is the revenue we would receive by constructing a billboard at location $X[i]$. Given that state law requires billboards to be at least 5 miles apart, give an efficient algorithm to compute the maximum revenue we can acquire by constructing billboards.