

CS 473: Fundamental Algorithms, Spring 2011

Discussion 4

February 8, 2011

1. RECURRENCES

Solve the following recurrences.

- $T(n) = 5T(n/4) + n$ and $T(n) = 1$ for $1 \leq n < 4$.
- $T(n) = 2T(n/2) + n \log n$
- $T(n) = 2T(n/2) + 3T(n/3) + n^2$

2. TREE TRAVERSAL.

Let T be a rooted binary tree on n nodes. The nodes have unique labels from 1 to n .

- Given the preorder and postorder node sequences for T , give a recursive algorithm to reconstruct a tree that satisfies the preorder and postorder sequences. Is this reconstruction unique?
- Given the preorder and inorder node sequences for T , give a recursive algorithm to reconstruct a tree that satisfies the preorder and inorder sequences. Is this reconstruction unique?

3. DIVIDE AND CONQUER.

Let $p = (x, y)$ and $p' = (x', y')$ be two points in the Euclidean plane given by their coordinates. We say that p dominates p' if and only if $x > x'$ and $y > y'$. Given a set of n points $P = \{p_1, \dots, p_n\}$, a point $p_i \in P$ is undominated in P if there is no

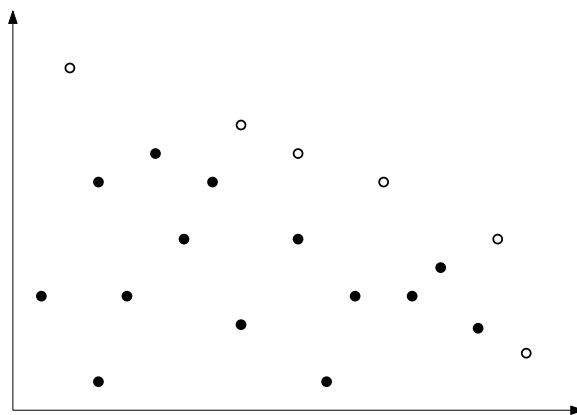


Figure 1: The undominated points are shown as unfilled circles.

other point $p_j \in P$ such that p_j dominates p_i . Describe an algorithm that given P outputs all the undominated points in P ; see figure. Your algorithm should run in time asymptotically faster than $O(n^2)$

4. CONVEX HULL.

You are given a set P of n points in the plane, and you would like to compute their convex-hull (i.e., that is the shortest perimeter polygon that contains all the points). To see how the convex-hull looks like, think about the plane as being a wood board, and place a nail at each point. Now, you shrink a rubber band around the points. The rubber shrinks into the convex-hull. Clearly, the vertices of the convex-hull are a subset of the input points. Show an $O(n \log n)$ time algorithm for computing the convex-hull. (Hint: Split the plane by a vertical line, compute the convex-hulls on both sides, and then figure out how to stitch the two convex-hulls together. To get a handle on this stitching problem, think about rotating a tangent around one of the convex-hulls.)