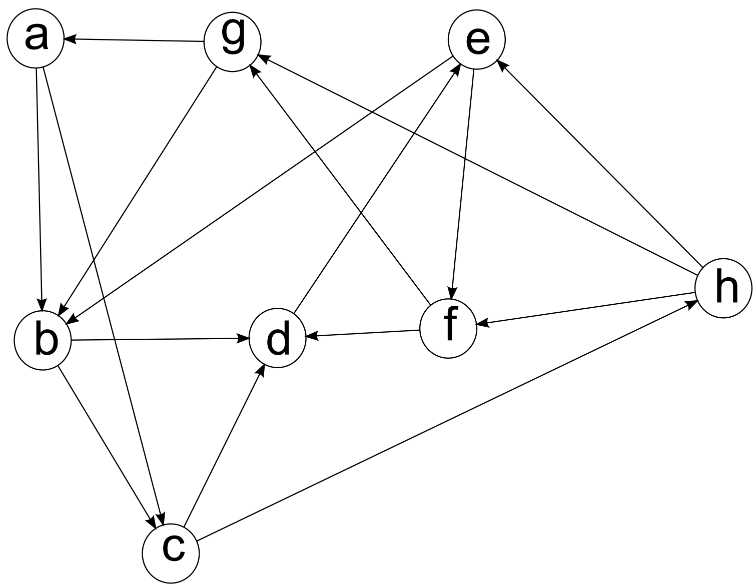
CS 473: Fundamental Algorithms, Spring 2011

Discussion 2

January 25, 2011

1. Consider the following graph.



   Draw the **DFS** tree rooted at D for the above graph. Use alphabetic ordering to break ties. Label the vertices of the tree with their pre(v) : post(v) time. Add in the remaining edges of the graph and label them as forward (F), backward (B), and cross (C) edges. Sort the vertices by their post visit order.

2. Let G be a directed graph and G^{SCC} its strong connected component meta-graph (which is a DAG). Prove or disprove the following. For any **DFS** of G the vertex with smallest post-visit number is in a sink component of G^{SCC}.

3. Let $G = (V, E)$ be an undirected graph with n vertices ($|V| = n$) and m edges ($|E| = m$). Give an $O(n)$ time algorithm to check if G has at least *two* distinct cycles and output them if it does. Assume that the graph is represented using adjacency lists. Note that m can be much larger than n so the algorithm should not check all edges. *Hint:* What is the structure of a minimal connected graph that has two cycles? Use DFS.

4. There are n light bulbs in a garden. These bulbs can be turned on manually by flipping on the switches at the light posts. Also, each light post can broadcast turn-on signals to some other pre-defined light posts in the garden, turning them on. When a light post is turned on, it will automatically broadcast a turn-on signal to its pre-defined light posts.

This signal broadcasting is directional. If a broadcasts to b, it is not necessarily true that b also broadcasts to a.

So one can manually flip on some of the switches to the light posts, and those light posts will broadcast a turn-on signal to other light posts. These will in turn be switched on and broadcast signals to their own pre-defined set of light posts, and so on.

Given each light post in the garden and the respective light posts to which they broadcast, derive a linear time algorithm for finding the minimum number of switches needed to be flipped to light up the whole garden. (Linear time means $O(n + m)$ where n is the number of light posts and m is the number of broadcast associations between them).

Source: *ACM ICPC 2010 World Finals Warmup 2*

Example Case: Number of lights : 5 , Number of broadcast associations: 4
Associations : 1=>2, 1=>3 3=>4, 5=>3
Answer: Minimum number of flips required : 2 ,
Turning on switches 1 and 5 should light up the whole garden

Hints:
(a) Model the problem using directed graphs.
(b) What is the solution if the graph in question is strongly connected?
(c) What is the solution in general?