HW8 out tonight → due next wed
Nov 11 deep deadline for grads

Applications of maxflows/mincuts
- Edge disjoint paths
- Vertex cut
- Bipartite matching

Transform input (algo)
Transform output (algo)
Time analysis in terms of original input
Proof of correctness (we don’t need this in HW, but you do)
Features in solutions ⇒ paths in max flow

Exam Scheduling
Input:
in classes
r rooms
t time slot
p proctor

E[1...n] enrollment
S[1...r] # seats
A[1...t,1...p] = availability
A(k,l) = True means proctor l is available at time k

• Every class needs to be scheduled
• E ≤ S for each exam
• ≤1 exam per room per time slot
• Each proctor can watch ≤5 exams, only when available

Output: room, time, proctor for every class

Set of tuples (i,j,k,l)
i - class one per exam
j - room
k - time
l - proctor
**Tuple Selection**

Finite sets \( X_1, X_2, \ldots, X_d \) of resources

- \( c(x) \) for each \( x \in X_i \) for all \( i \)
- \( c(x,y) \) for each \( x \in X_i \) and \( y \in X_{i+1} \) for all \( i \)

**Output:** largest set of tuples \( (x_1, \ldots, x_d) \in X_1 \times \cdots \times X_d \)

s.t. each \( x \in X_i \) appears \( \leq c(x) \) times

\[ x \in X_i, y \in X_{i+1} \leq c(x,y) \]

All constraints must be between adjacent pairs \( X_i, X_{i+1} \)

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**Algorithm:**

Construct \( G \)

\[ f^* = \max \text{flow}(G) \]

- if \( |f^*| < n \) return \( \text{FALSE} \)
- decompose \( f^* \) into paths
- write each path as tuple

\( O(N^3) \) time

\( N = \text{total input size} \)
Disjoint-Path Cover

Find smallest set of vertex-disjoint paths that cover every vertex of G.

Given n envelopes height hi width wi
put i inside j ⇔ hi < hj and wi < wj
Nest envelopes into as few sets as possible.

MATCH nodes in G to their successors
Select every path has a last vertex w/ no succ
Assign min #paths = max #successors

Build bipartite graph G'=(V', E')
V' = L U R
L = copy of U
R = copy of U

"E' = E"
E' = \( \{(u_L, v_R) \mid u \rightarrow v \in E\} \)

Find max matching in G'
\downarrow successors in G
\downarrow paths in G
#paths in G = #V - #M

Matching time:
O(|V'E'|) = O(|VE|)
= O(v^3) for nesting envelopes
Given a directed graph $G$ not a dag

**Cycle cover**

Cover the edges of $G$ with cycles edge-disjoint

For each edge, select its success or on the same cycle

$V' = E \cup F$

$v \rightarrow w, E' = \{(u,v) \rightarrow (v,w)\}$

$uv$ and $vw \in E$

Perfect Max matching

$O(V'E') = O(E^2\sqrt{V})$

$E' \leq E^2$

$E' \leq E \cdot V$