Monday October 31 — Midterm 2

HW 4567 + today!

Thursday — review session

Conflict — register by Fri

No problem 3

Edge-disjoint paths

Given directed $G=(V,E)$ vertices s.t.

Find max # paths from s to t not sharing edges

Algorithm

Assign capacity 1 to every edge

Compute max $(s,t)$-flow $f^*$ ← Ford-Fulkerson $O(VE)$

Decompose $f^*$ into paths

$O(VE)$ time $\leftarrow$ Peng et al. 2022 $O(E^{1.52})$

Undirected?

Still $O(VE)$ time

Vertex-disjoint?

Vertex capacities (in addition to edge caps)

Feasible: $D \leq F(e) \leq c(e)$

$+ \sum_{u \rightarrow v} F(u \rightarrow v) \leq c(v)$

1. Change algorithm
2. Change graph
More generally: Find max # paths
\[ \leq A \text{ through each edge} \]
\[ \leq B \text{ through each vertex} \]
\[ O(VE) \text{ time} \]
Given a bipartite graph

Find a maximum matching

= max # edges, no 2 sharing a vertex

Add source $s$ edges $s \rightarrow L$
Add target $t$ edges $R \rightarrow t$
Direct $L \rightarrow R$
$\forall e \in E$
$c(e) = 1$

Max Flow $\rightarrow$ Flow Decomposition

$\text{Integer Max Flow} \rightarrow \text{Matching} = \bigoplus e | F(e) = 1$

Alternating path

Augmenting path

$O(VE)$ time
Max Matching (G)

\[ M \leftarrow \emptyset \]
while there is an alternating path in G
\[ P \leftarrow \text{any alt. path} \]
\[ M \leftarrow M \cup P \]
return M

\[ O(E+V) \]
\[ O(V) \]
\[ O(V^2 + EV) \]

Berge (1957)
Jacobi (1836)

Permute rows and columns
all positive along diagonal