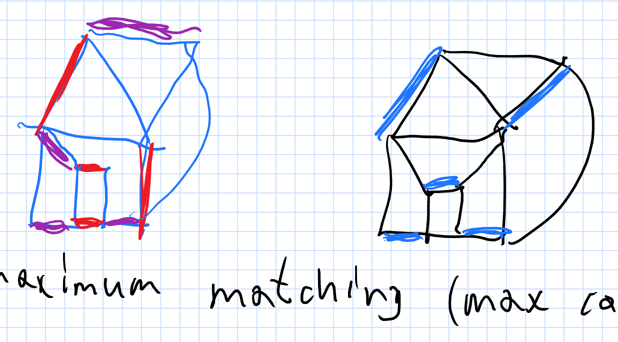
L17: Approx alg

Matching

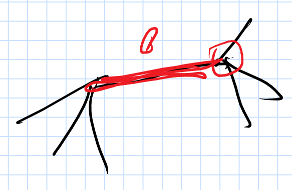
$G=(V,E)$ $n=|V|$ $m=|E|$

M is a matching

$M\subseteq E$ s.t. no pair of edges share an endpoint.



Compute maximum matching (max cardinality matching).



Lemma
The greedy algorithm that picks edges and delete their vertices output a matching M s.t.

$$|M| \geq |\text{Maximum Matching}|/2$$

Proof:

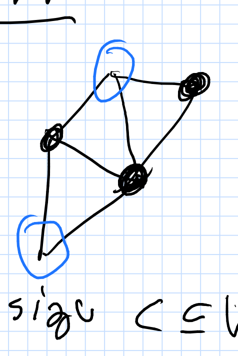
$\frac{1}{2}$-approximation

$$\forall G \quad \frac{Alg(G)}{Opt(G)} \geq \frac{1}{2}$$

$\min_G \frac{Alg(G)}{Opt(G)}$ — approx quality of this algorithm.

approx ration for maximization problem

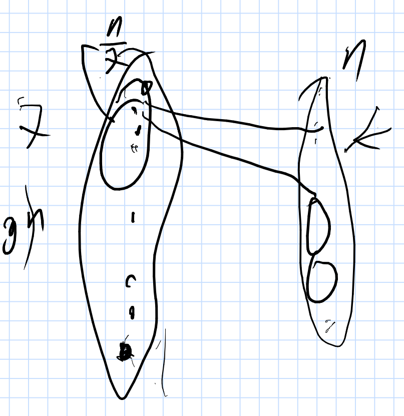
Vertex Cover



Find min size $C\subseteq V$ s.t. all $e\in E(G)$
$e\cap C\neq\emptyset$

Theorem
The greedy algorithm pick max degree vertex to the vertex cover outputs a cover that in the worst case have size $\Omega(opt \cdot \log n)$

Proof



$O(n\log n)$

for $i=1,\dots,n-1$ $\lfloor \frac{n}{i} \rfloor$ vertices on the left connected to i vertices on the right

$$\sum \frac{n}{i} = \Omega(n\log n)$$

2 for the price of one

compute greedy maximal matching M, $k=|M|$
- VC must have size at least k
- $V(M)$ provide a VC of size $2k$

$$k \leq opt \leq \underline{2k}$$

2approx to VC.

G: graph
α = size of optimal vertex cover
α is small compared to n.

$$O\left(\binom{n}{\alpha} n^2\right) \qquad \binom{n}{\alpha}\leq n^\alpha$$

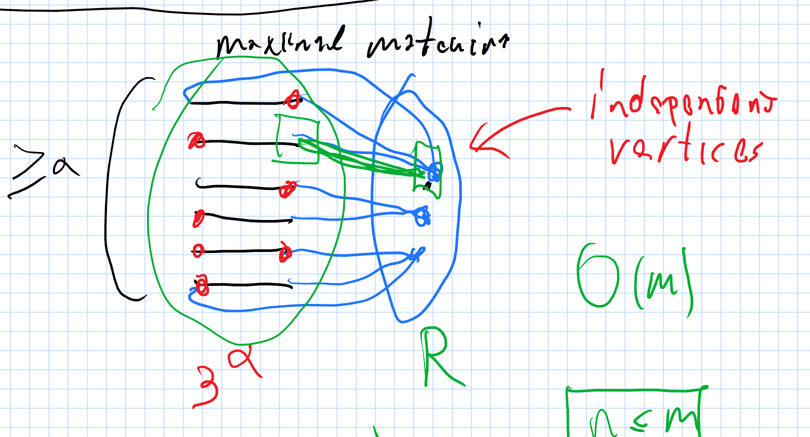
$$O\left(\sum_{i=1}^{\alpha} \binom{n}{i} n^2\right) = O\left(n^{\alpha+3}\right)$$

Q: Can we get alg for VC with running time

$$c^\alpha n^{O(1)}$$

c some constant

FPT = Fixed parameter tractable algorithms.



$O(m)$

$\boxed{n\leq m}$

$$O(3^\alpha m) \ll O(n^\alpha)$$

TSP

K_n: complete graph
$uv\in K_n$ $w(uv)$: time to travel b/w uv.

Q: Compute cheapest tour that visits all vertices of the graph exactly once

Lemma TSP can not be approximated.

proof
H: instance of Hamiltonian cycle.

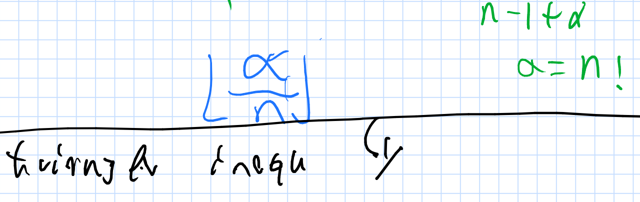
$H=([n],E)$ $[n]=\{1,2,\dots n\}$

$$\forall e\in E(K_n) \quad w(e) = \begin{cases} 1 & e\in E(H) \\ \alpha & e\notin E(H)\end{cases}$$

$\alpha = \infty$

$\exists$ HC in H
$\Rightarrow$ TSP of cost n

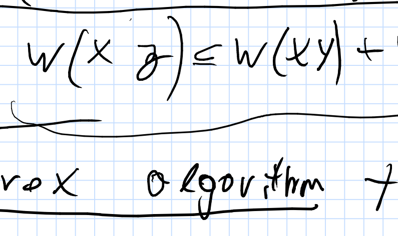
no HC in H
$\Rightarrow$ TSP cost is at least $(n-1)1+\alpha$



$\alpha = n!$

$\left\lfloor \frac{\alpha}{n} \right\rfloor$

TSP with triangle inequality

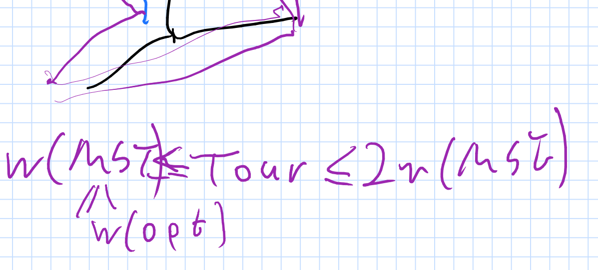


$$\boxed{w(xz)\leq w(xy)+w(yz)}$$

2-approx algorithm for $TSP_{\Delta\leq}$

MST

$w(MST) \leq w(opt\ TSP)$



$$w(MST) \leq Tour \leq 2w(MST)$$
$\quad \parallel$
$w(opt)$

2-aprox.