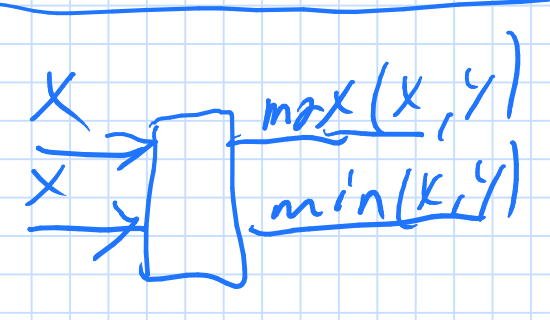
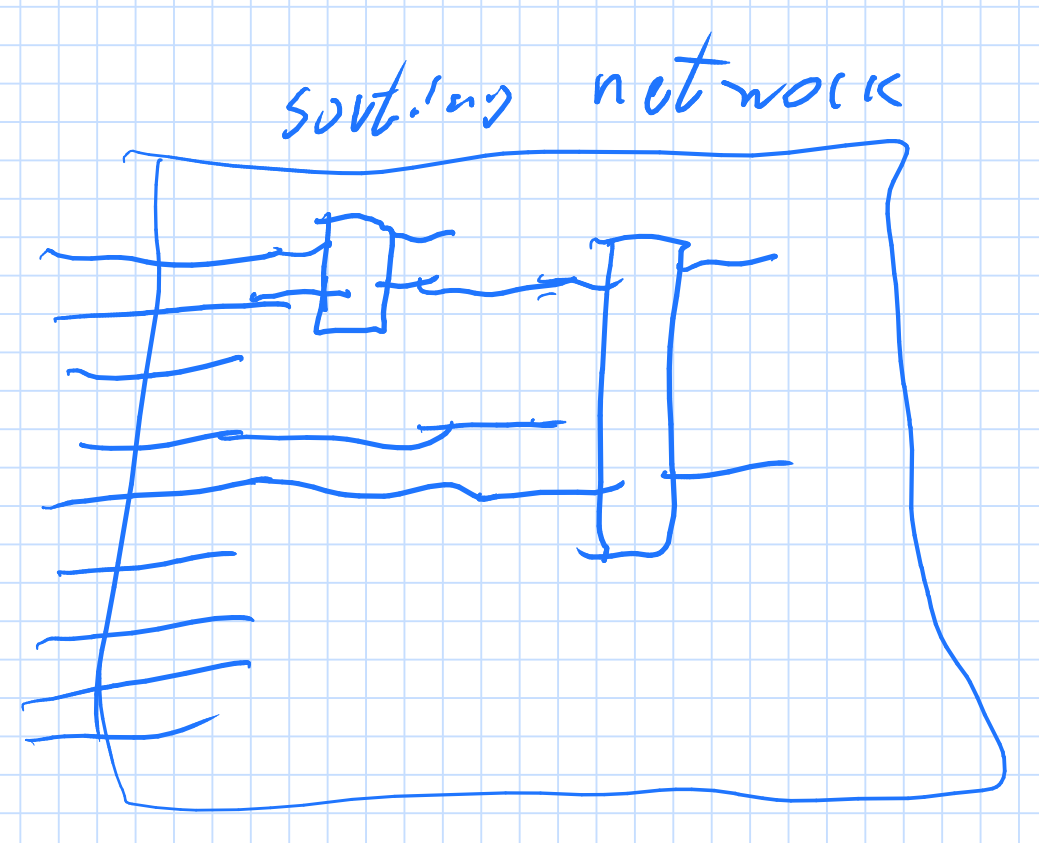
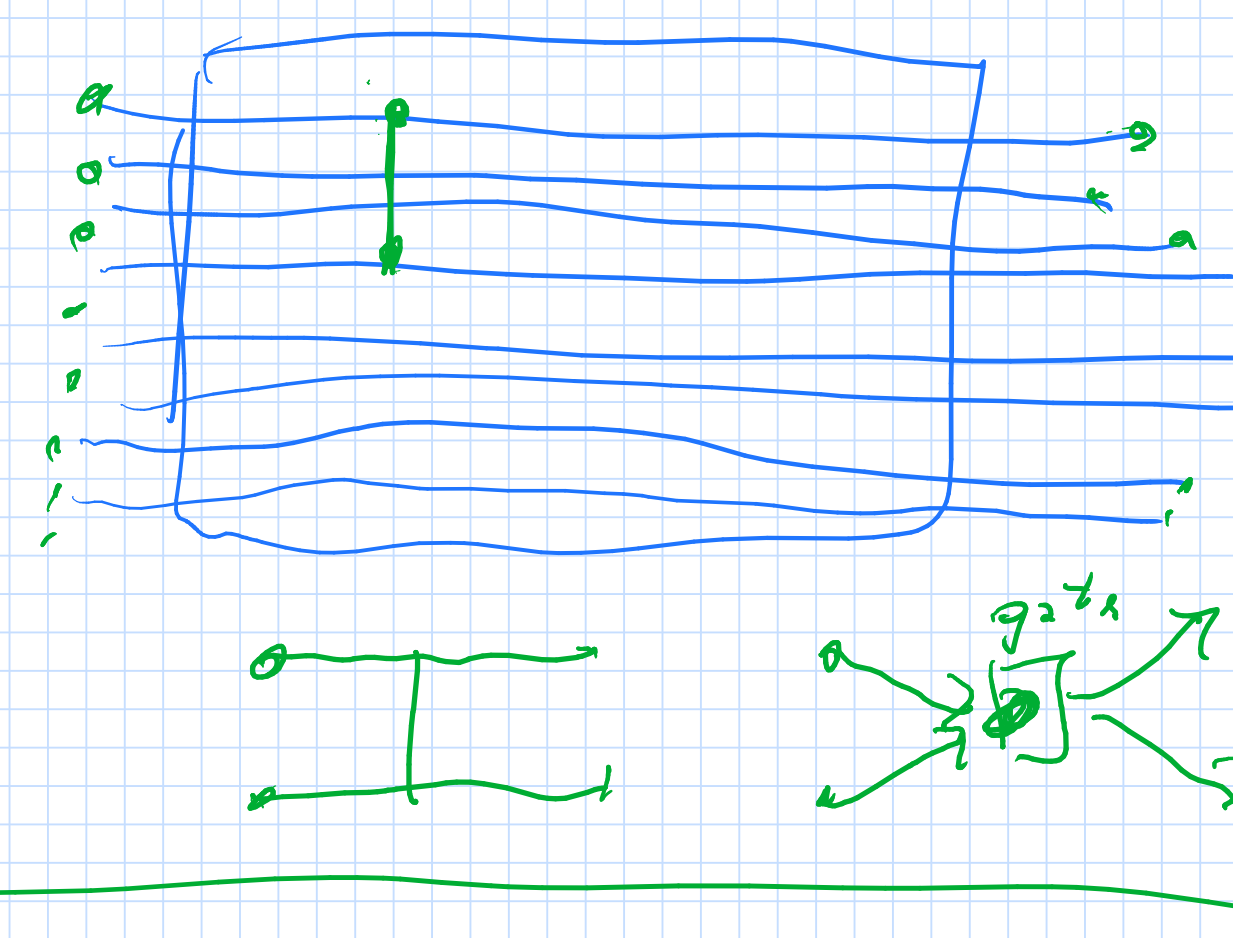
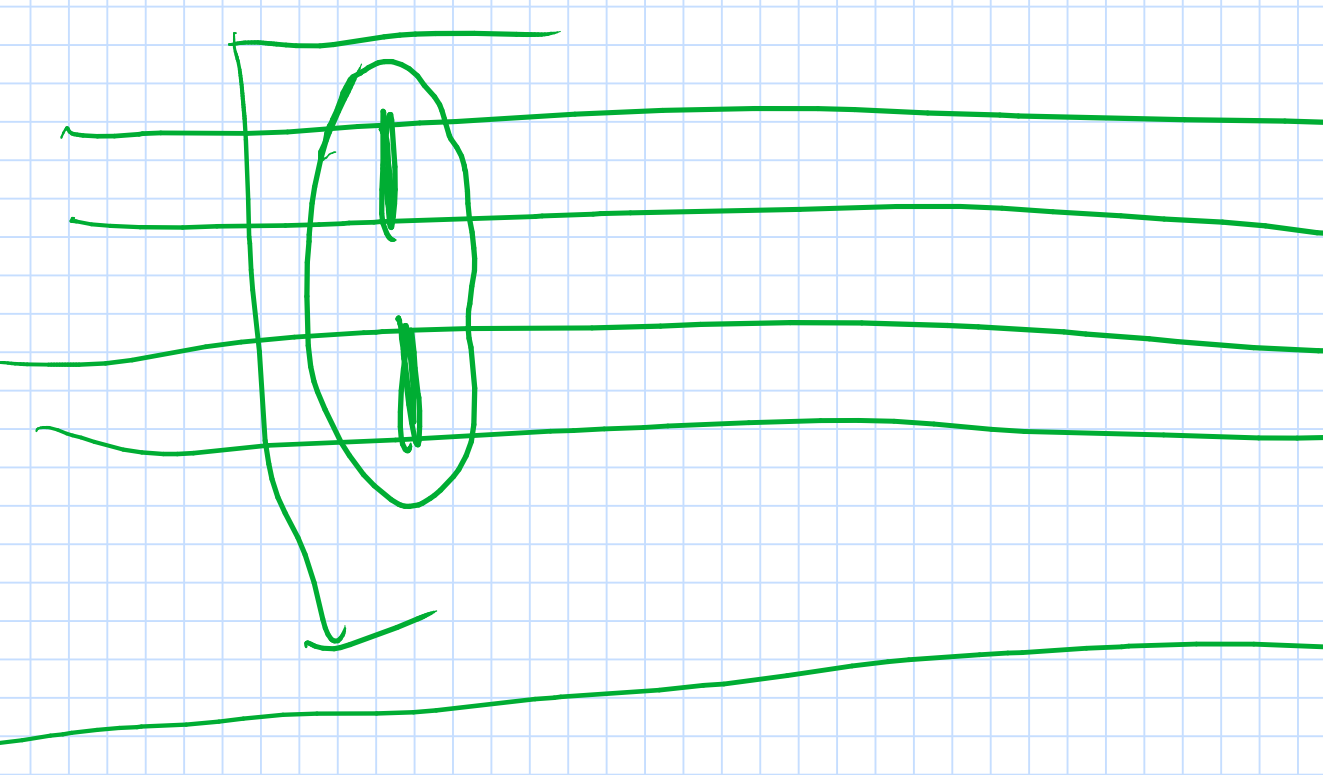
473. L10: Sorting networks

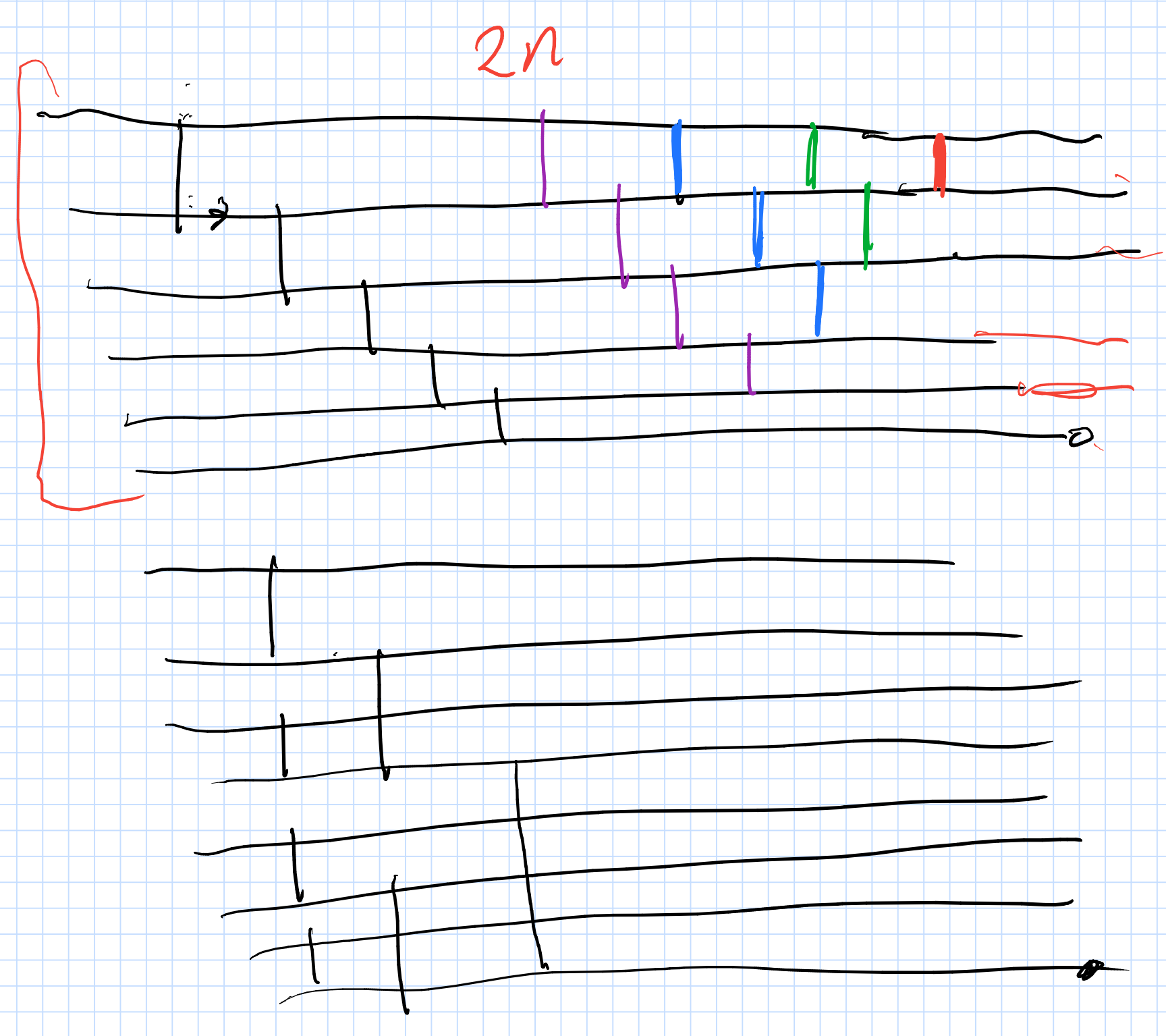


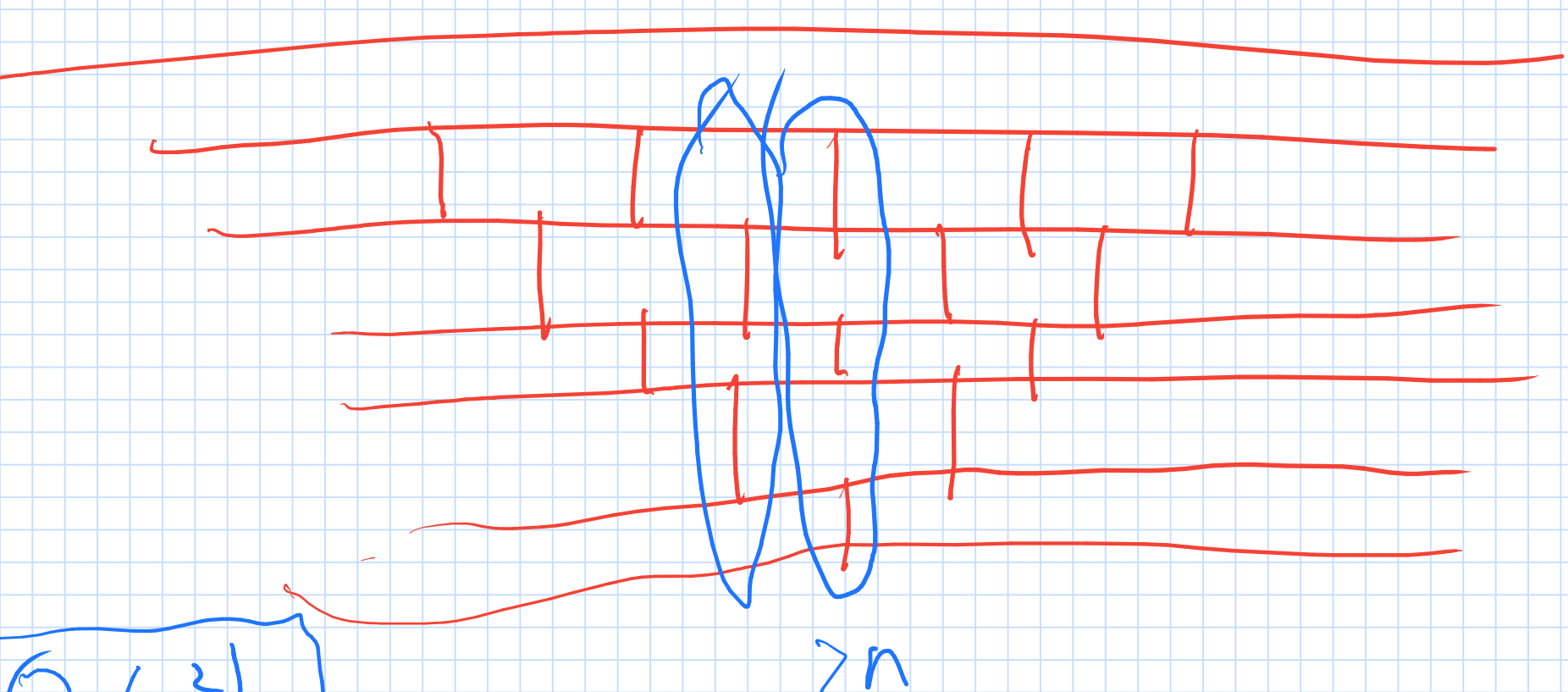


sorting network
⇒ DAG









$O(n^2)$

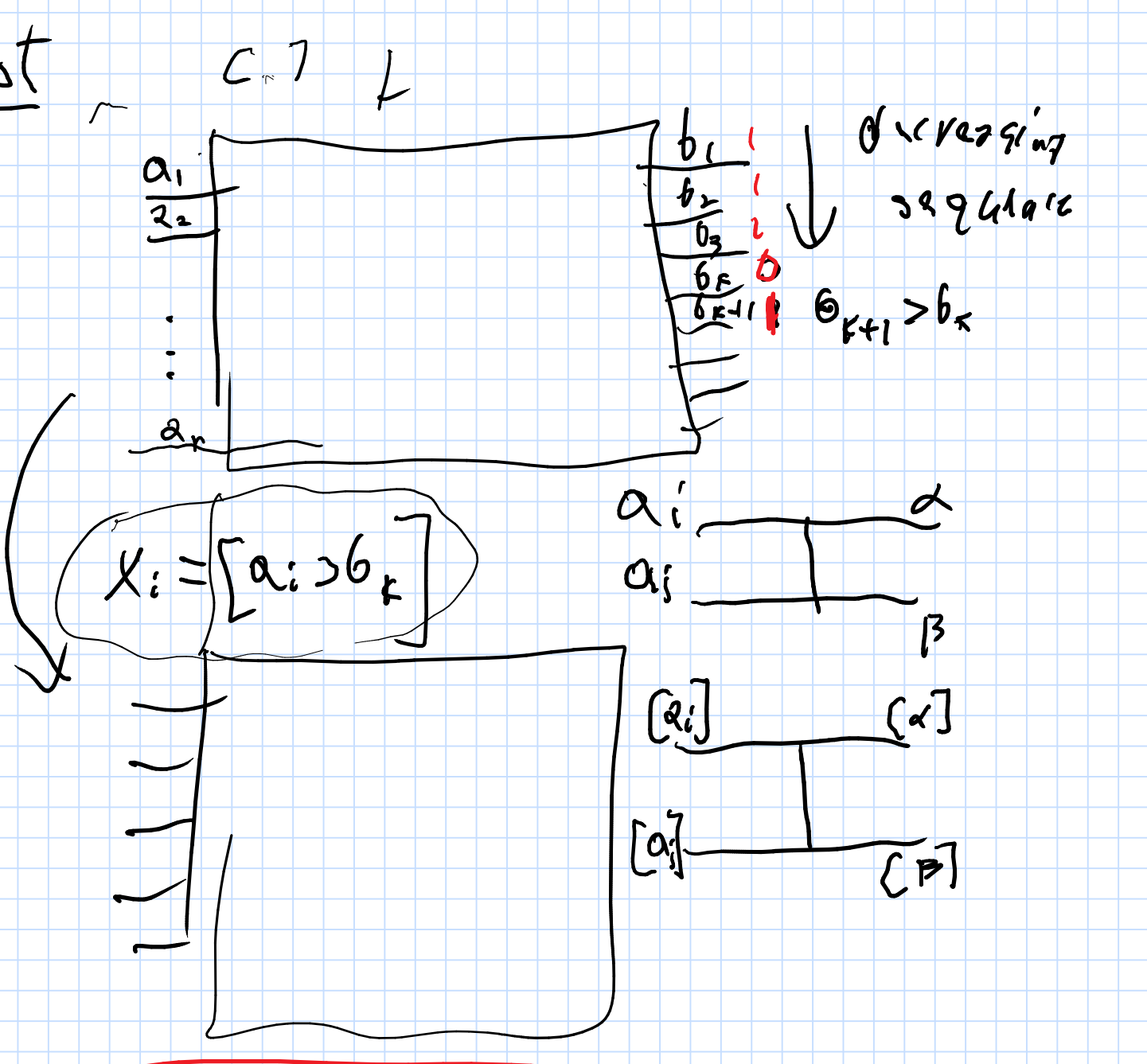
AKS sorting networks
$O(n \log n)$ gates
depth $O(\log n)$
$n > 2^{8000}$ ←

Bitonic sorting networks
$O(n \log^2 n)$ # gates
$O(\log^2 n)$ depth

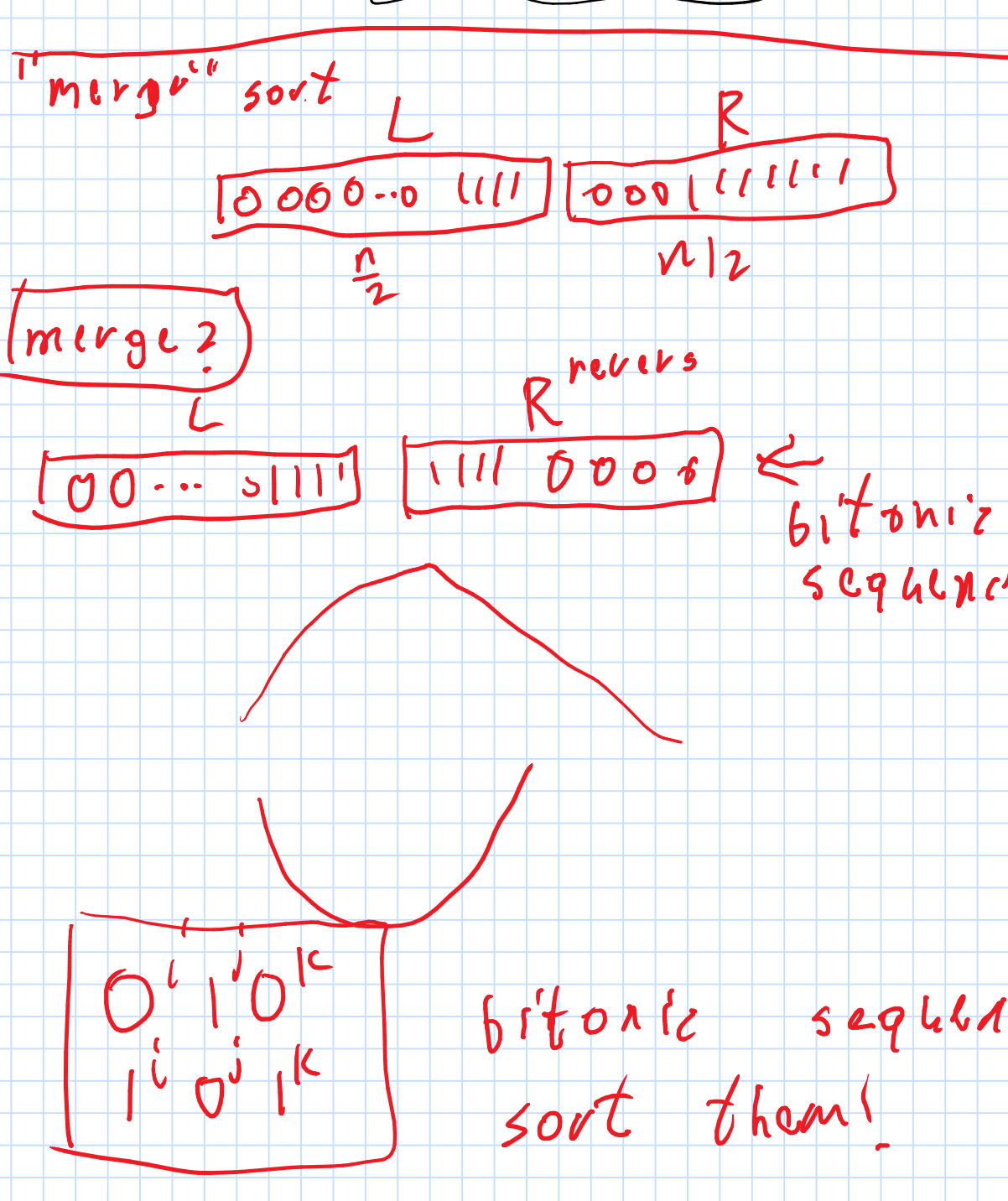
zero-one principle

Lemma
A sorting network sort correctly all inputs ⟺ it sort correctly all binary sequences.

Proof

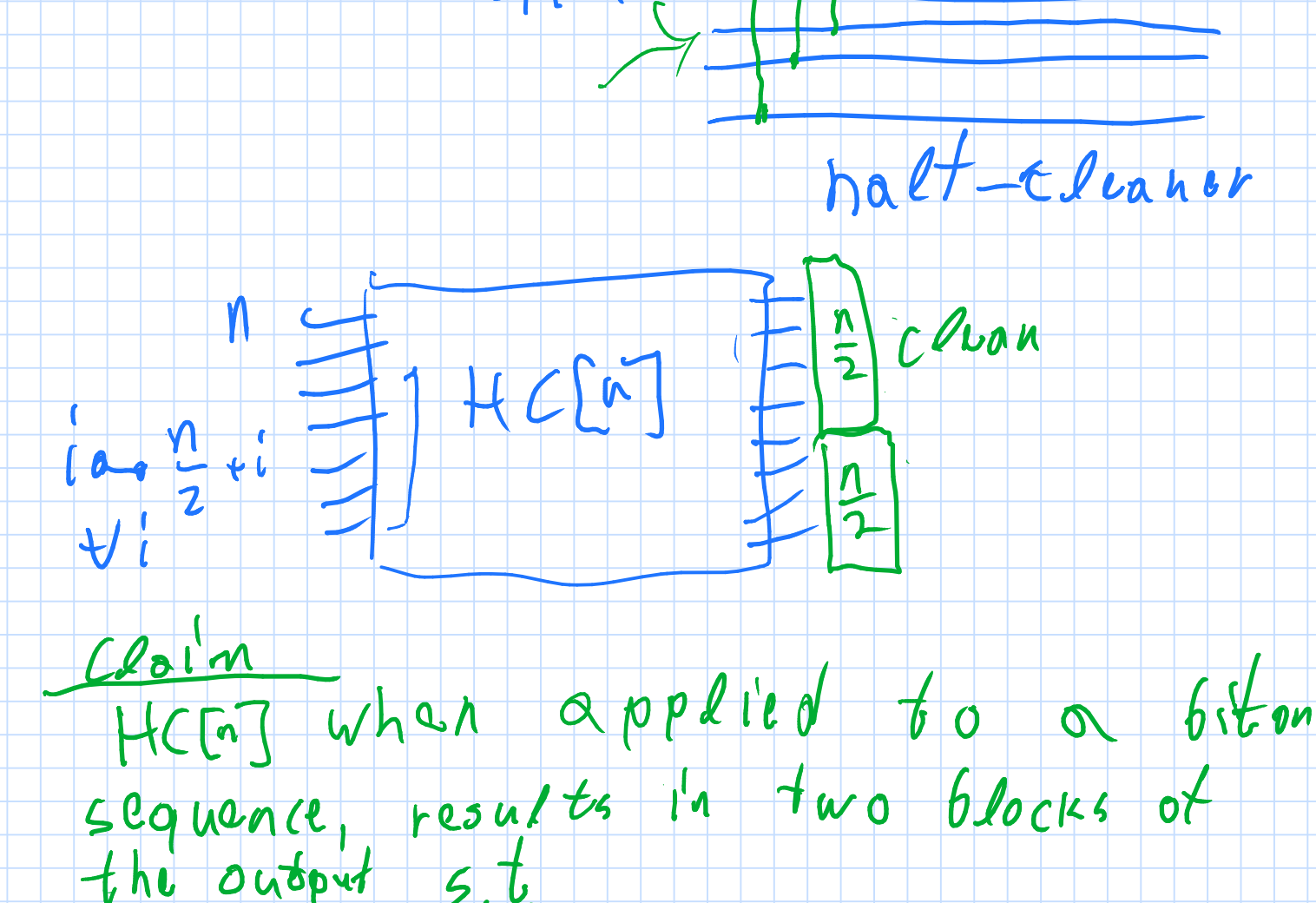


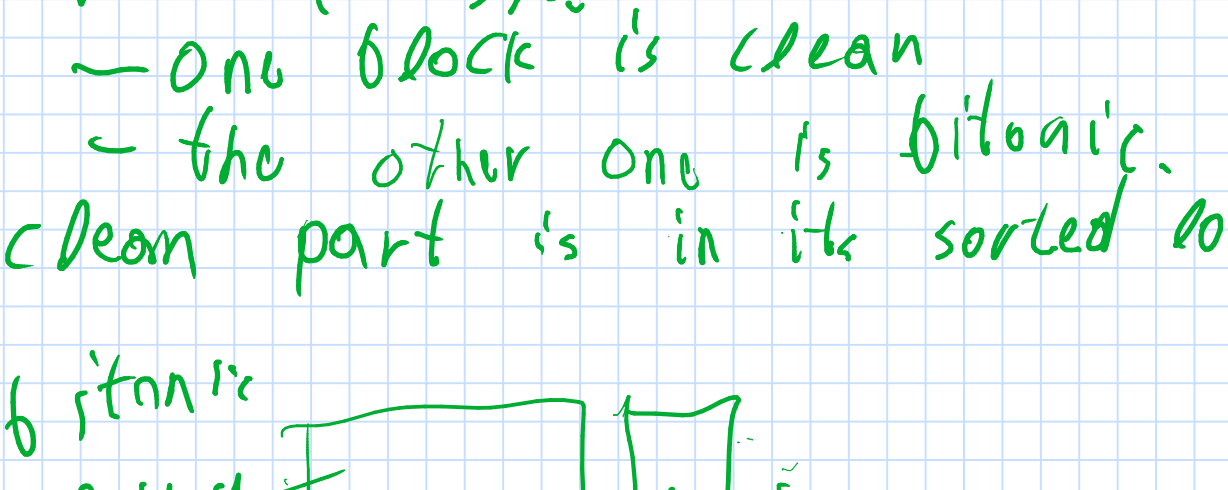
"merge" sort



$0^i 1^j 0^k$
$1^i 0^j 1^k$

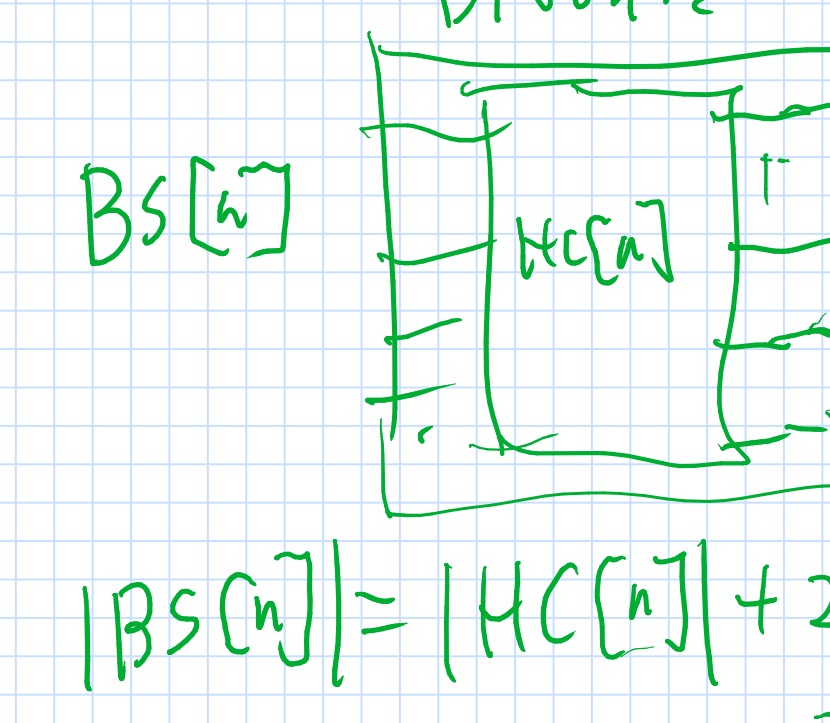
bitonic sequences
sort them!



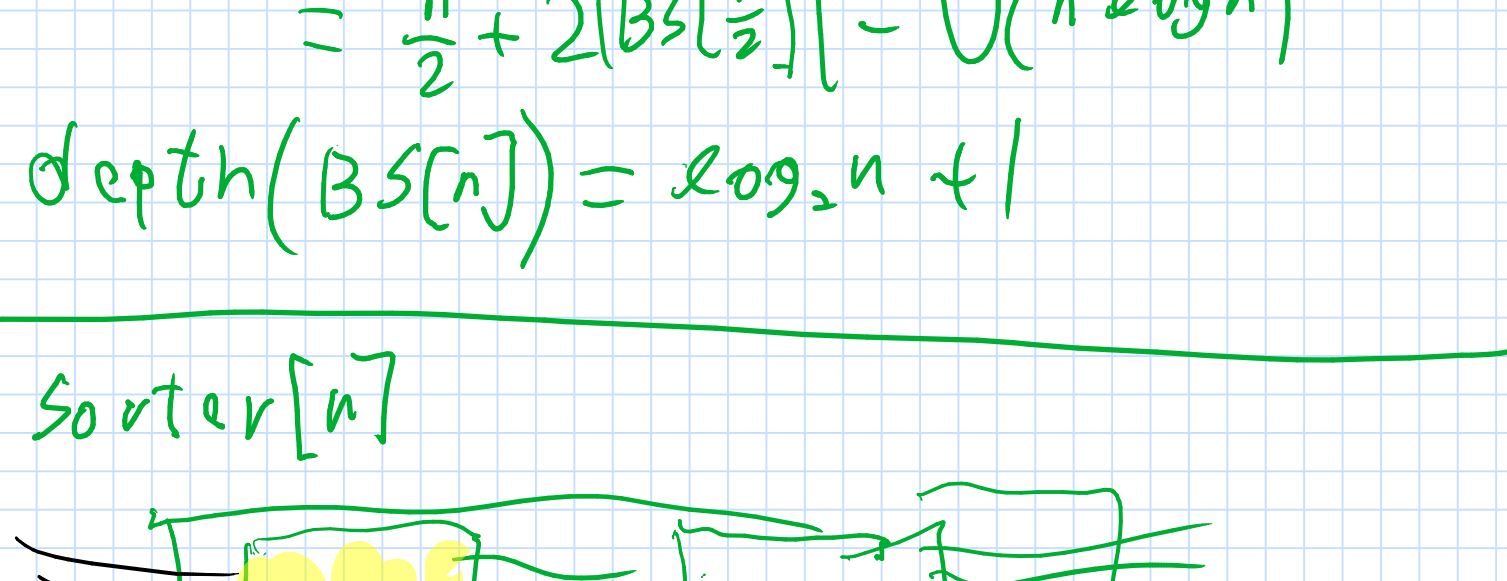


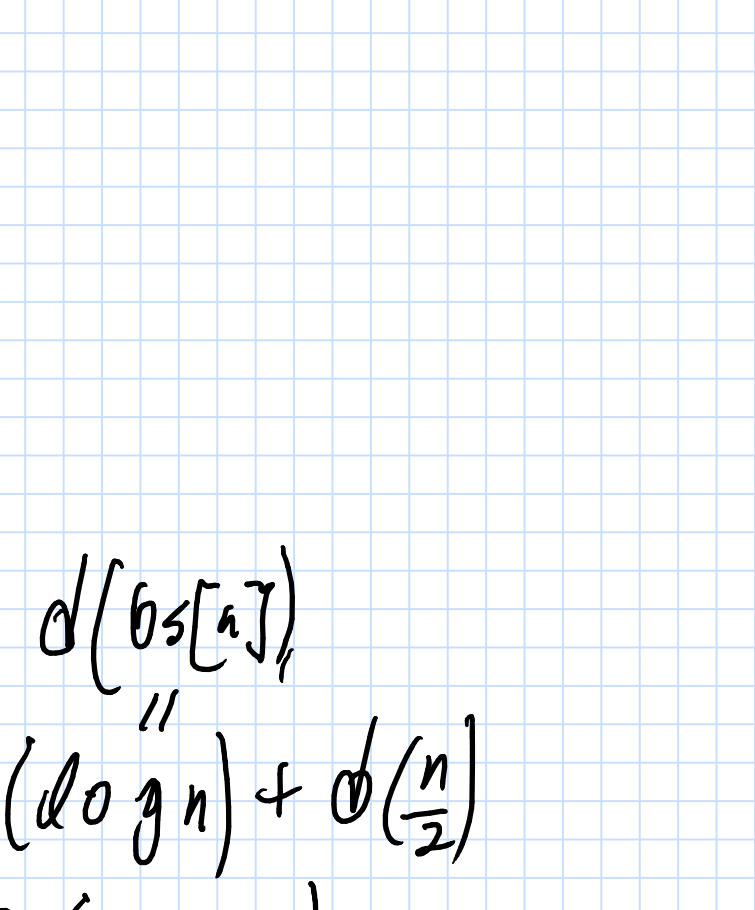
Claim
HC[n] when applied to a bitonic sequence, results in two blocks of the output s.t.
- one block is clean
- the other one is bitonic.

clean part is in its sorted location.



Bitonic sorter

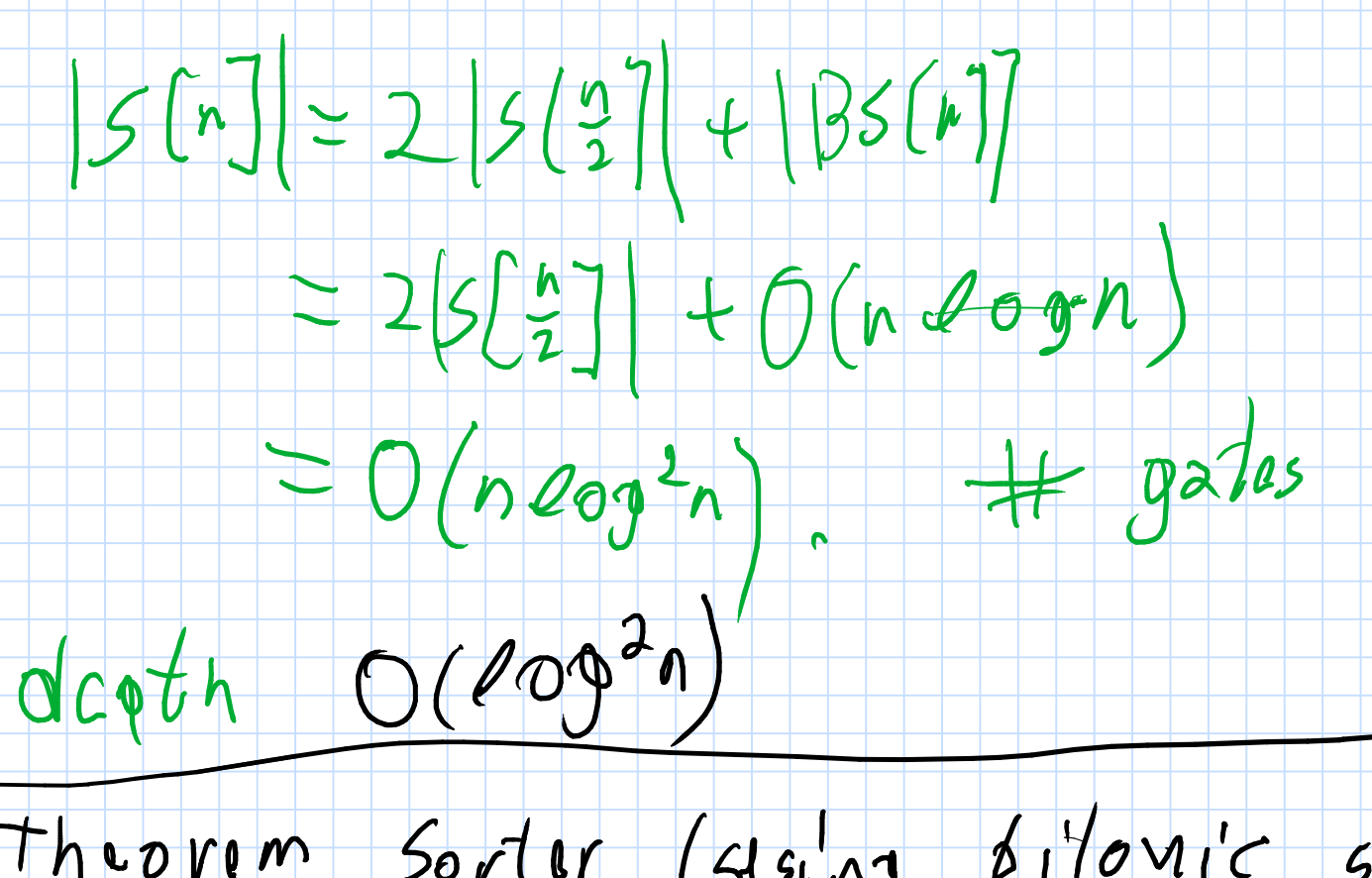




$$|BS[n]| = |HC[n]| + 2|BS[\tfrac{n}{2}]| = \tfrac{n}{2} + 2|BS[\tfrac{n}{2}]| = O(n \log n)$$

$$depth(BS[n]) = \log_2 n + 1$$

Sorter[n]



$$d(n) = \underbrace{O(\log n)}_{d(BS[n])} + d(\tfrac{n}{2}) = O(\log^2 n)$$

$$|S[n]| = 2|S[\tfrac{n}{2}]| + |BS[n]| = 2|S[\tfrac{n}{2}]| + O(n \log n) = O(n \log^2 n) \quad \text{\# gates}$$

depth $O(\log^2 n)$

Theorem Sorter (using bitonic sorter) has depth $O(\log^2 n)$ and $O(n \log^2 n)$ gates.