473 Dynamic programing   9/7/21

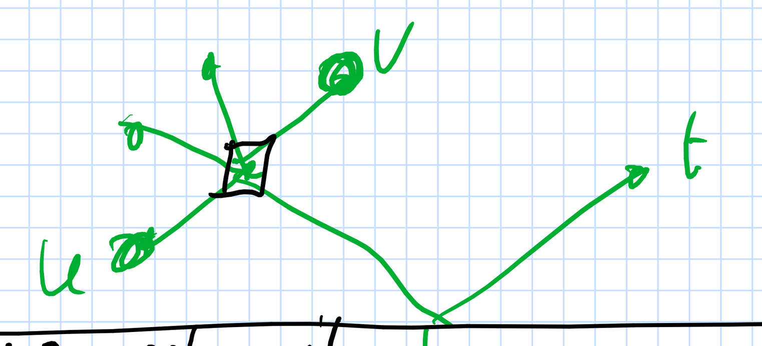
$G=(V,E)$

$X \subseteq V$: set of k vertices

s,t: two vertices

weights on the edges.

Problem: Compute shortest ~~path~~ walk from s to t that visits all the vertices of X.



- guess ordering of sites in X being visited. $(k!) \approx 2^{\Theta(k\log k)}$

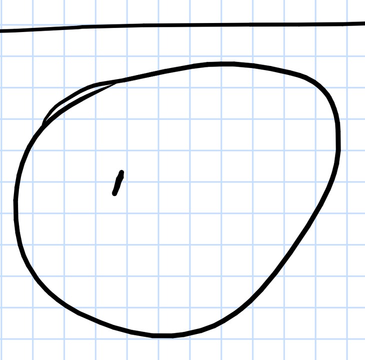
- $x_1, x_2, \dots, x_k$ ordering

compute shortest path from:

$s \to x_1$ shortest path

$x_1 \to x_2, \dots, x_i \xrightarrow{sh} x_{i+1} \dots \to x_k \to t$

$k!$



G

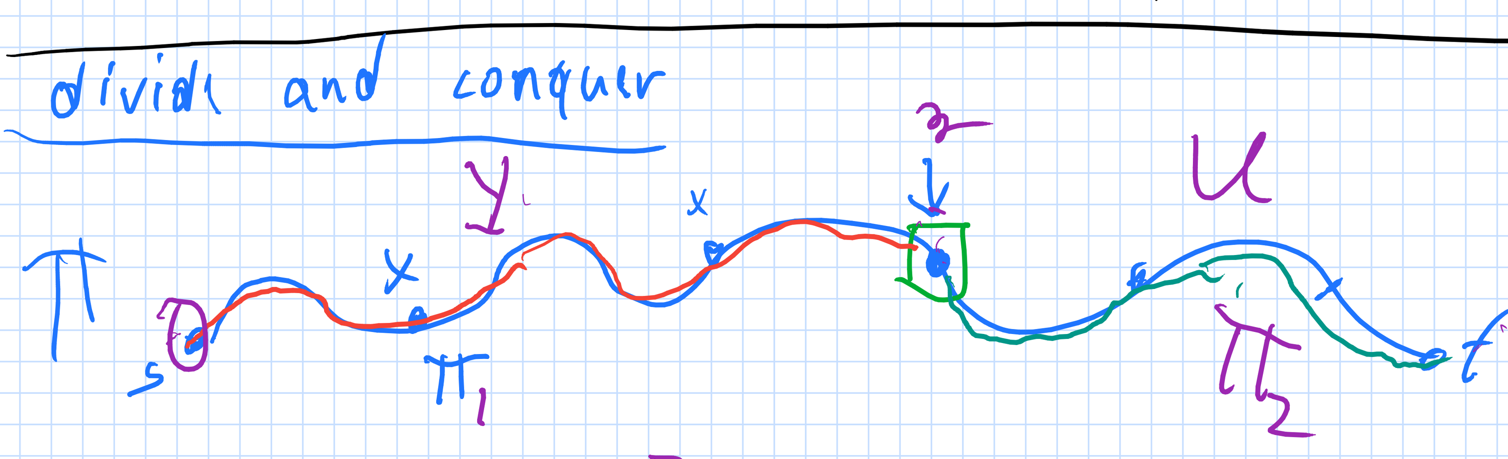
$O((n\log n + m)n)$

$= O(n^2\log n + nm)$ time

$+ O(k! \cdot k)$

$= O(n^2\log n + nm + \underbrace{k!\cdot k}_{bad})$

divide and conquer



$Y \cup \{z\} \cup U = X$

$|Y| < k \quad |U| < k$

$Y \cup U$

$g(Y,s,t)$: prices of shortest walk from s to t s.t. all the sites in $Y \subseteq X$ are being visited.

$\subseteq X \quad |X| = k$

$g(\boxed{2^k}, \boxed{k+2}, \boxed{k+2}) \qquad 2^k$

$$g(U,s,t) = \begin{cases} d_G(s,t) & |U| = \emptyset \\ \min\limits_{u\in U} \min\limits_{\substack{Z\subseteq U \\ u\notin Z}} \left(g(Z\setminus\{u\}, s, u) + g(\underbrace{\ \ }_{U\setminus(Z\cup\{u\})}, u, t) \right) & O(k\cdot 2^k) \end{cases}$$

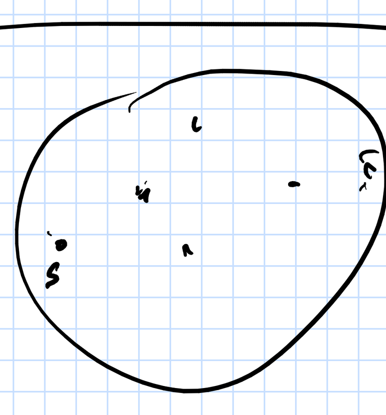
Memoization

$g(X,s,t)$

# of distinct calls $2^k \cdot (k+2)^2 = O(2^k k^2)$

Time for each recursive call that computes a value that was not computed before $O(k\cdot 2^k)$

Overall running time is $O(2^k \cdot k^2 \cdot k \cdot 2^k) = O(2^{2k} k^3)$



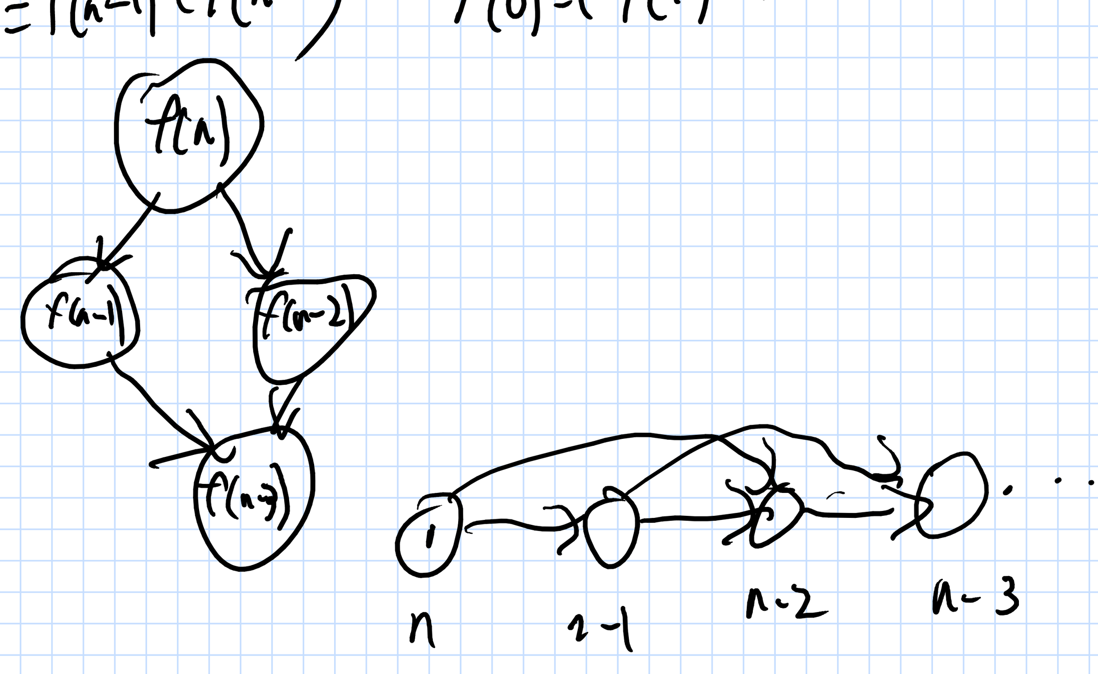
$G \quad O((n\log n + m)(k+1))$

$O(kn\log n + km + 2^k k^3) \qquad O(m + k!\cdot k)$

$2^k k^3 \lll \underbrace{k!\,k}_{2^{k\log k}}$

configuration space

$f(n) = f(n-1) + f(n-2) \qquad f(0)=1 \; f(1)=1$



edit distance

PEARE 2

PEACE

0 0 0 1 1

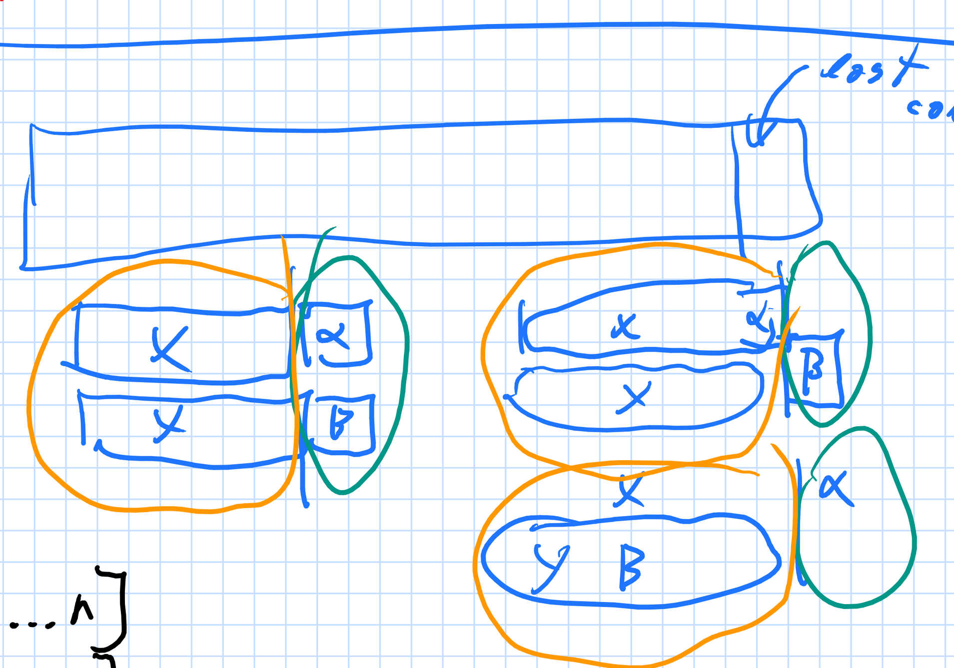
P|E|A|R|E $= 2$

P|E|A|C|E

ins

P|E|A|R|P|E|A|C|E $= 9$

del



$S[1\dots n]$

$T[1\dots m]$

$ed(S[1\dots i], T[1\dots j])$;

if $i=0$ then j

if $j=0$ then i

$\min(ed(S[1\dots i-1], T[1\dots j-1]) + cost(S[i], T[j]),$

$ed(S[1\dots i-1], T[1\dots j]) + 1,$

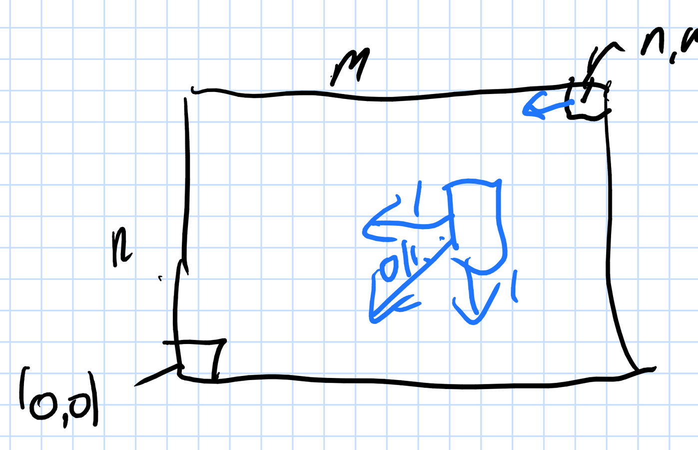
$ed(S[1\dots i], T[1\dots j-1]) + 1))$ Memoization

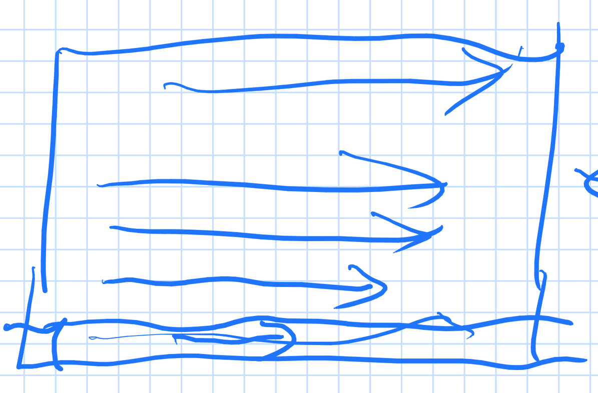
$e(i,j)$ = edit distance of $S[1\dots i]$ to $T[1\dots j]$

$(i,j) \to (i-1,j) \quad i=0,\dots n$

$(i,j) \searrow (i-1,j-1) \quad j=0,\dots m$

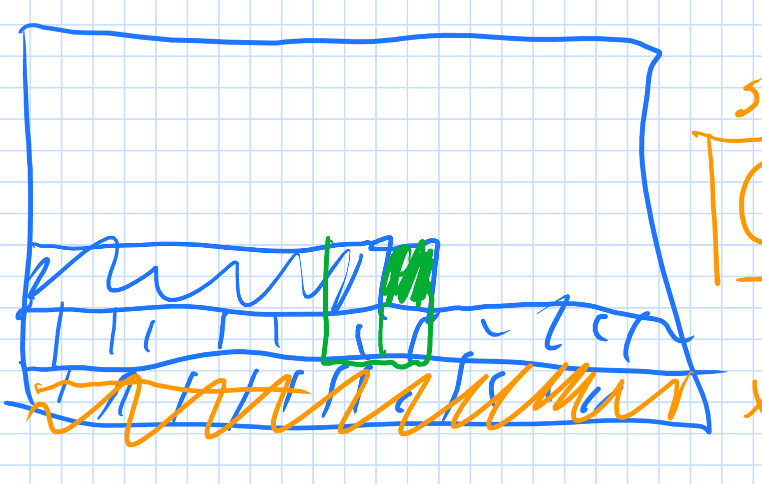
$(i,j) \downarrow (i,j-1)$





$\leftarrow O(nm)$

Running time



space

$O(n+m)$