cs473: Algorithms	Assigned: *Thu., Nov. 21, 2019*
Problem Set #10	
Prof. Michael A. Forbes Prof. Chandra Chekuri	Due: *Thu., Dec. 5, 2019 (10:00am)*

For problems that ask to prove that a given problem X is NP-hard, a full-credit solution requires the following components:

- Specify a known NP-hard problem Y, taken from the problems listed in Erickson's notes.
- Describe a polynomial-time algorithm for Y, using a black-box polynomial-time algorithm for X as a subroutine. Most NP-hardness reductions have the following form: given an arbitrary instance of Y, describe how to transform it into an instance of X, pass this instance to a black-box algorithm for X, and finally, describe how to transform the output of the black-box subroutine to the final output solving the original instance of Y. A diagram can be helpful.
- Prove that your reduction is correct. As usual, correctness proofs for NP-hardness reductions usually have two components, representing that the answer is true/false, or representing that the answer is too-large/too-small.

All (non-optional) problems are of equal value.

1. The directed Hamiltonian-path problem seeks to decide whether a given directed graph $G = (V, E)$ has a path that visits each vertex exactly once. Suppose you have a black-box algorithm for solving the directed Hamiltonian-path problem (note that this algorithm only answers 'yes' or 'no'). Using this black-box algorithm, describe a polynomial-time algorithm that, given a directed graph $G = (V, E)$, outputs a Hamiltonian-cycle in G if it has one, or returns 'no' otherwise.

   Note: You are allowed to use the algorithm solving the directed Hamiltonian-path problem more than once.

2. An instance of SUBSETSUM consists of n non-negative integers $a_1, a_2, \ldots, a_n$ and a non-negative target integer B. The goal is to decide if there is a subset of the n numbers whose sum is exactly B. The 2PARTITION problem is the following: given n (not necessarily non-negative) integers $a_1, a_2, \ldots, a_n$, is there a subset S such that the sum of the numbers in S is equal to $\frac{1}{2}\sum_{i=1}^{n} a_i$. Describe an efficient reduction from SUBSETSUM to 2PARTITION.

   Note: One can also show that 2PARTITION reduces to SUBSETSUM, but this requires slight additional work as the SUBSETSUM problem does not allow negative a_i.

3. Given an undirected graph $G = (V, E)$ a subset $S \subseteq V$ is a *dominating set* for G if for all $v \in V$, we have that $v \in S$ or there is a neighbor of v in S. The DOMINATINGSET problem is the following: given G and an integer k, does G have a dominating set of size $\leq k$?

   (a) (**optional**, *not* for submission) Reduce DOMINATINGSET to SETCOVER.

   (b) Reduce SETCOVER to DOMINATINGSET.