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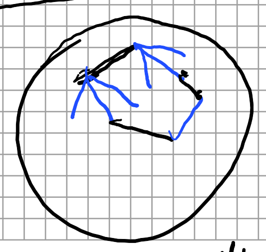
— Matching

$G = (V, E)$ n vertices, m edges, undirected



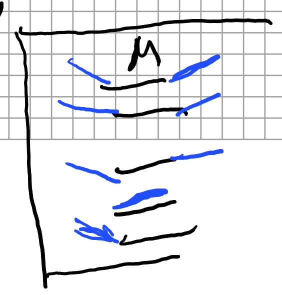
$O(n+m)$

M: Matching

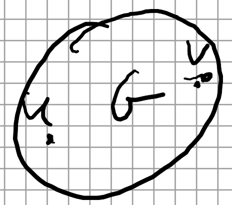
$|M| \geq \frac{|opt|}{2}$

maximal

maximum matching

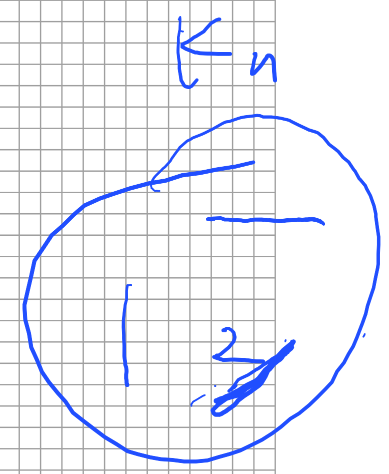


Diameter in a graph



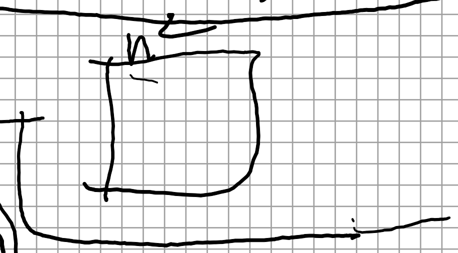
$d(u,v)$

$\mathrm{diam}(G) = \max_{u,v \in V(G)} d(u,v)$



(1, 2 - graph)

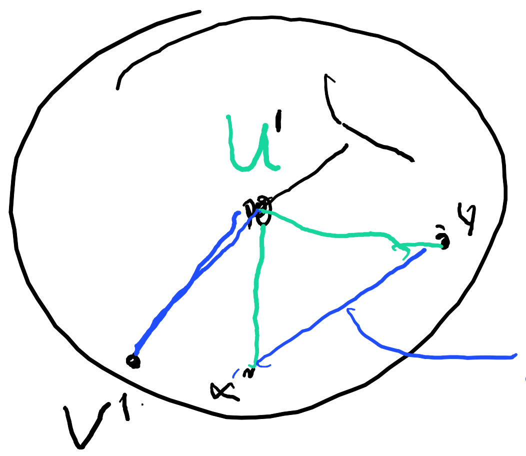
APSP $O(n^3)$



$O(n^2 \log n + nm)$

$u', v' \in V(G)$

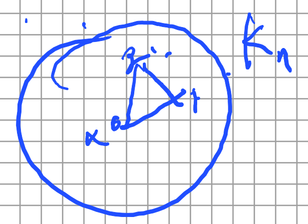
$\frac{\mathrm{diam}(G)}{2} \le d(u', v') \le \mathrm{diam}(G)$



$\mathrm{diam}(G) \le 2d(u', v')$

$O(n \log n + m)$.

TSP with the triangle ineq

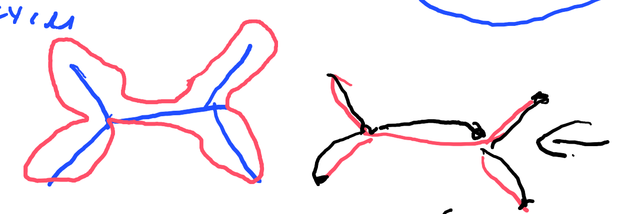


$w(x,z) \leq w(x,y) + w(y,z)$

- Eulerian cycle/Graph

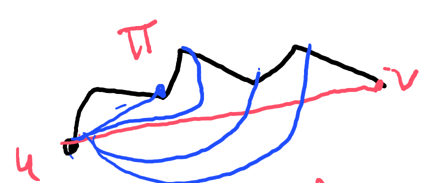
- G is connected
⇒ All degrees are even.

G: $\underbrace{\text{TSP}}_{\text{cycle}} \geq w(\text{TSP}) \geq w(\text{MST})$



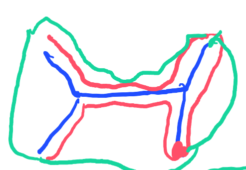
$2w(\text{MST})$

Computed a cycle of $w \leq 2w(\text{MST})$.



$d(\pi) \geq w(u,v)$

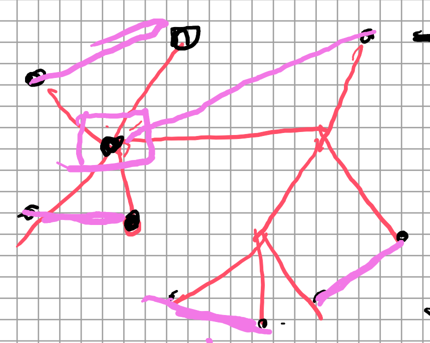
C



2-aprox

$\boxed{\frac{3}{2}\text{-approx}}$

3/2-approx for TSP

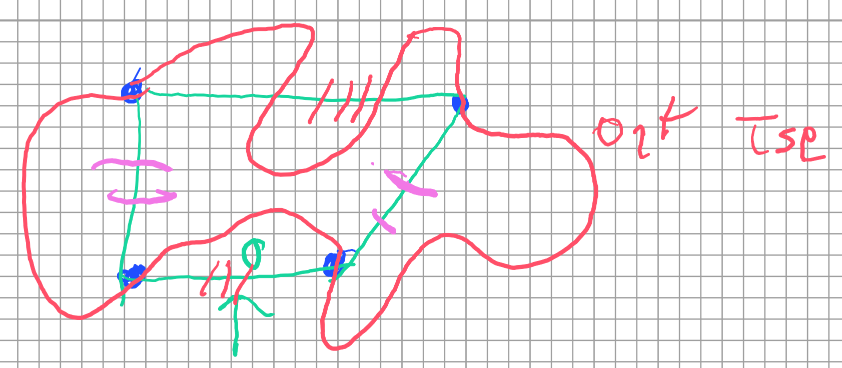


- # of odd degree vertices in a graph even.

$$\sum d(v) = 2|G|$$

- Conenta
  min cost matching for all odd degree vertices.

Min cost matching is polynomial time solvable



$w(\sigma) \le w(TSP)$

min weight matching of odd vertices $\le \frac{w(\sigma)}{2} \le \frac{w(TSP)}{2}$

$W(sol) \le w(MST) + w(M)$

$\le \frac{3}{2} w(TSP)$. $\blacksquare$

Christophides

Max 3SAT

$(x_i + x_j + x_k)(\overline{x_k} + x_e + x_w) \cdots$

Q: Compute assignment that satisfies as many clauses as possible.

$x_1, \ldots, x_n$ variables in F given formula

$x_i \leftarrow 0/1$ with probability half.

$$E\left[\begin{array}{l}\text{expected}\\ \text{number of}\\ \text{satisfied clauses}\end{array}\right] = E\left[\sum_{c \in F} Z_c\right] = \sum_{c \in F} E[Z_c] = m \cdot \frac{7}{8}$$

$\boxed{\frac{7}{8}}$

m = clauses in F

$$P[x + y + z \text{ is satisfied}] = \frac{7}{8}$$

$8 = 2^3$

$Z_c = 1 \iff$ the clause c is satisfied

$$E[Z_c] = P[Z_c = 1] = \frac{7}{8}$$