1

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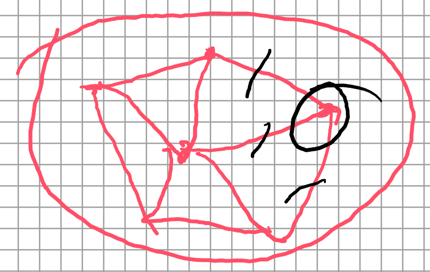
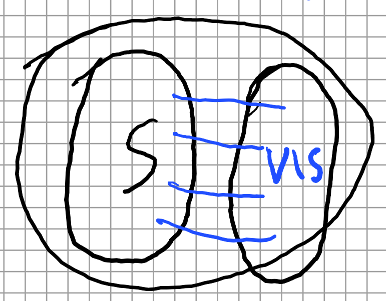
min cut

$(S, V \setminus S)$





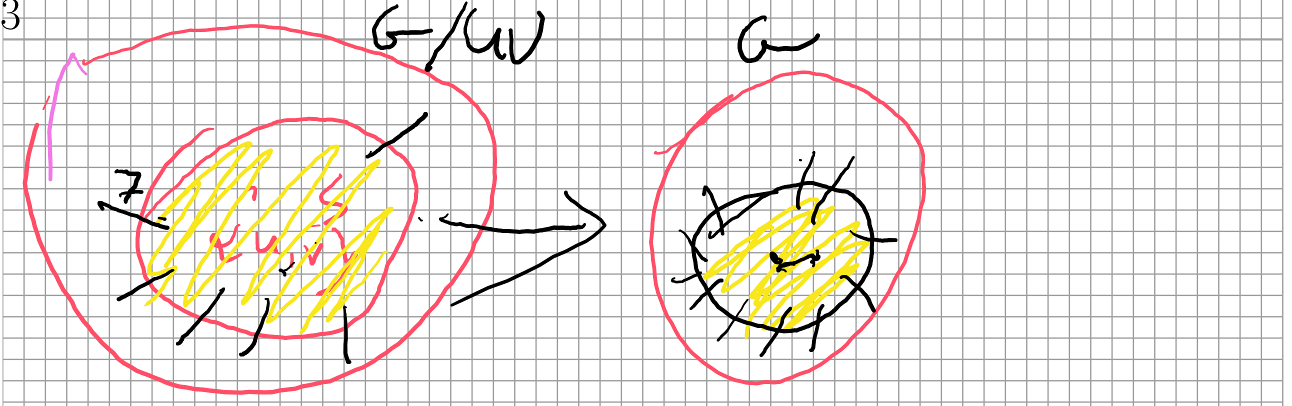
$O(2^n m)$

max cut

NP-Hard

Contraction

3

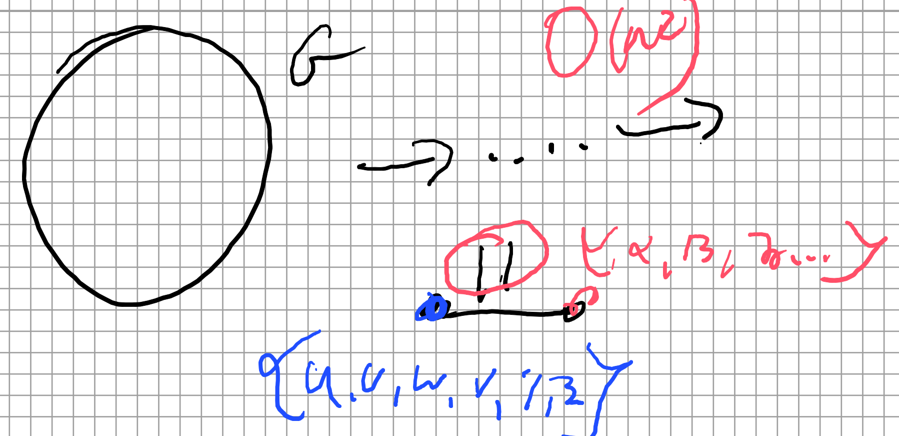


$n-1$

$2^{n-1}-2$ $\qquad$ 2^n-2

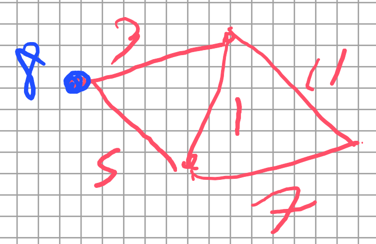
<u>Obs:</u> If uv is not in the min cut of G then

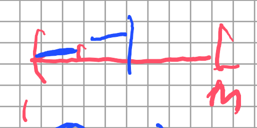
$$\text{mincut}(G/uv) = \text{mincut}(G)$$



Contraction: $O(n)$ time.

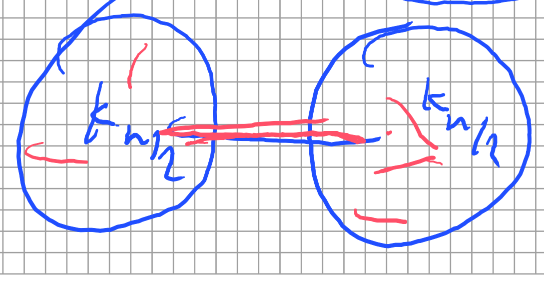
How to pick an edge?





$O(m)$

$O(n)$ carefully



$\frac{1}{n^2}$

Claim

If G has n vertices and a min cut of size k

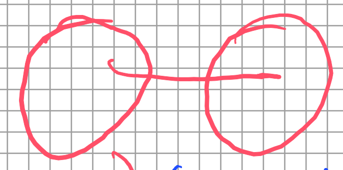
$$\Longrightarrow |E(G)| \geq \frac{kn}{2}.$$

Proof:

$$|E(G)| = \sum_{v \in V(G)} d(v) / 2 \geq \frac{kn}{2}. \quad \blacksquare$$

$P = \text{Prob}\begin{pmatrix}\text{hitting} \\ \text{min cut}\end{pmatrix} = \frac{k}{|E(G)|} \le \frac{k}{kn/2} = \frac{2}{n}.$

$\ge 1 - \frac{2}{n}$



$\left(1 - \frac{2}{n}\right)\left(1 - \frac{2}{n-1}\right)\left(1 - \frac{2}{n-2}\right)\left(1 - \frac{2}{n-3}\right) \cdots$

$\|$

$\frac{n-2}{n} \cdot \frac{n-3}{n-1} \cdot \frac{n-4}{n-2} \cdot \frac{n-5}{n-3} \cdots \frac{2}{4} \cdot \frac{1}{3}$

$\alpha = \frac{2}{n(n-1)}$

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α = probability of success

$\geq \frac{2}{n(n-1)}$

alg succeeds with pr α — $O(n^4)$

then repeat $1/\alpha$

$(1-\alpha)^{1/\alpha} \leq (e^{-\alpha})^{\frac{1}{\alpha}} = e^{-1} = \frac{1}{e} < \frac{1}{2}$ — $O(n^2)$ times

$1 - x \leq exp(-x)$ — $O(n^2)$

$$\left(1 - \frac{2}{n(n-1)}\right)^{cn^2 \log n}$$

$$\le \exp\left(-\frac{2cn^2 \log n}{n(n-1)}\right)$$

$$\le \frac{1}{n^{O(1)}}$$

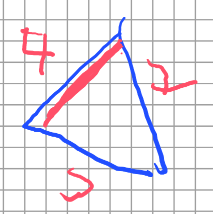
$$O(n^4 \log n).$$

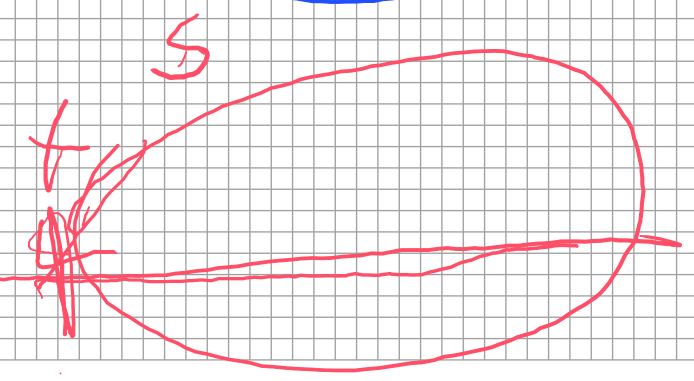
$\blacksquare$

$n \rightarrow n-1 \rightarrow n-2 \; \cdots \rightarrow 3 \rightarrow 2$

$\frac{n-2}{n}$

$\frac{3}{5} \quad \frac{2}{4} \quad \frac{1}{3}$





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$n \longrightarrow t$

$\frac{n-2}{n} \cdot \frac{n-3}{n-1} \cdots \frac{t-1}{t+1}$

$= \frac{(t-2)(t-1)}{n(n-1)} = \frac{1}{2}$

$\frac{t^2}{n^2} \quad t = \frac{n}{\sqrt{2}}$

FastCut(G) ← bottom cases

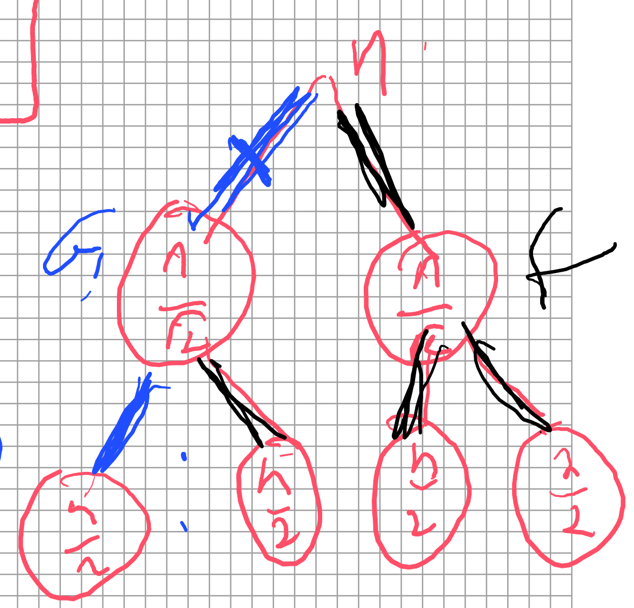
$G_1 \leftarrow$ Contract$(G, \frac{n}{\sqrt{2}})$

$G_2 \leftarrow$ Contract$(G, \frac{n}{\sqrt{2}})$

$C_1 \leftarrow$ FastCut(G_1)

$C_2 \leftarrow$ FastCut(G_2)

return $\min(C_1, C_2)$



$$T(n) = O(n^2) + 2T(n/\sqrt{2})$$

$$= O(n^2 \log n)$$

T_h a full binary tree of height h.

Color edges of T_h with blue and red colors

Q: What is the probability to a blue path from the root to a leaf of T_h.

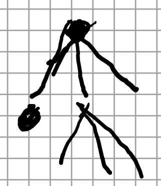
$h = O(\log n)$

Claim $\exists$ blue path in T_h. with prob $\geq \frac{1}{h+1}$..

$O(\log^2 n)$ times

$O(n^2 \log^3 n)$ ✓

Galton Watson process



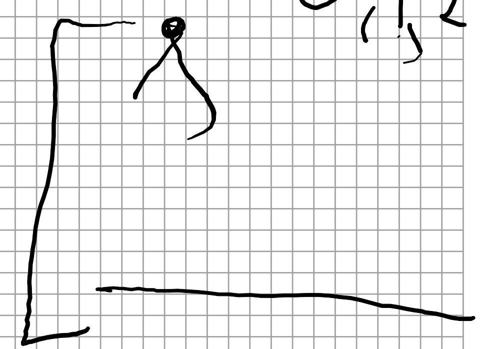
X distribution

$E[X]<1$

$E[X]>1$

$E[X]=1$

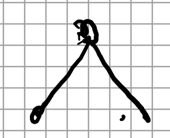
n



$0,1,2$

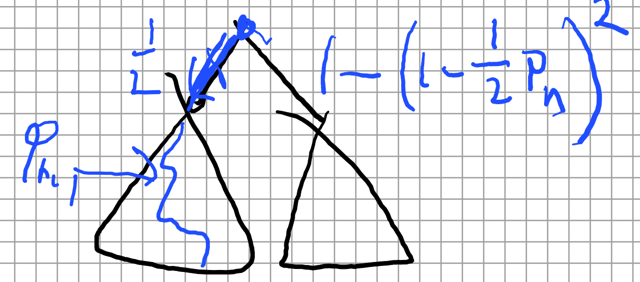
P_h $\quad$ $P_0 = 1$

$h = 1$



$P_h = \frac{3}{4}$

h



$$P_h = 1 - \left(1 - \frac{P_{h-1}}{2}\right)^2 = P_{h-1} - \frac{P_{h-1}^2}{4} \geq \frac{1}{h+1}$$

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