473 lecture: 09/20/18

Thursday, September 20, 2018 12:18 PM

QuickSort

n: Input

RT: Running time

$$E[RT\ \text{QuickSort}] = O(n \log_2 n)$$

$$P\left[\text{QS performs} \geq 10 n \log n \text{ comparisons}\right] \leq \frac{1}{n^4}$$

High probability bound

$$P[\sim] = o(1)$$

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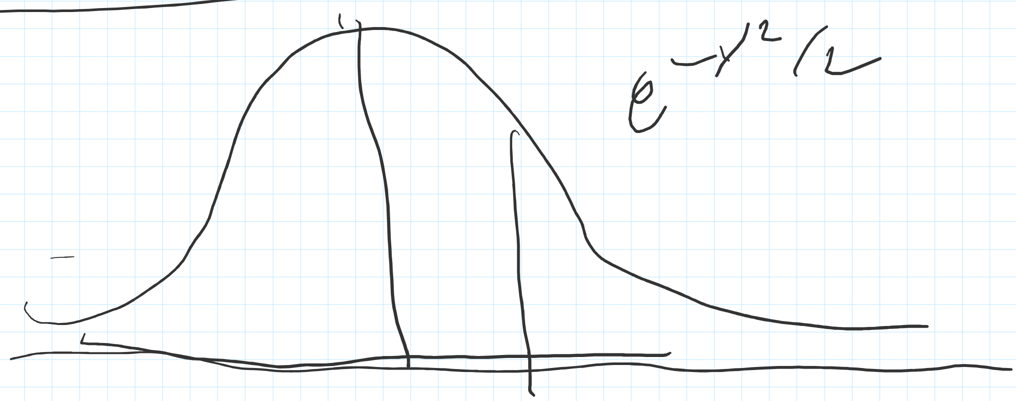
input n — ran alg

$$E[RT(I)]$$

n

I

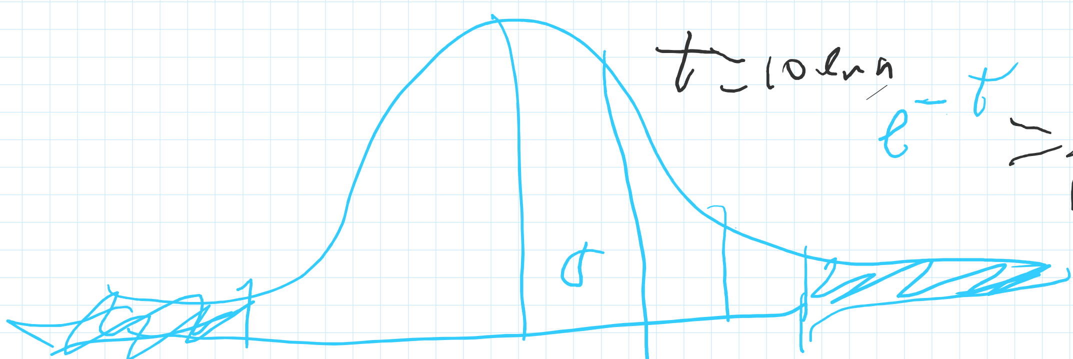
$$E[RT(n)] = \max_{|I|=n} E[RT(I)]$$



$P[X \geq \frac{n}{2} + t\sigma] \leq \exp(-t)$

$n/2 - \sqrt{n}, \; n/2 + \sqrt{n}$

$t = 10 \ln n \qquad e^{-t} = \frac{1}{n^{10}}$



$\frac{n}{2} - t\sigma \qquad \frac{n}{2} \qquad \frac{n}{2} + t\sigma$

$$\left[\frac{n}{2} - 10\sqrt{n}\ln n, \; \frac{n}{2} + 10\sqrt{n}\ln n\right]$$

Chernoff inequality

$X_i \in \{-1, +1\}$

$i = 1 \ldots n$ independent.

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$$S = \sum_{i=1}^{n} x_i \quad E[S] = 0$$

$$\Pr[S \geq \Delta] \leq \exp\left(-\frac{\Delta^2}{2n}\right)$$

$$\Pr[S \leq -\Delta]$$

$$\Delta = \sqrt{4 n \log n}$$

$$Y_0 = 1$$

$$Y_i = \begin{cases} (1+\beta) Y_{i-1} & 1/2 \\ (1-\beta) Y_{i-1} & 1/2 \end{cases}$$

$$E[Y_i \mid Y_{i-1}] = \left(\frac{1-\beta}{2} + \frac{1+\beta}{2}\right) Y_{i-1}$$

$$= Y_{i-1}$$

$$E[E[X \mid Y]] = E[X]$$

$$E[Y_i] = E\left[E[Y_i \mid Y_{i-1}]\right]$$
$$= E[Y_{i-1}] = Y_0 = 1$$

60% win $0.6n$

$$\underbrace{(1+\beta)^{0.6n}(1-\beta)^{0.4n}}_{} = c^n$$

$\beta = 0.1$ $\qquad \cancel{<}$ > 1

$X_1, \ldots, X_n \in \{0, 1\}$

$S = \sum X_i$

$P[S \le \frac{n}{4}] \le$ exponentially small

Proof $Y_0 = 1$

$$Y_i = \begin{cases} Y_{i-1} & X_i = 0 \\ Y_{i-1}/2 & X_i = 1 \end{cases}$$

$$E[Y_i \mid Y_{i-1}] = \frac{1}{2} Y_{i-1} + \frac{1}{2}\frac{Y_i}{2}$$

$$= \frac{3}{4} Y_{i-1}$$

$$E[Y_i] = E[E[Y_i \mid Y_{i-1}]] = \left(\frac{3}{4}\right)^i$$

$$E[Y_n] = \left(\frac{3}{4}\right)^n$$

$$P[S \leq 0.1n] = P\left[Y_n \geq \frac{1}{2^{0.1n}}\right]$$

$$\left(\frac{3}{4}\right)^4 = \frac{81}{256} \leq \frac{1}{3}$$

$$P\left[Y_n \geq \frac{1}{2^{n/10}}\right] \leq \frac{E[Y_n]}{2^{n/10}}$$

$$P[Y \geq t] \leq \frac{E[Y]}{t}$$ Markov inequality

$$=$$

$$\left(\frac{3}{4}\right)^n / 2^{n/10} \leq \frac{\left(\frac{1}{3}\right)^{n/4}}{2^{n/10}} < 2^{-\frac{n}{4}+\frac{n}{10}}$$

$$\leq 2^{-n/8}$$