Algorithms

NP - Completeness

Hardness

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input → Alg → output

2 SAT

2 CNF

Input: Boolean formula

output: Can it be satisfied.

(clause) $\wedge$ ( )

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$\underbrace{(x \vee y)}_{\vee} \wedge \underbrace{(\bar{y} \vee z)}$ n variables

m clauses

F $\cancel{x}=0$ false

$y=1$

$z=1$ Satisfying assignment

$(x \vee y \vee z)$ X 3CNF

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Algorithm

– Stupid!

– F n vars m clauses

2CNF

$x_1, x_2, \dots, x_n$ $O(2^n m)$

Try all possible assignments

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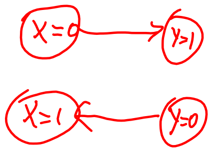
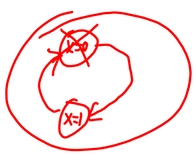
$(x \vee y)$



2n vertices

2m edges



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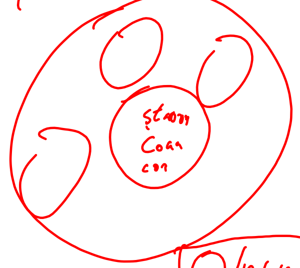
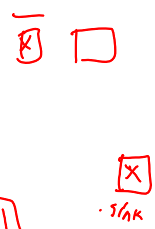
Graph



DAG



$\boxed{O(n+m)}$

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2SAT linear time

3SAT 3CNF

$(\bar{w} \vee x \vee \bar{z}) \wedge (x \vee y \vee \bar{z}) \wedge$

$x=0$

TRY EVERYTHING

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Brute force search

$O(2^n m)$ – Can we do better

P=NP ?

We don't know

$O(1.74^n m)$

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2SAT / 3SAT

linear 2^{cn} c is some constant

alg lower-bounds

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SETH

Strong exponential time hypothesis

3SAT (kSAT for k sufficiently large) can not be solved in time faster than 2^{cn}

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$O(n^c)$ $\downarrow$ 2^m 2^{cn}

c log n cn Log

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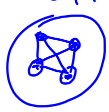
3SAT

CLIQUE

G, k

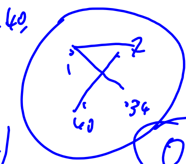


Brute force solving

1, 7, 40, 34

k

$O(k^2)$



Yes

$O(n^2)$

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	Solving	Verifying a yes
3SAT	1.7^n	assignment $O(n)$
CLIQUE	$\binom{n}{k} k^2$ $\approx 2^n$	list of vertices $O(k^2)$ a clique.

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NP: Not Deterministic Polynomial

All decision problems s.t. a yes/True answer can be verified in polynomial time.

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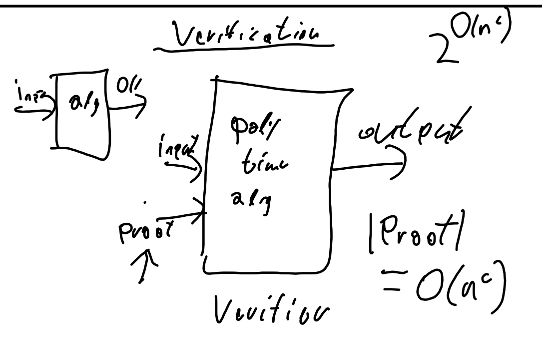
Verification



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$$P \overset{?}{\underset{\neq}{\subseteq}} NP \subseteq EXP$$

exponential time ⇓ EXP

NP ⇓ 3SAT, CLIQUE

$2^{\text{polynomial}}$

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Cook-Levin Theorem

If 3SAT can be solved in polynomial time then all the problems in NP can be solved in polynomial time!

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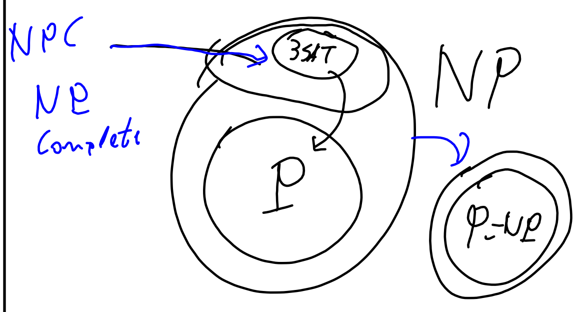

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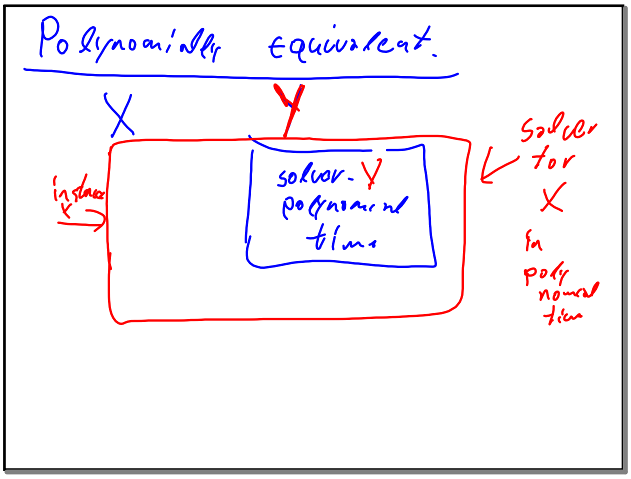

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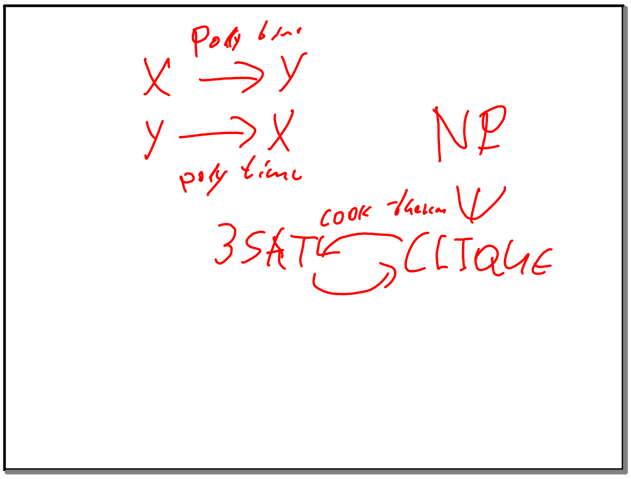

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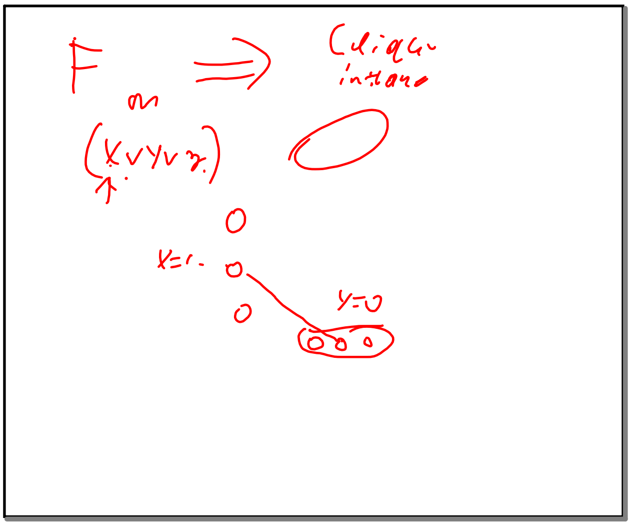

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