CS 473: Algorithms, Fall 2016
HW 2 (due Tuesday, September 13th at 8pm)

This homework contains three problems. **Read the instructions for submitting homework on the course webpage**.

Collaboration Policy: For this home work, each student can work in a group with upto three members. Only one solution for each group needs to be submitted. Follow the submission instructions carefully.

1. Given an undirected graph $G = (V, E)$ we defined its *square*, denoted by square(G) as the graph $G' = (V, E')$ where $uv \in E'$ iff there is a path of length at most 2 between u and v in G. That is, $uv \in E'$ if $uv \in E$ or if there is a node w such that $uw, wv \in E$. In class we saw an algorithm for the maximum weight independent set problem in a tree $T = (V, E)$. Design and analyze an algorithm for the maximum weight independent set problem for the square of a given tree $T = (V, E)$.

2. Let $G = (V, E)$ be a directed graph and let k be an integer. Describe an efficient algorithm that given two nodes $s, t \in V$ checks whether there is an *s*-*t walk* in G that contains at least k distinct nodes. Note that it is important that we ask for a walk and not a simple path for otherwise the problem would be NP-Complete.

   - Develop an algorithm when G is a DAG.
   - Develop an algorithm for the general case using the strong-component meta-graph of a given directed graph which is a DAG. (See Chapter 3 of Dasgupta etal book if you are unfamiliar with this notion.) *Hint:* What is the answer if G is strongly connected?

3. Consider the following multi-processor scheduling problem. The input consists of n jobs $J_1, J_2, \ldots, J_n$ and m identical machines $M_1, M_2, \ldots, M_m$. Each job J_i has a non-negative size s_i. The goal is to assign the jobs to the machines to minimize the maximum load over all machines. The load of a machine is the sum of the sizes of the jobs assigned to it. This is an NP-Hard problem. However, here we will consider the setting where there are only 3 distinct job sizes $\{a, b, c\}$. That is, $s_i \in \{a, b, c\}$ for $1 \leq i \leq n$. Describe a polynomial-time algorithm for this problem. You need not answer this part but can you obtain a polynomial time algorithm when the number of distinct job sizes is at most k where k is some fixed constant?

The remaining problems are for self study. Do *NOT* submit for grading.

- Did you do some of the DP problems mentioned in the previous home work? Below you will find more challenging ones.
- Given a graph $G = (V, E)$ a matching is a set of edges $E' \subseteq E$ such that no two edges in E' share an end point. Describe an efficient algorithm that given a tree $T = (V, E)$ and non-negative weights $w : E \to \mathbb{R}_+$ finds a maximum weight matching in T.
- See Homework 2 from Spring 2016 taught by Jeff Erickson. `https://courses.engr.illinois.edu/cs473/sp2016/hw/hw2.pdf`
- See Homeworks 2 and 3 from Fall 2015 taught by Sariel Har-Peled. All the problems are on dynamic programming. They are a bit challenging. `https://courses.engr.illinois.edu/cs473/fa2015/w/hw/hw_02.pdf` and `https://courses.engr.illinois.edu/cs473/fa2015/w/hw/hw_03.pdf`.
- There are several nice shortest path problems in both text books and at the end of Jeff's notes on shortest paths. Check them out.
- Given a directed graph $G = (V, E)$ with non-negative edge lengths $\ell : E \to \mathbb{R}_+$, describe an algorithm that finds the shortest cycle in G that contains a specific node s. Describe an algorithm to find the shortest cycle containing s with at most k edges. Describe an algorithm to find a cycle C with minimum *average* length where the average length of a cycle C is defined as $\ell(C)/|C|$.