HW 8 (due Monday, 6pm, November 9, 2015)

NEW CS 473: Theory II, Fall 2015 Version: **1.0**

Collaboration Policy: This homework can be worked in groups of up to three students.
Submission is online on moodle.

1. (25 PTS.) LP I

Let L be a linear program given in slack form, with n nonbasic variables N, and m basic variables B. Let N' and B' be a different partition of $N \cup B$, such that $|N' \cup B'| = |N \cup B|$. Show a polynomial time algorithm that computes an equivalent slack form that has N' as the nonbasic variables and B' as the basic variables. How fast is your algorithm?

2. (25 PTS.) Multi mini flowy.

In the *minimum-cost multicommodity-flow problem*, we are given a directed graph $G = (V, E)$, in which each edge $(u, v) \in E$ has a nonnegative capacity $c(u, v) \geq 0$ and a cost $\alpha(u, v)$. As in the multicommodity-flow problem (Chapter 29.2, CLRS), we are given k different commodities, $K_1, K_2, \ldots, K_k$, where commodity i is specified by the triple $K_i = (s_i, t_i, d_i)$. Here s_i is the source of commodity i, t_i is the sink of commodity i, and d_i is the demand, which is the desired flow value for commodity i from s_i to t_i. We define a flow for commodity i, denoted by f_i, (so that $f_i(u, v)$ is the flow of commodity i from vertex u to vertex v) to be a real-valued function that satisfies the flow-conservation, skew-symmetry, and capacity constraints. We now define $f(u, v)$, the ***aggregate flow*** to be sum of the various commodity flows, so that $f(u, v) = \sum_{i=1}^{k} f_i(u, v)$. The aggregate flow on edge (u, v) must be no more than the capacity of edge (u, v).
The cost of a flow is $\sum_{u,v \in V} f(u, v)\alpha(u, v)$, and the goal is to find the feasible flow of minimum cost. Express this problem as a linear program.

3. (25 PTS.) Some calculations required,

Provide *detailed* solutions for the following problems, showing each pivoting stage separately.
maximize $6x_1 + 8x_2 + 5x_3 + 9x_4$
subject to
$2x_1 + x_2 + x_3 + 3x_4 \leq 5$
$x_1 + 3x_2 + x_3 + 2x_4 \leq 3$
$x_1, x_2, x_3, x_4 \geq 0.$

4. (25 PTS.) Some calculations required,

minimize $4x_{12} + 6x_{13} + 9x_{14} + 2x_{23} + 7x_{24} + 3x_{34}$
subject to
$x_{12} + x_{13} + x_{14} \geq 1$
$-x_{12} + x_{23} + x_{24} = 0$
$-x_{13} - x_{23} + x_{34} = 0$
$x_{14} + 3x_{24} + x_{34} \leq 1$
$x_{12}, x_{13}, \ldots, x_{34} \geq 0.$