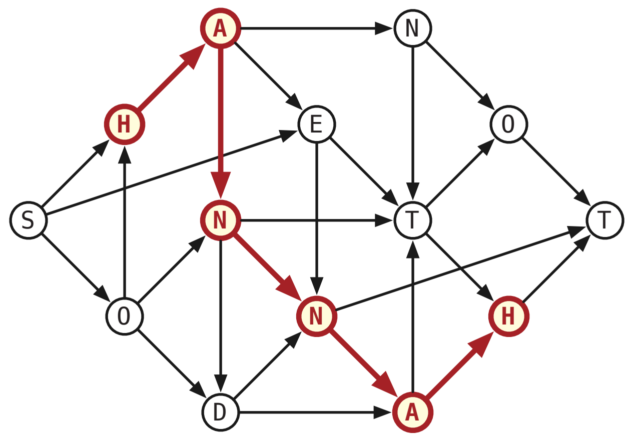
1. Suppose we are given a directed acyclic graph G with labeled vertices. Every path in G has a label, which is a string obtained by concatenating the labels of its vertices in order. Recall that a *palindrome* is a string that is equal to its reversal.

   Describe and analyze an algorithm to find the length of the longest palindrome that is the label of a path in G. For example, given the graph below, your algorithm should return the integer 6, which is the length of the palindrome **HANNAH**.



2. Let G be a connected directed graph that contains both directions of every edge; that is, if $u{\to}v$ is an edge in G, its reversal $v{\to}u$ is also an edge in G. Consider the following non-standard traversal algorithm.

```
SpaghettiTraversal(G):
  for all vertices v in G
      unmark v
  for all edges u→v in G
      color u→v white
  s ← any vertex in G
  Spaghetti(s)
```

```
Spaghetti(v):
  mark v                              ⟨⟨"visit v"⟩⟩
  if there is a white arc v→w
      if w is unmarked
          color w→v green
      color v→w red                   ⟨⟨"traverse v→w"⟩⟩
      Spaghetti(w)
  else if there is a green arc v→w
      color v→w red                   ⟨⟨"traverse v→w"⟩⟩
      Spaghetti(w)
  ⟨⟨else every arc v→w is red, so halt⟩⟩
```

   We informally say that this algorithm "visits" vertex v every time it marks v, and it "traverses" edge $v{\to}w$ when it colors that edge red. Unlike our standard graph-traversal algorithms, Spaghetti may (in fact, *will*) mark/visit each vertex more than once.

   The following series of exercises leads to a proof that Spaghetti traverses each directed edge of G exactly once. Most of the solutions are very short.

   (a) Prove that no directed edge in G is traversed more than once.

   (b) When the algorithm visits a vertex v for the kth time, exactly how many edges into v are red, and exactly how many edges out of v are red? *[Hint: Consider the starting vertex s separately from the other vertices.]*

(c) Prove each vertex v is visited at most $\deg(v)$ times, except the starting vertex s, which is visited at most $\deg(s)+1$ times. This claim immediately implies that SpaghettiTraversal(G) terminates.

(d) Prove that when SpaghettiTraversal(G) ends, the last visited vertex is the starting vertex s.

(e) For every vertex v that SpaghettiTraversal(G) visits, prove that all edges incident to v (either in or out) are red when SpaghettiTraversal(G) halts. *[Hint: Consider the vertices in the order that they are marked for the first time, starting with s, and prove the claim by induction.]*

(f) Prove that SpaghettiTraversal(G) visits every vertex of G.

(g) Finally, prove that SpaghettiTraversal(G) traverses every edge of G exactly once.

CS 473 Fall 2013 — Homework 7 Problem 1

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1. Describe and analyze an algorithm to find, given a dag G with labeled vertices, the length of the longest palindrome that is the label of a path in G.

CS 473 Fall 2013 — Homework 7 Problem 2

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2. Prove (in seven steps) that SPAGHETTITRAVERSAL(G) traverses each edge in G exactly once.