

1. A *longest common subsequence* of a set of strings  $\{A_i\}$  is a longest string that is a subsequence of  $A_i$  for each  $i$ . For example, `alrit` is a longest common subsequence of strings

`algorithm` and `altruistic`.

Given two strings  $A[1..n]$  and  $B[1..n]$ , describe and analyze a fast dynamic programming algorithm that computes the length of a longest common subsequence of the two strings.

2. Describe and analyze a fast dynamic programming algorithm that computes the length of a longest common subsequence of three strings  $A[1..n]$ ,  $B[1..n]$ , and  $C[1..n]$ .
3. A *lucky-10 number* is a string  $D[1..n]$  of digits from 1 to 9 (no zeros), such that the  $i$ -th digit and the last  $i$ -th digit sum up to 10; in another words,  $D[i] + D[n - i + 1] = 10$  for all  $i$ . For example,

`3141592648159697` and `11599`

are both lucky-10 numbers. Given a string of digits  $D[1..n]$ , describe and analyze a dynamic programming algorithm that computes the length of a longest lucky-10 subsequence of the string. [Hint: Try to use your solution to problem 1 **directly**.]

4. **To think about later:** Can you solve problem 1 in  $O(n)$  space?