CS 473: Fundamental Algorithms, Fall 2011

Homework 5 (due Monday, 23:55:00, October 10, 2011)

Collaboration Policy & submission guidelines: See homework 1.
Each student individually have to also do **quiz 5** online.

Version: **1.11**

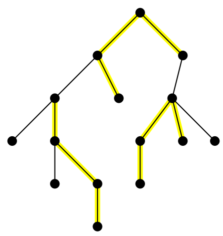
1. (40 PTS.) Snakes on a tree.

   Let $G = (V, E)$ be an undirected graph with weights on the edges. That is for an edge $uv \in E$, we have a weight $w(uv)$ associated with uv. A ***k-snake*** in a graph is a simple path with exactly k edges in it. Two k-snakes are disjoint if they do not share any vertex. A collection M of k-snakes is a ***k-snake packing*** if all pairs of snakes of M are disjoint (for $k = 1$ the set M is a matching in the graph). The total weight of a k-snake packing is the total weight of the edges used by the snakes.

   We are interested in the problem of computing the maximum weight k-snake packing in G. In general, this problem seems hard. Fortunately for the tree case this is much easier.

   Describe an efficient algorithm (i.e., provide pseudo-code, etc), as fast as possible, for computing the maximum weight k-snake packing when G is a rooted tree. Your algorithm should be recursive and use memoization to achieve efficiency. (You can not assume G is a binary tree - a node might have arbitrary number of children.) What is the running time of your algorithm as function of $n = |V(G)|$ and k?)

   For example, the following shows a tree with a possible 3-snake packing.



2. (30 PTS.) All your internet base are belong to us.

   You are given a road somewhere out there. The road is straight, and there are n farm houses along it. You are given the distance x_i of the ith house from the beginning of the road. (You can assume $x_1 \leq x_2 \leq \ldots \leq x_n$.) You need to place k stations along the road, such that you connect every house to one of the stations. The price of connecting a station to a house is their distance.

   (A) Prove that if n is odd and $k = 1$ then the optimal placement for the single station is at $x_{(n-1)/2+1}$.
   (B) Prove that if n is even and $k = 1$ then the optimal placement for the single station is at either $x_{(n)/2}$ or $x_{(n)/2+1}$ (and both locations have exactly the same price).
   (C) Describe an algorithm, as fast as possible, just that given $x_1, \ldots, x_n$, and an integer k, outputs the locations of the best placements for the k stations (sorted by increasing order), and the price of wiring all the houses to these stations. How fast is your algorithm? (specifically, what is the running time if $k = O(1)$ or $k = n/2$.)

3. (30 PTS.) And you thought this was a theory class.

Implement your algorithm from the previous part in C or C++. The input to your program will be

$$\begin{array}{c} n \\ k \\ x_1 \\ \vdots \\ x_n \end{array}$$

(Given on the standard input.) To simplify things you can assume all these numbers are integers. The output should be:

(A) Price of the solution (on its own line).
(B) The location of the k stations (each on its own line) – in increasing order.
(C) For each house, the index of the station (out of the k stations) it is connected to (each on its own line).

You need to submit a single C (or C++) file that contains your program (which must compile on its own and run efficiently). We will provide several input/output examples on the class webpage. We would also provide some help code to implement memoization.[1]

[1] We will use MOSS to verify that programs are not copied...