4.1 Recurrences

Solve the following recurrences.

(A) \( T(n) = 5T(n/4) + n \) and \( T(n) = 1 \) for \( 1 \leq n < 4 \).

(B) \( T(n) = 2T(n/2) + n \log n \)

(C) \( T(n) = 2T(n/2) + 3T(n/3) + n^2 \)

4.2 Tree Traversal.

Let \( T \) be a rooted binary tree on \( n \) nodes. The nodes have unique labels from 1 to \( n \).

(A) Given the preorder and postorder node sequences for \( T \), give a recursive algorithm to reconstruct a tree that satisfies the preorder and postorder sequences. Is this reconstruction unique?

(B) Given the preorder and inorder node sequences for \( T \), give a recursive algorithm to reconstruct a tree that satisfies the preorder and inorder sequences. Is this reconstruction unique?

4.3 Divide and Conquer.

Let \( p = (x, y) \) and \( p' = (x', y') \) be two points in the Euclidean plane given by their coordinates. We say that \( p \) dominates \( p' \) if and only if \( x > x' \) and \( y > y' \). Given a set of \( n \) points \( P = \{p_1, \ldots, p_n\} \), a point \( p_i \in P \) is undominated in \( P \) if there is no other point \( p_j \in P \) such that \( p_j \) dominates \( p_i \). Describe an algorithm that given \( P \) outputs all the undominated points in \( P \); see figure. Your algorithm should run in time asymptotically faster than \( O(n^2) \)

Figure 1: The undominated points are shown as unfilled circles.
4.4 Convex hull.

You are given a set $P$ of $n$ points in the plane, and you would like to compute their convex-hull (i.e., that is the shortest perimeter polygon that contains all the points). To see how the convex-hull looks like, think about the plane as being a wood board, and place a nail at each point. Now, you shrink a rubber band around the points. The rubber shrinks into the convex-hull. Clearly, the vertices of the convex-hull are a subset of the input points. Show an $O(n \log n)$ time algorithm for computing the convex-hull. (Hint: Split the plane by a vertical line, compute the convex-hulls on both sides, and then figure out how to stitch the two convex-hulls together. To get a handle on this stitching problem, find closest points in the $x$-axis between the two hulls, and climb up to the stitching bridges.)