This homework contains four problems. **Read the instructions for submitting homework on the course webpage.** In particular, **make sure** that you write the solutions for the problems on separate sheets of paper; the sheets for each problem should be stapled together. Write your name and netid on each sheet.

**Collaboration Policy:** For this homework, Problems 1-3 can be worked in groups of up to 3 students each. Problem 0 should be answered in Compass as part of the assessment HW2-Online and should be done individually.

1. **(30 pts)** You are given an array $A$ of $n$ distinct integers, and an integer $k$ such that $1 \leq k \leq n$. The **square distance** between a pair of integers $x, y$ is defined as the quantity $(x-y)^2$. Your goal is to design an $O(n)$ time algorithm to find $k$ elements in $A$ with the smallest square distance to the median (i.e. the element of rank $\lfloor n/2 \rfloor$ in $A$). For instance, if $A = [9, 5, -3, 1, -2]$ and $k = 2$, then the median element is 1, and the 2 elements in $A$ with the smallest square distance to the median are $\{1, -2\}$. If $k = 3$, then you can output either $\{1, -2, -3\}$ or $\{1, -2, 5\}$. **Hint:** Use the linear time Selection algorithm.

2. **(30 pts)** Describe an $O(n \log n)$ time algorithm to compute the longest decreasing subsequence of a given sequence of $n$ numbers. (**Hint:** use an appropriate data structure to speed up the dynamic programming algorithm.)

3. **(30 pts)** Let $T$ be a rooted tree with integer lengths on its edges (they can be positive, negative or zero). Describe an algorithm that finds among all paths from a node in $T$ to one of its descendants the one with the smallest length. The instance of the problem given below has the smallest path $E - F - H$ of length $-9$. 

![Diagram of a tree with edge lengths]