1. Draw the DFS tree rooted at A for the above graph. Use alphabetic ordering to break ties. Label the vertices of the tree with their \( \text{pre}(v) : \text{post}(v) \) time. Add in the remaining edges of the graph and label them as forward (F), backward (B), and cross (C) edges. Sort the vertices by their post visit order.

2. Let \( G \) be a directed graph and \( G^{\text{SCC}} \) its strong connected component meta-graph (which is a DAG). Prove or disprove the following. For any DFS of \( G \) the vertex with smallest post-visit number is in a sink component of \( G^{\text{SCC}} \).

3. Let \( G = (V, E) \) be an undirected graph with \( n \) vertices (\(|V| = n \)) and \( m \) edges (\(|E| = m \)). Give an \( O(n) \) time algorithm to check if \( G \) has at least two distinct cycles and output them if it does. Assume that the graph is represented using adjacency lists. Note that \( m \) can be much larger than \( n \) so the algorithm should not check all edges. \textit{Hint}: What is the structure of a minimal connected graph that has two cycles? Use DFS.

4. There are \( n \) light bulbs in a garden. These bulbs can be turned on manually by flipping on the switches at the light posts. Also, each light post can broadcast turn-on signals to some other pre-defined light posts in the garden, turning them on. When a light post is turned on, it will automatically broadcast a turn-on signal to its pre-defined light posts.

This signal broadcasting is directional. If \( a \) broadcasts to \( b \), it is not necessarily true that \( b \) also broadcasts to \( a \).

So one can manually flip on some of the switches to the light posts, and those light posts will broadcast a turn-on signal to other light posts. These will in turn be switched on and broadcast signals to their own pre-defined set of light posts, and so on.
Given each light post in the garden and the respective light posts to which they broadcast, derive a linear time algorithm for finding the minimum number of switches needed to be flipped to light up the whole garden. (Linear time means $O(n + m)$ where $n$ is the number of light posts and $m$ is the number of broadcast associations between them).

**Source:** *ACM ICPC 2010 World Finals Warmup 2*

**Example Case:** Number of lights : 5, Number of broadcast associations: 4
Associations : 1=>2, 1=>3, 3=>4, 5=>3
**Answer:** Minimum number of flips required : 2,
Turning on switches 1 and 5 should light up the whole garden

**Hints:**
(a) Model the problem using directed graphs.
(b) What is the solution if the graph in question is strongly connected?
(c) What is the solution in general?