CS 473: Algorithms, Fall 2009
HW 1 (due Tuesday, September 8 in class)

This homework contains three problems. **Read the instruction for submitting homework on the course webpage**. In particular, *make sure* that you write the solutions for the problems on separate sheets of paper. Write your name and netid on each sheet.

Collaboration Policy: For this home work, students can work in groups of up to 3 members each. Each group submits only one written solution. Indicate your group members on the homework (netids are needed). You will be in this group for the next three homeworks.

Read the course policies before starting the homework.

1. (30pts) Obtain tight asymptotic bounds for functions below that satisfy the specified recurrences. Justify your answers. A full formal proof is not necessary.
   - (20 pts) $A(n) = 7A(n/4) + A(3n/4) + n^2 \log n$. $A(n) = O(1)$ for $n < 4$.
   - (10 pts) $B(n) = B(2n/3) + \sqrt{n}$. $B(n) = O(1)$ for $n < 3$.
2. (30 pts) Let $p = (x, y)$ and $p' = (x', y')$ be two points in the Euclidean plane given by their coordinates. We say that p dominates p' if and only if $x > x'$ *and* $y > y'$. Given a set of n points $P = \{p_1, \ldots, p_n\}$, a point $p_i \in P$ is undominated in P if there is no other point $p_j \in P$ such that p_j dominates p_i. Describe an algorithm that given P outputs all the undominated points in P; see figure. An $O(n^2)$ time algorithm will not get you any points.

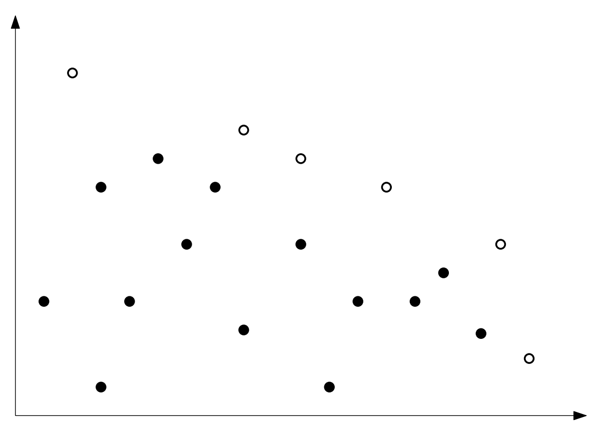

Figure 1: The undominated points are shown as unfilled circles.

3. (40 pts) Given a sequence $a_1, a_2, \ldots, a_n$ of n distinct numbers, an *inversion* is a pair $i < j$ such that $a_i > a_j$. Note that a sequence has no inversions if and only if it is sorted in ascending order. The text book describes (see Chapter 5) an $O(n \log n)$ algorithm to count the number of inversions in a given sequence. We consider two generalizations.
   - (25 pts) Call a pair $i < j$ a *significant* inversion if $a_i > 2a_j$. Describe an $O(n \log n)$ time algorithm to count the number of significant inversions in a given sequence. This is problem 5.2 from the text book.

- (15 pts) Consider a further generalization. In addition to the sequence $a_1, \ldots, a_n$ we are given weights $w_1, \ldots, w_n$ where $w_i \geq 1$ for each i. Now call a pair $i < j$ a significant inversion if $a_i > w_j a_j$. Describe an $O(n \log^2 n)$ time algorithm to count the number of significant inversions given the sequences a and w. Extra credit (10 pts) if you can obtain an $O(n \log n)$ running time.