1. Recall the problem of finding a set of undominated points. Given an input set of points $P = \{p_1, p_2, \ldots, p_n\}$ in the plane (with distinct coordinates), a point $p$ is undominated if there are no other points above and to the right of $p$. Describe a method to maintain the set of undominated points as new points are added to the set. Specifically, describe and analyze a data structure that stores the undominated points of $P$, and an algorithm $\text{INSERT}(x, y)$ that adds the point $p = (x, y)$ to $P$ and returns TRUE or FALSE to indicate whether the set of undominated points has changed. Your data structure should use $O(n)$ space, and your $\text{INSERT}$ algorithm should run in $O(\log n)$ amortized time.

2. Given a string of letters $Y = y_1y_2 \ldots y_n$, a segmentation of $Y$ is a partition of its letters into contiguous blocks of letters (also called words). Each word has a quality that can be computed by a given oracle (e.g., you can call $\text{QUALITY}$ (“meet”) to get the quality of the word “meet”). The quality of a segmentation is equal to the sum over the qualities of its words. Each call to the oracle takes linear time in terms of the argument; that is $\text{QUALITY}(S)$ takes $O(|S|)$ time.

Using the given oracle, give an algorithm that takes a string $Y$ and computes a segmentation of maximum total quality.

3. Consider two distinct horizontal lines $l_1$ and $l_2$ in the plane. There are $n$ points on $l_1$ with distinct $x$-coordinates $A = a_1, a_2, \ldots, a_n$ and there are $n$ points on $l_2$ with distinct $x$-coordinates $B = b_1, b_2, \ldots, b_n$. Design an algorithm to compute, given $A$ and $B$, a largest set $S$ of non-intersecting line segments subject to the following restrictions:

(a) Any segment in $S$ connects $a_i$ to $b_i$ for some $i$ ($1 \leq i \leq n$).
(b) Any two segments in $S$ do not intersect.