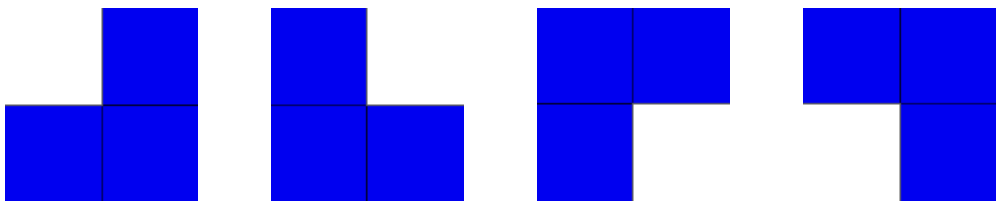
CS 473: Algorithms, Fall 2009
HBS 0

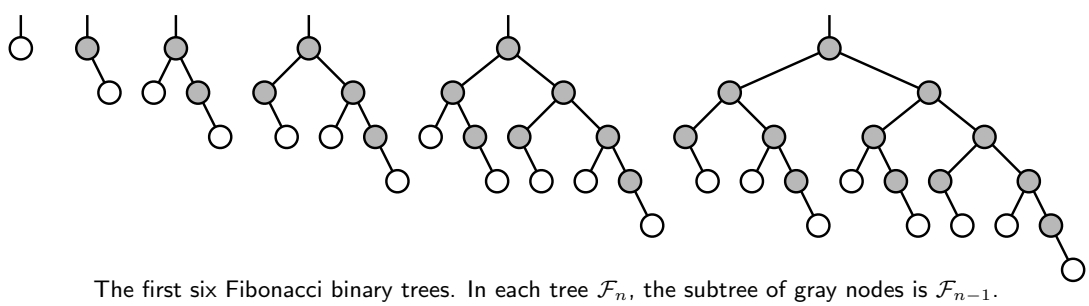
1. A k-coloring of a graph G is a labeling $f : V(G) \to S$ from vertices to colors where $|S| = k$. A k-coloring is proper if adjacent vertices are assigned different colors. A graph is k-colorable if it has a proper k-coloring. Prove that any graph G has a proper $(\Delta + 1)$-coloring where Δ is the maximum degree of a vertex of G (no vertex has more than Δ neighbors). For example, any cycle is 3-colorable as $\Delta = 2$ for cycles.

2. You are given a $2^n \times 2^n$ chessboard with a single square removed. Prove that you can tile the entire chessboard (minus the missing square) using copies of the 2×2 L's shown below.



3. The nth *Fibonacci binary tree* $\mathcal{F}_n$ is defined recursively as follows:

   - $\mathcal{F}_1$ is a single root node with no children.
   - For all $n \geq 2$, $\mathcal{F}_n$ is obtained from $\mathcal{F}_{n-1}$ by adding a right child to every leaf and adding a left child to every node that has only one child.



The first six Fibonacci binary trees. In each tree $\mathcal{F}_n$, the subtree of gray nodes is $\mathcal{F}_{n-1}$.

   (a) Prove that the number of leaves in $\mathcal{F}_n$ is precisely the nth Fibonacci number: $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$.

   (b) How many nodes does $\mathcal{F}_n$ have?

   (c) (*) What is the depth of $\mathcal{F}_n$'s most shallow leaf?