1. A $k$-coloring of a graph $G$ is a labeling $f : V(G) \rightarrow S$ from vertices to colors where $|S| = k$. A $k$-coloring is proper if adjacent vertices are assigned different colors. A graph is $k$-colorable if it has a proper $k$-coloring. Prove that any graph $G$ has a proper $(\Delta + 1)$-coloring where $\Delta$ is the maximum degree of a vertex of $G$ (no vertex has more than $\Delta$ neighbors). For example, any cycle is 3-colorable as $\Delta = 2$ for cycles.

2. You are given a $2^n \times 2^n$ chessboard with a single square removed. Prove that you can tile the entire chessboard (minus the missing square) using copies of the $2 \times 2$ L’s shown below.

3. The $n$th Fibonacci binary tree $F_n$ is defined recursively as follows:
   - $F_1$ is a single root node with no children.
   - For all $n \geq 2$, $F_n$ is obtained from $F_{n-1}$ by adding a right child to every leaf and adding a left child to every node that has only one child.

   (a) Prove that the number of leaves in $F_n$ is precisely the $n$th Fibonacci number: $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 2$.
   (b) How many nodes does $F_n$ have?
   (c) (*) What is the depth of $F_n$’s most shallow leaf?