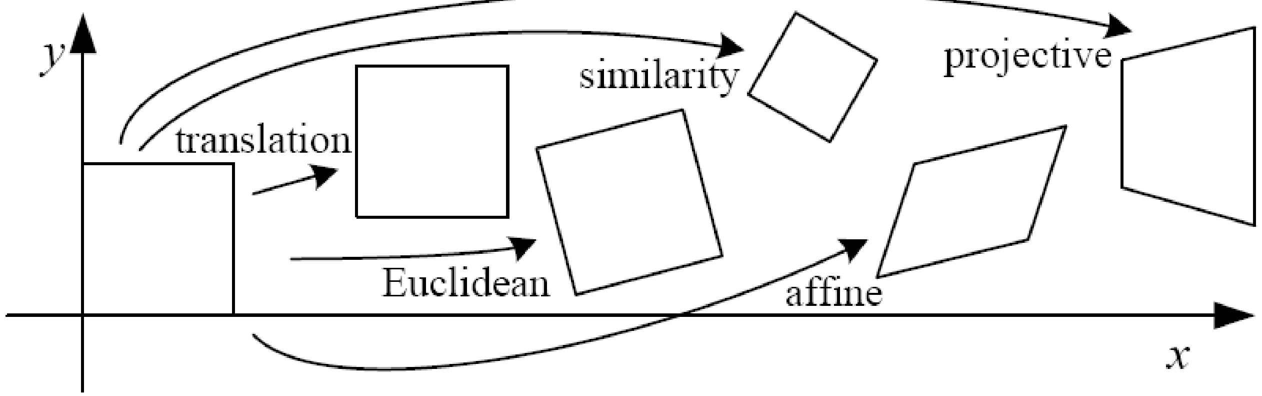
Image Morphing

Computational Photography

Derek Hoiem, University of Illinois

Many slides from Alyosha Efros

2D image transformations



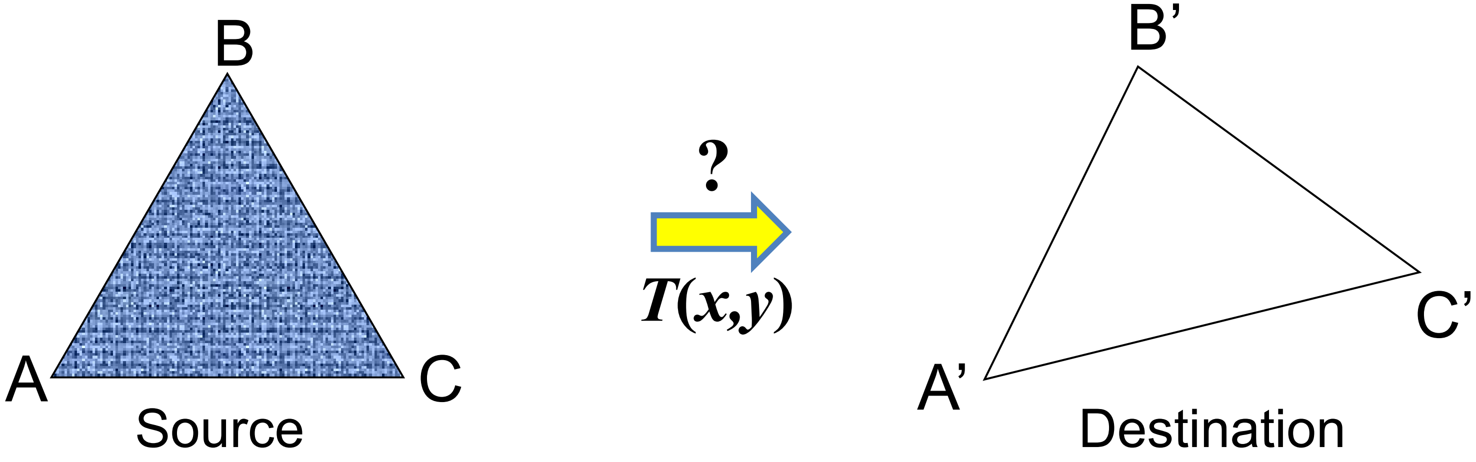
Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+\cdots$	
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+\cdots$	
similarity	$\begin{bmatrix} s\boldsymbol{R} & \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+\cdots$	
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+\cdots$	
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines	

These transformations are a nested set of groups

- Closed under composition and inverse is a member

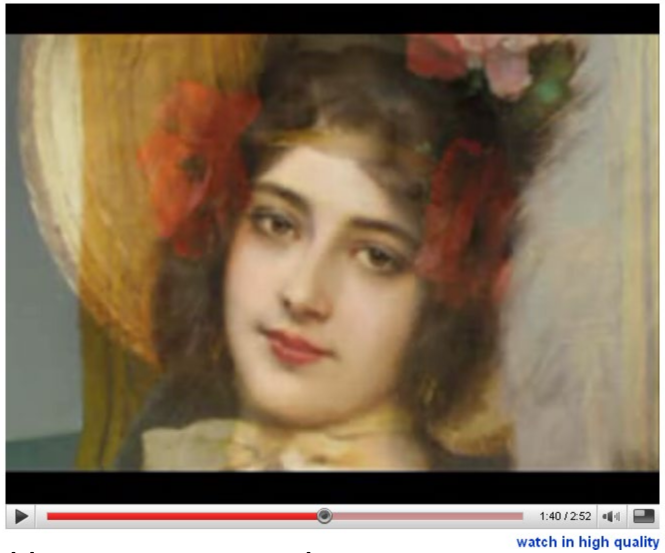
Take-home Question

Suppose we have two triangles: ABC and A’B’C’. What transformation will map A to A’, B to B’, and C to C’? How can we get the parameters?



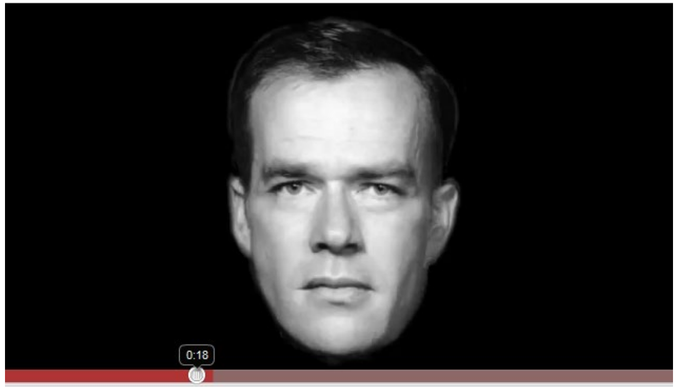
Today: Morphing

Women in art



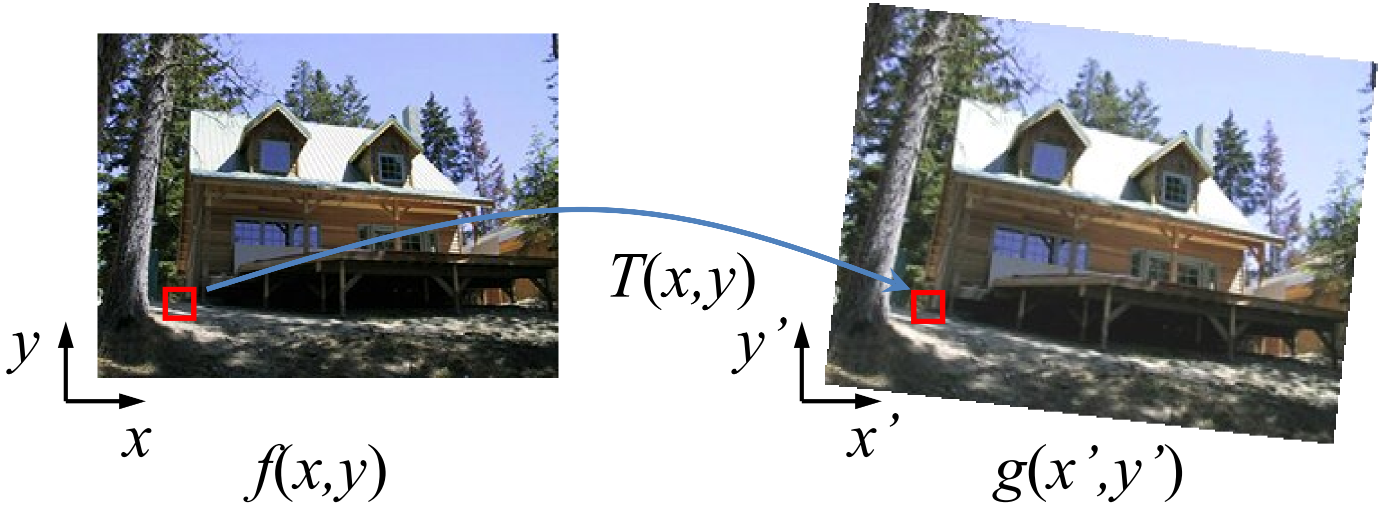
http://youtube.com/watch?v=nUDIoN-_Hxs

Aging



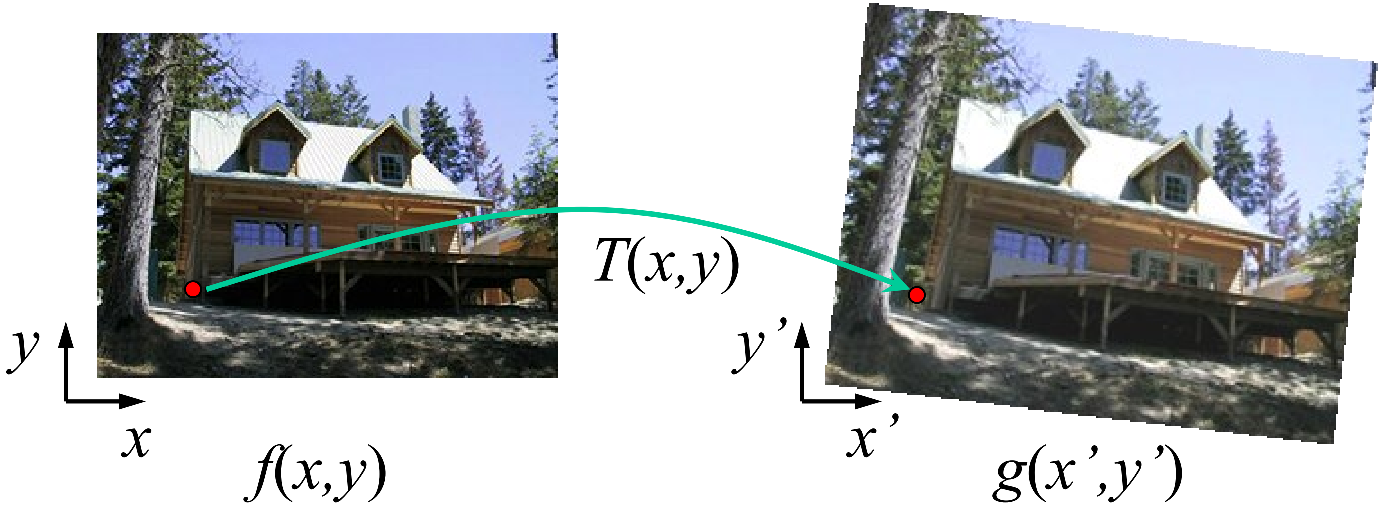
http://www.youtube.com/watch?v=L0GKp-uvjO0

Texturing in transformed coordinates



Given a coordinate transform *(x',y')* = *T*(*x,y*) and a source image *f*(*x,y*), how do we compute a transformed image *g(x',y')* = *f*(*T*(*x,y*))?

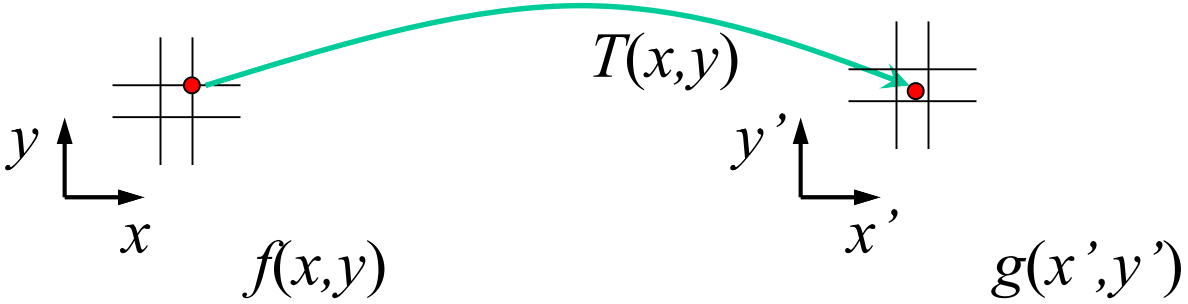
Forward mapping



Send each pixel $f(x,y)$ to its corresponding location $(x',y') = T(x,y)$ in the second image

Forward mapping

What is the problem with this approach?



Send each pixel $f(x,y)$ to its corresponding location

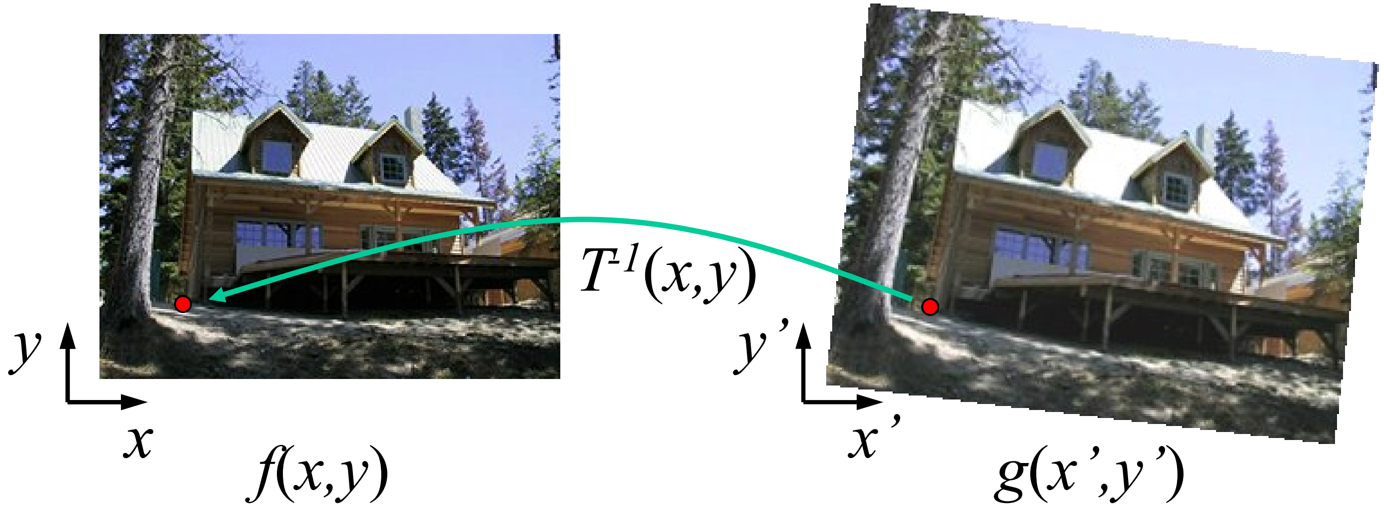
$(x',y') = T(x,y)$ in the second image

Q: what if pixel lands "between" two pixels?

A: distribute color among neighboring pixels (x',y')

- Known as "splatting"

Inverse mapping

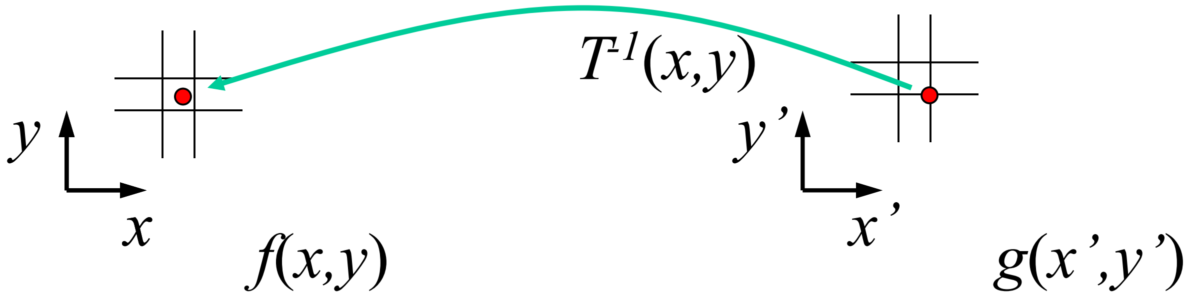


Get each pixel $g(x',y')$ from its corresponding location

$(x,y) = T^{-1}(x',y')$ in the first image

Q: what if pixel comes from "between" two pixels?

Inverse mapping



Get each pixel $g(x',y')$ from its corresponding location

$$(x,y) = T^{-1}(x',y') \text{ in the first image}$$

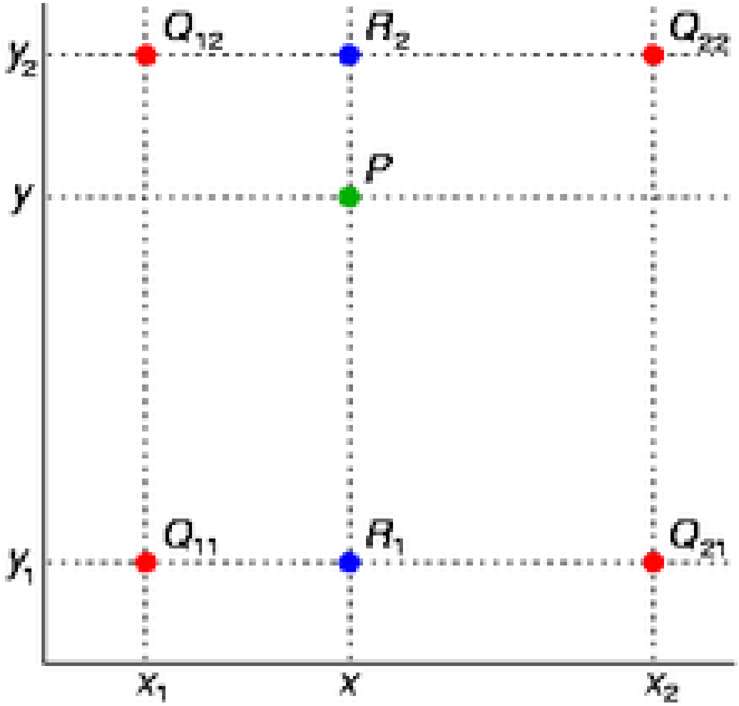
Q: what if pixel comes from “between” two pixels?

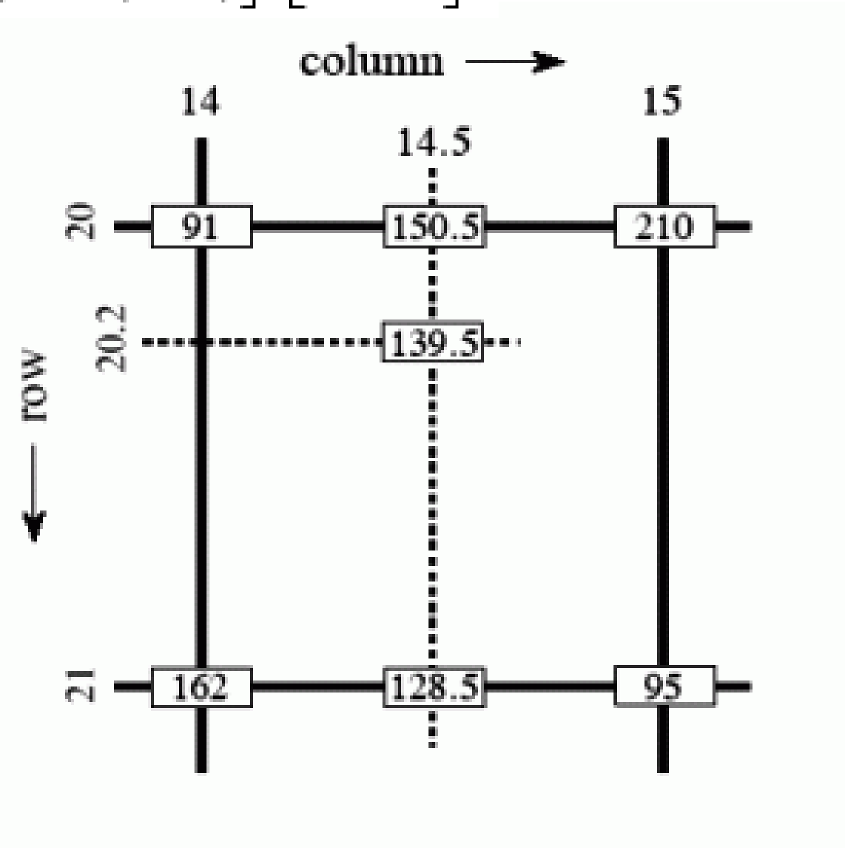
A: *Interpolate* color value from neighbors

- nearest neighbor, bilinear, Gaussian, bicubic
- E.g. `interpolate.interp2` or `ndimage.map_coordinates` in Python `scipy`

Bilinear Interpolation

$$f(x,y) \approx \begin{bmatrix}1-x & x\end{bmatrix} \begin{bmatrix}f(0,0) & f(0,1) \\ f(1,0) & f(1,1)\end{bmatrix} \begin{bmatrix}1-y \\ y\end{bmatrix}.$$





http://en.wikipedia.org/wiki/Bilinear_interpolation

Forward vs. inverse mapping

Q: which is better?

A: Usually inverse—eliminates holes

- however, it requires an invertible warp function

Morphing = Object Averaging

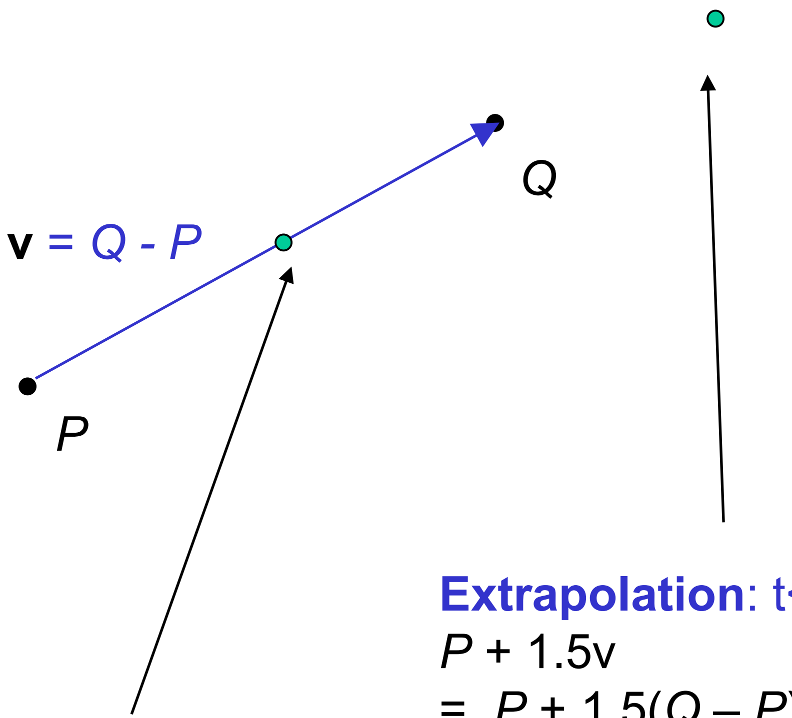
  

The aim is to find "an average" between two objects

- Not an average of two images of objects…
- …but an image of the average object!
- How can we make a smooth transition in time?
  - Do a "weighted average" over time t

Averaging Points

What's the average of P and Q?



Extrapolation: t<0 or t>1

$P + 1.5v$

$= P + 1.5(Q - P)$

$= -0.5P + 1.5\,Q$ $(t=1.5)$

Linear Interpolation

New point: $P + t^*(Q-P)$

Or equivalently: $(1-t)P + tQ$

$0<t<1$

$P + 0.5v$

$= P + 0.5(Q - P)$

$= 0.5P + 0.5\,Q$

P and Q can be anything:

- points on a plane (2D) or in space (3D)
- Colors in RGB (3D)
- Whole images (m-by-n D)… etc.

Idea #1: Cross-Dissolve

Interpolate whole images:

$Image_{halfway} = (1\text{-}t)^*Image_1 + t^*image_2$

This is called **cross-dissolve** in film industry

But what if the images are not aligned?

Idea #2: Align, then cross-disolve

Align first, then cross-dissolve

- Alignment using global warp – picture still valid

Image credit: Efros

Dog Averaging

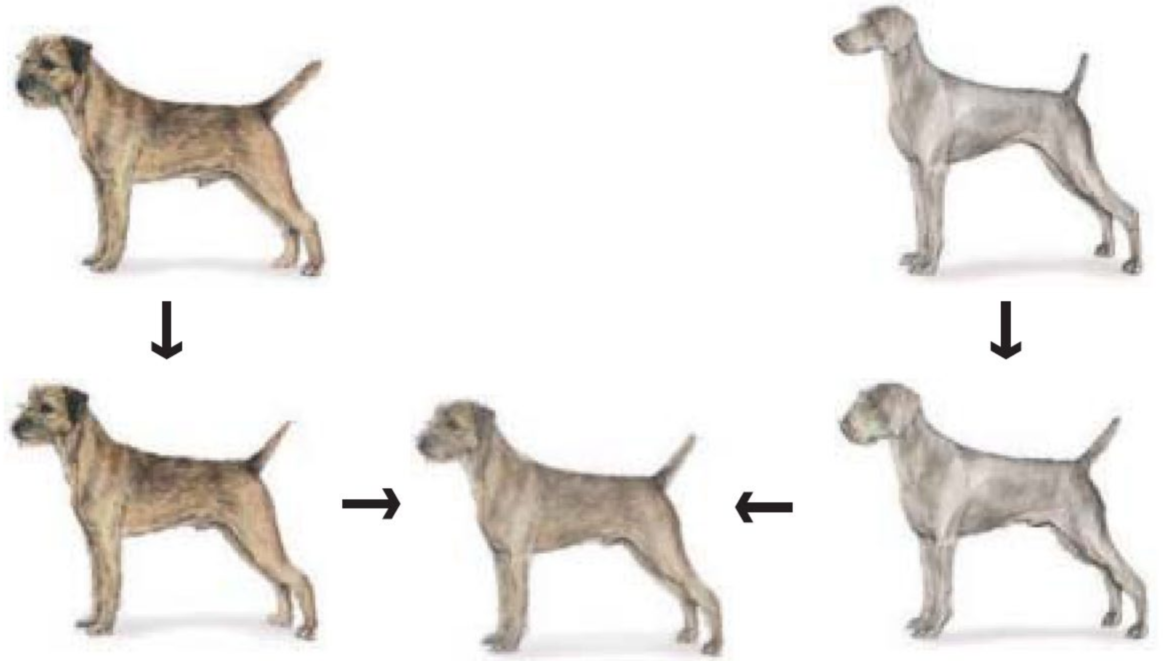
What to do?

- Cross-dissolve doesn’t work
- Global alignment doesn’t work
  - Cannot be done with a global transformation (e.g. affine)
- Any ideas?

Feature matching!

- Nose to nose, tail to tail, etc.
- This is a local (non-parametric) warp

Idea #3: Local warp, then cross-dissolve



Morphing procedure

For every frame t,

1. Find the average shape (the “mean dog”☺)
   - local warping
2. Find the average color
   - Cross-dissolve the warped images

Local (non-parametric) Image Warping

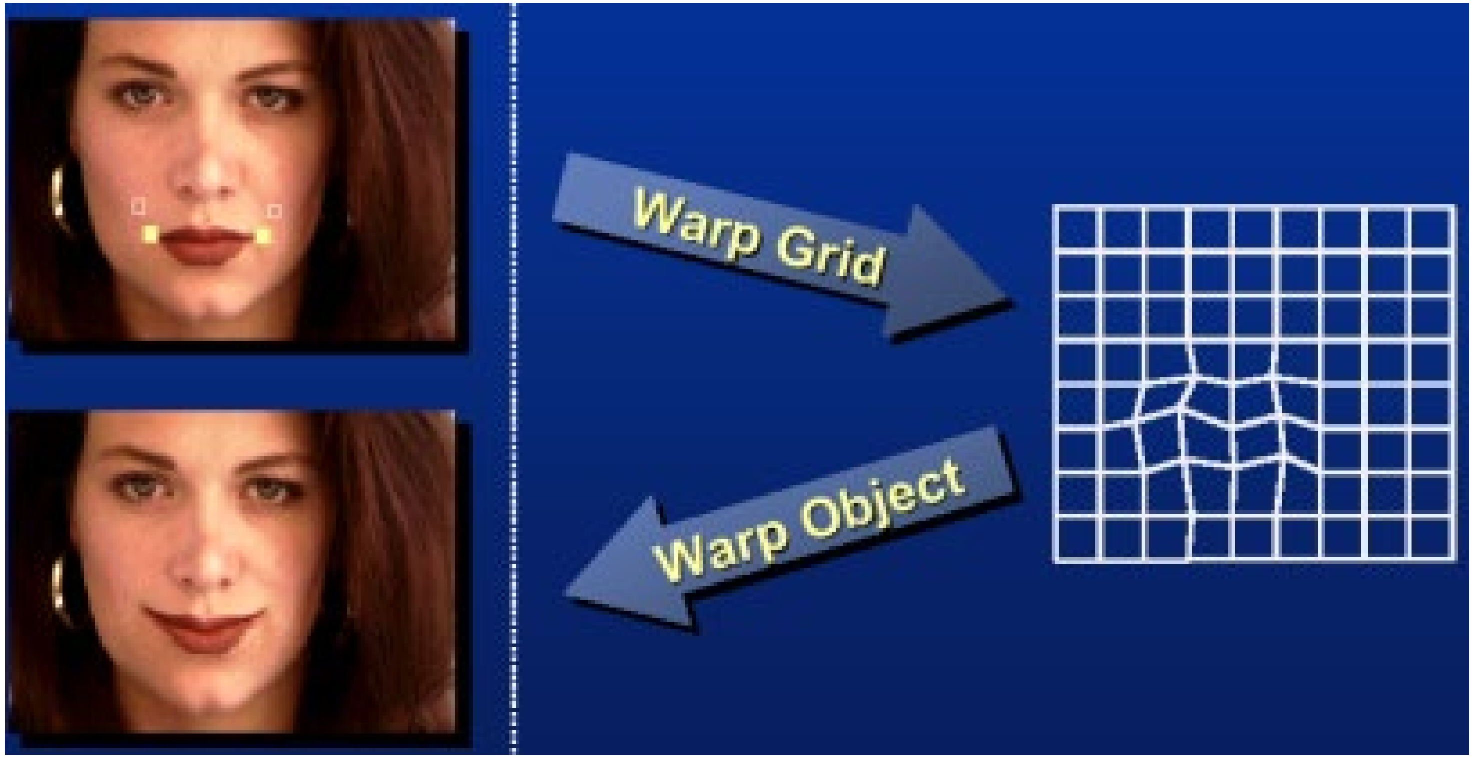
Need to specify a more detailed warp function

- Global warps were functions of a few (2,4,8) parameters
- Non-parametric warps u(x,y) and v(x,y) can be defined independently for every single location x,y!
- Once we know vector field u,v we can easily warp each pixel (use backward warping with interpolation)

Image Warping – non-parametric

Move control points to specify a spline warp

Spline produces a smooth vector field

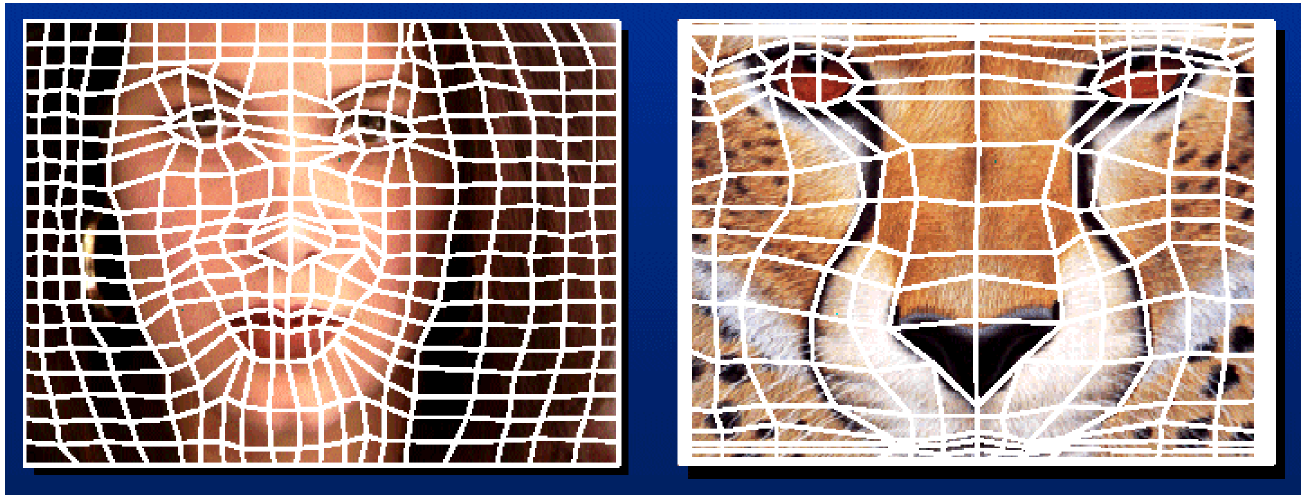


Warp specification - dense

How can we specify the warp?

Specify corresponding *spline control points*

- *interpolate* to a complete warping function



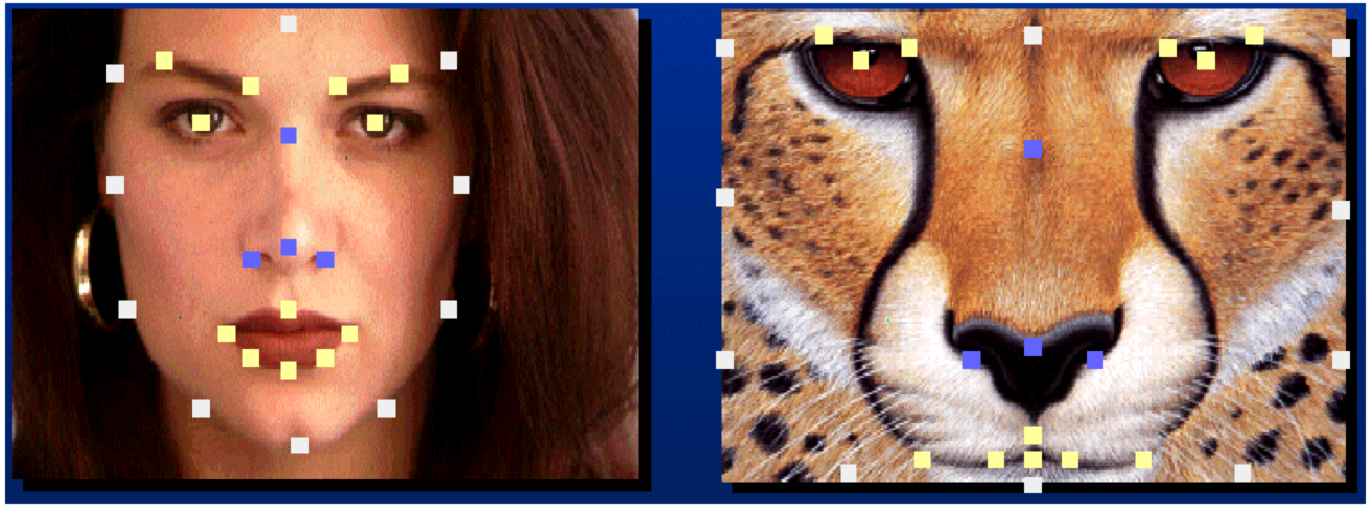
But we want to specify only a few points, not a grid

Warp specification - sparse

How can we specify the warp?

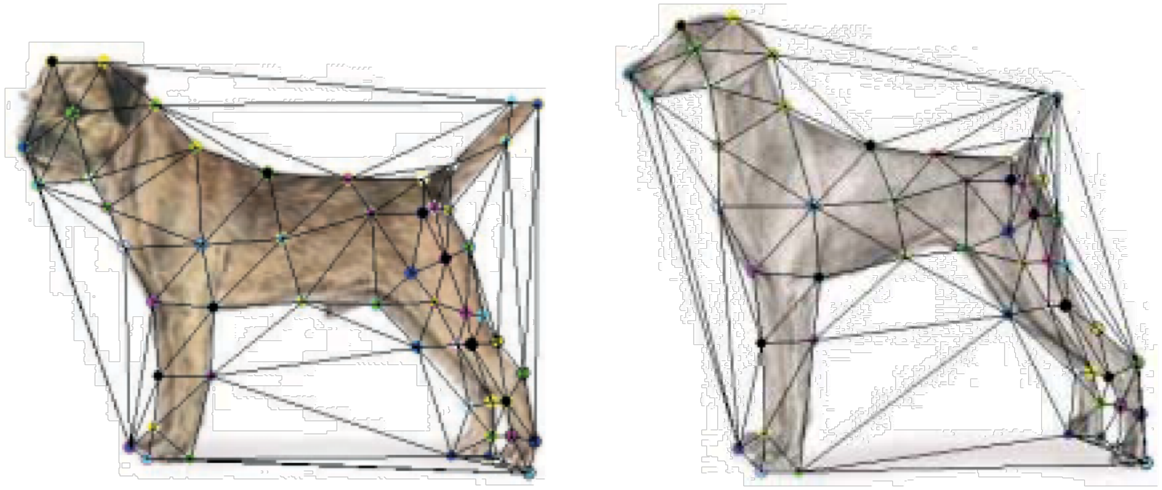
Specify corresponding *points*

- *interpolate* to a complete warping function
- How do we do it?



How do we go from feature points to pixels?

Triangular Mesh

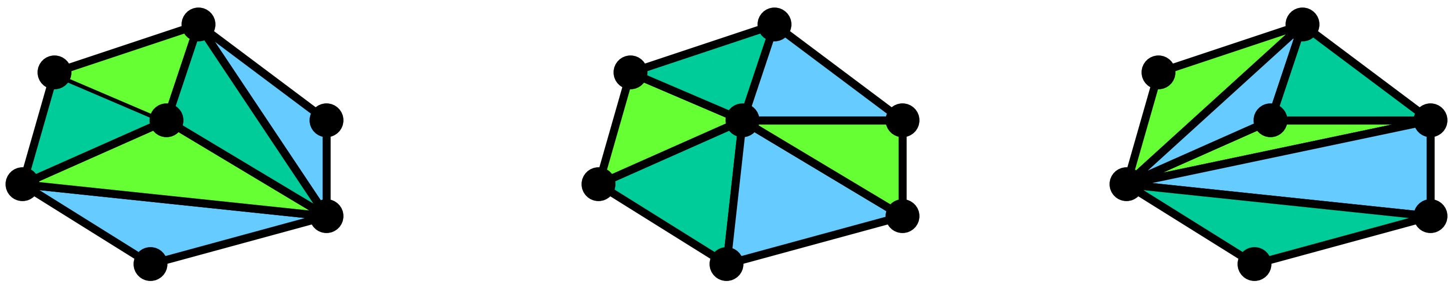


1. Input correspondences at key feature points
2. Define a triangular mesh over the points
   - Same mesh (triangulation) in both images!
   - Now we have triangle-to-triangle correspondences
3. Warp each triangle separately from source to destination
   - Affine warp with three corresponding points (just like take-home question)

Triangulations

A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

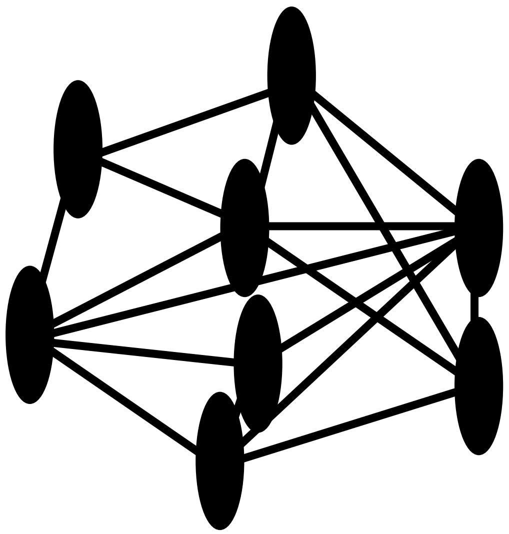
There are an exponential number of triangulations of a point set.



An O(n^3) Triangulation Algorithm

Repeat until impossible:

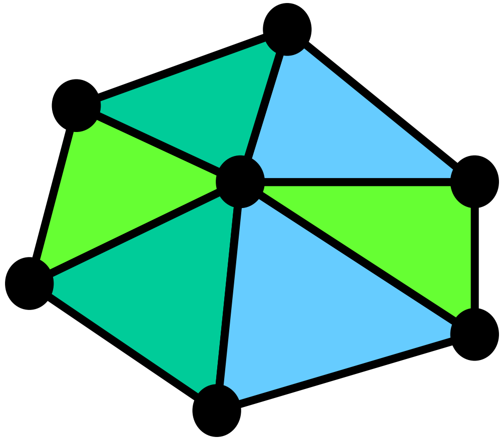
- Select two sites.
- If the edge connecting them does not intersect previous edges, keep it.



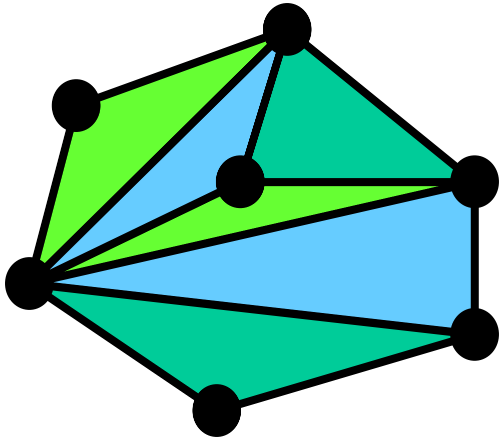
"Quality" Triangulations

Let $\alpha(T_i) = (\alpha_{i1}, \alpha_{i2}, .., \alpha_{i3})$ be the vector of angles in the triangulation T in increasing order:

- A triangulation T_1 is "better" than T_2 if the smallest angle of T_1 is larger than the smallest angle of T_2
- Delaunay triangulation is the "best" (maximizes the smallest angles)



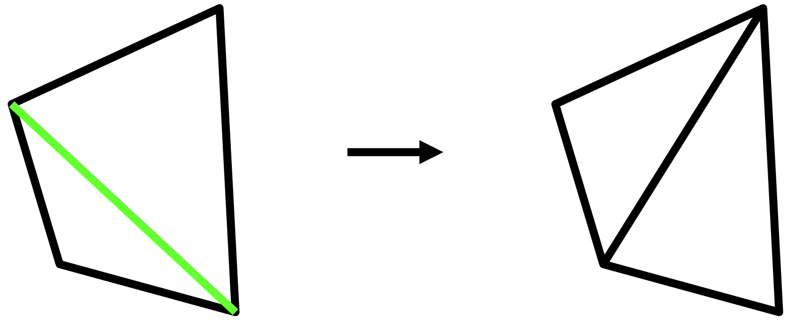
good



bad

Improving a Triangulation

In any convex quadrangle, an *edge flip* is possible.
If this flip *improves* the triangulation locally, it also improves the global triangulation.

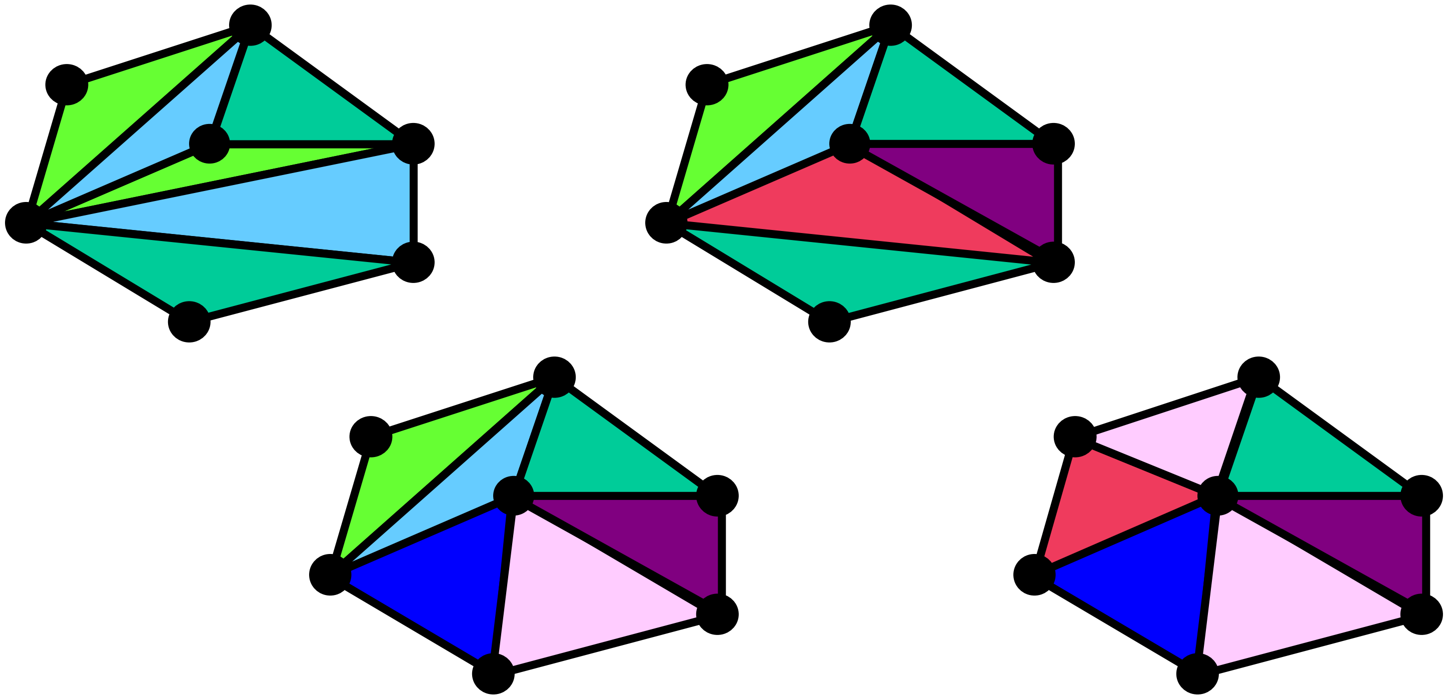


If an edge flip improves the triangulation, the first edge is called "*illegal*".

Naïve Delaunay Algorithm

Start with an arbitrary triangulation. Flip any illegal edge until no more exist.

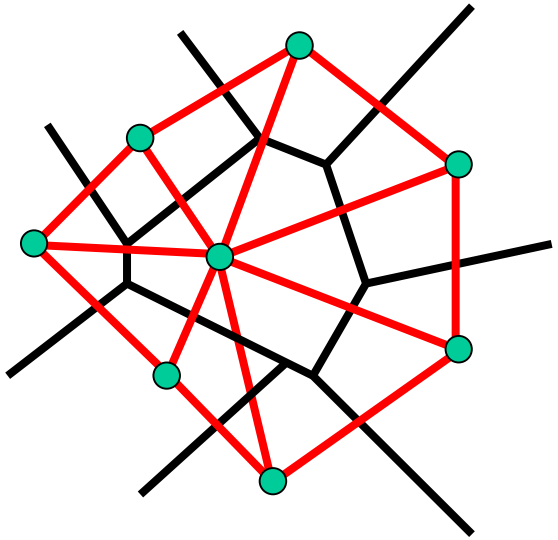
Could take a long time to terminate.



Delaunay Triangulation by Duality

Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.

- The DT may be constructed in O(*n*log*n*) time

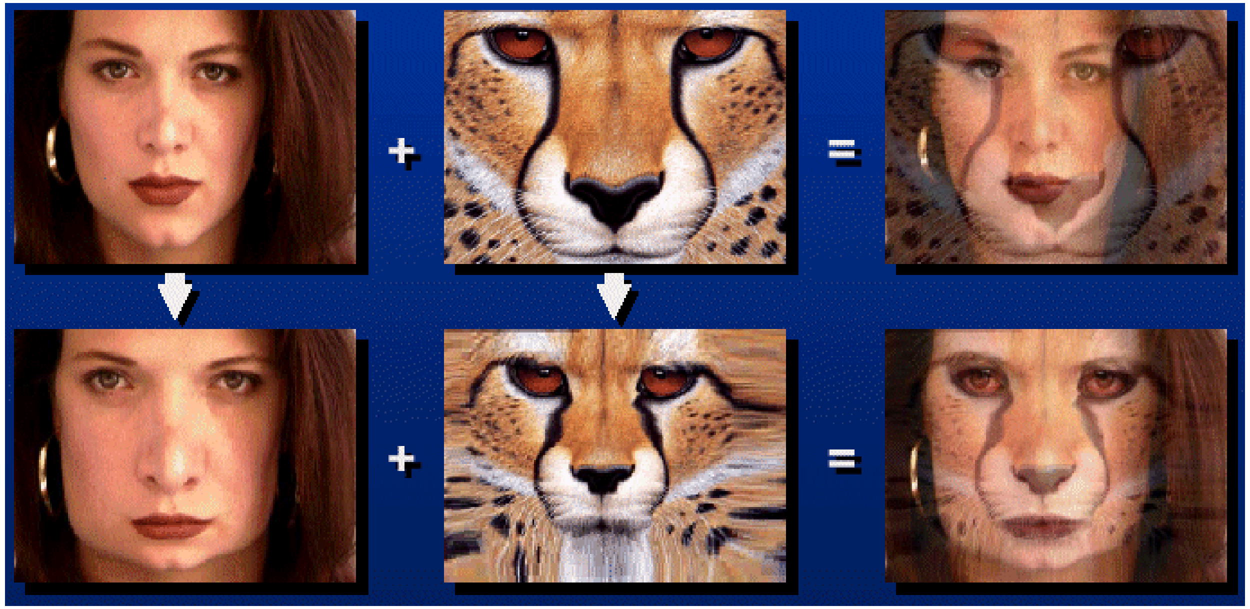


Demos: http://www.cs.cornell.edu/home/chew/Delaunay.html
http://alexbeutel.com/webgl/voronoi.html

Image Morphing

How do we create a morphing sequence?

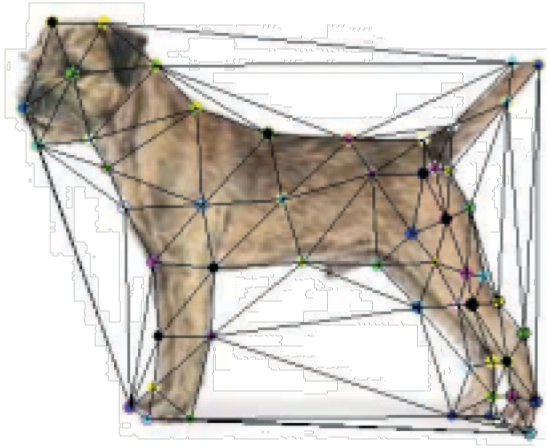
1. Create an intermediate shape (by interpolation)
2. Warp both images towards it
3. Cross-dissolve the colors in the newly warped images

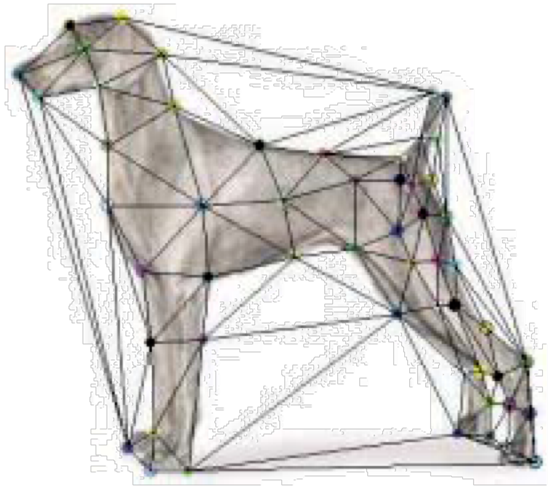


Warp interpolation

How do we create an intermediate shape at time t?

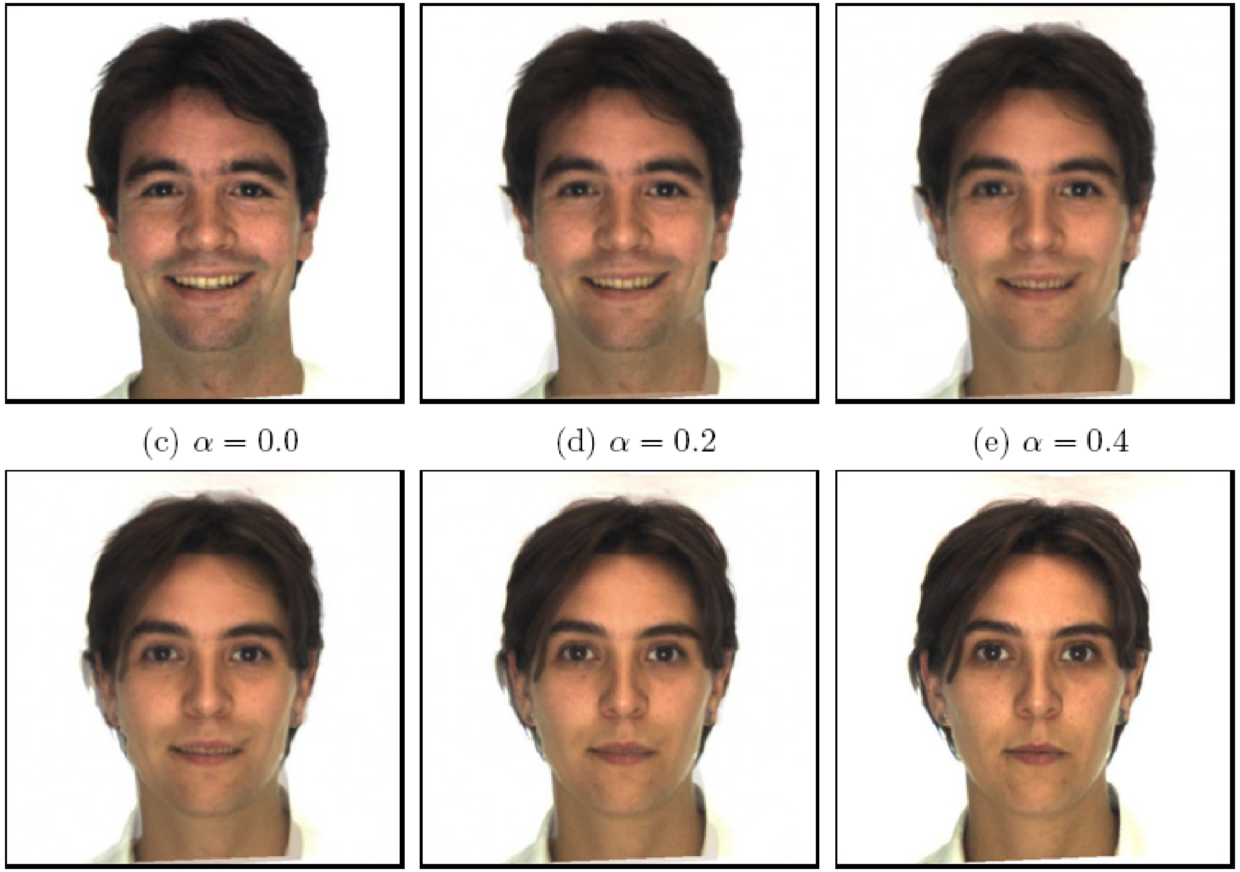
- Assume t = [0,1]
- Simple linear interpolation of each feature pair
  - (1-t)*p1+t*p0 for corresponding features p0 and p1





Morphing & matting

Extract foreground first to avoid artifacts in the background

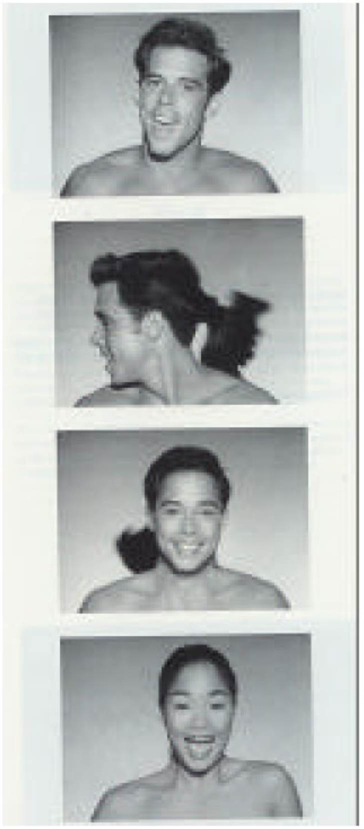


(c) $\alpha = 0.0$ (d) $\alpha = 0.2$ (e) $\alpha = 0.4$

(f) $\alpha = 0.6$ (g) $\alpha = 0.8$ (h) $\alpha = 1.0$

Slide by Durand and Freeman

Dynamic Scene



Black or White (MJ):
http://www.youtube.com/watch?v=R4kLKv5gtxc

Willow morph: http://www.youtube.com/watch?v=uLUyuWo3pG0

Summary of morphing

1. Define corresponding points
2. Define triangulation on points
   - Use same triangulation for both images
3. For each t = 0:step:1
   a. Compute the average shape (weighted average of points)
   b. For each triangle in the average shape
      - Get the affine projection to the corresponding triangles in each image
      - For each pixel in the triangle, find the corresponding points in each image and set value to weighted average (optionally use interpolation)

   c. Save the image as the next frame of the sequence

Next classes

Pinhole camera: start of perspective geometry

Single-view metrology: measure 3D distances from an image