

Single-view Metrology and Cameras

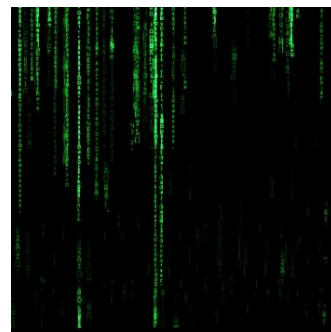
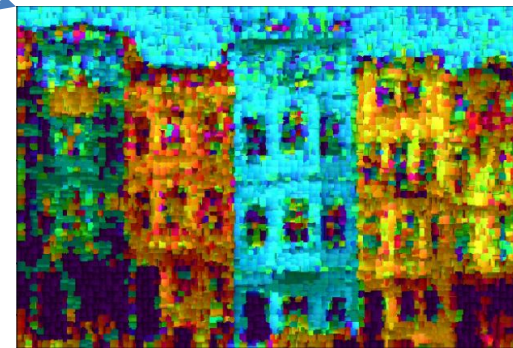


Computational Photography
Derek Hoiem, University of Illinois

Project 2 Results

- Incomplete list of great project pages/results

- Samuel Guo →
- Kshitij Gupta →
- Ethan Lord
- Charan Sakaran
- Yicheng Sun
- Jaewook Yeom
- Aayush Raj →
- Ananya Yammanuru →
- Ching-Hua Yu (stop sign hole fill) →
- Kush Gupta
- Lehan Yao →





Yicheng Sun

Jaewook Yeom

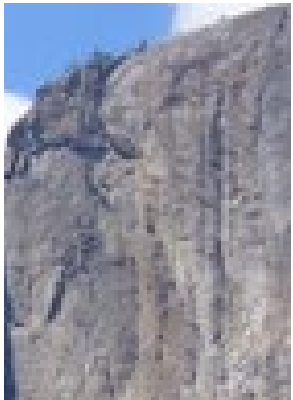
Orig



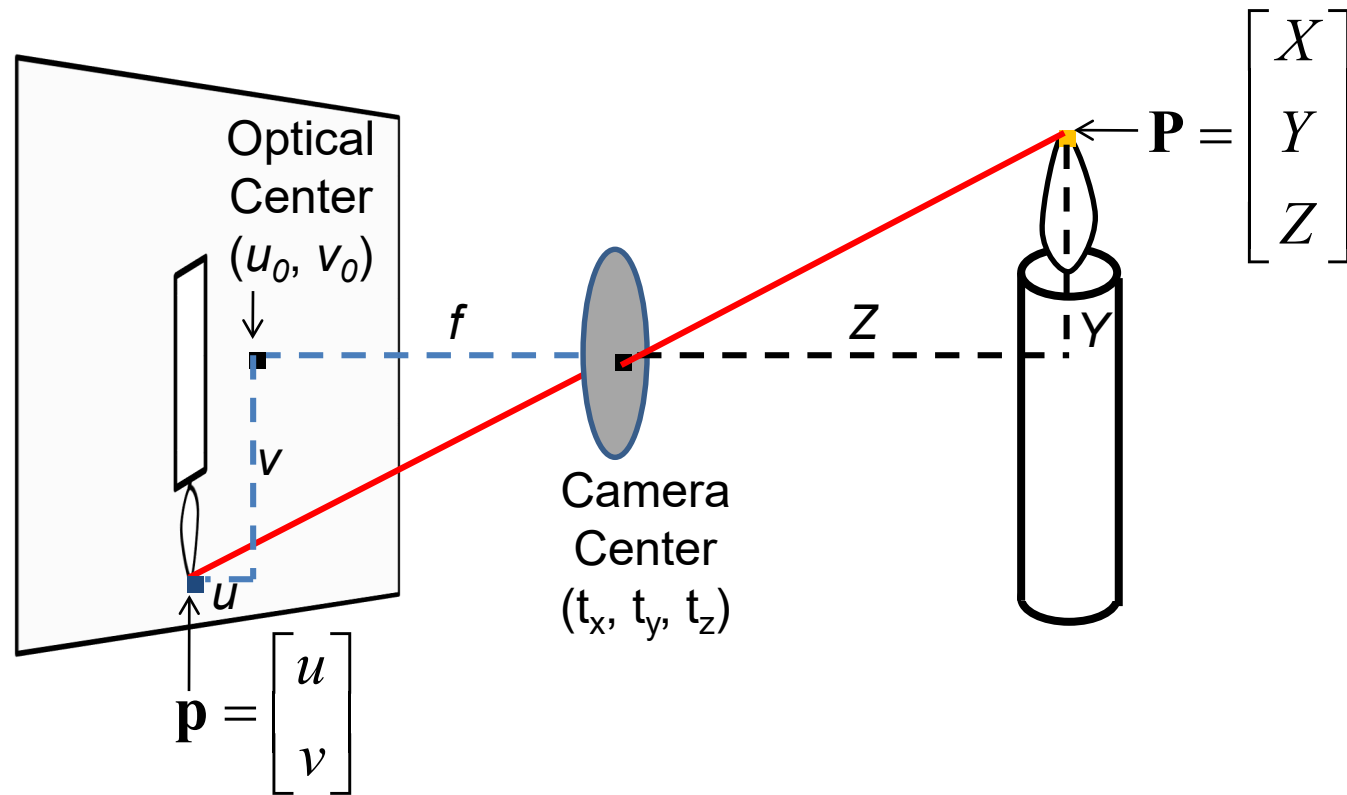
Hole



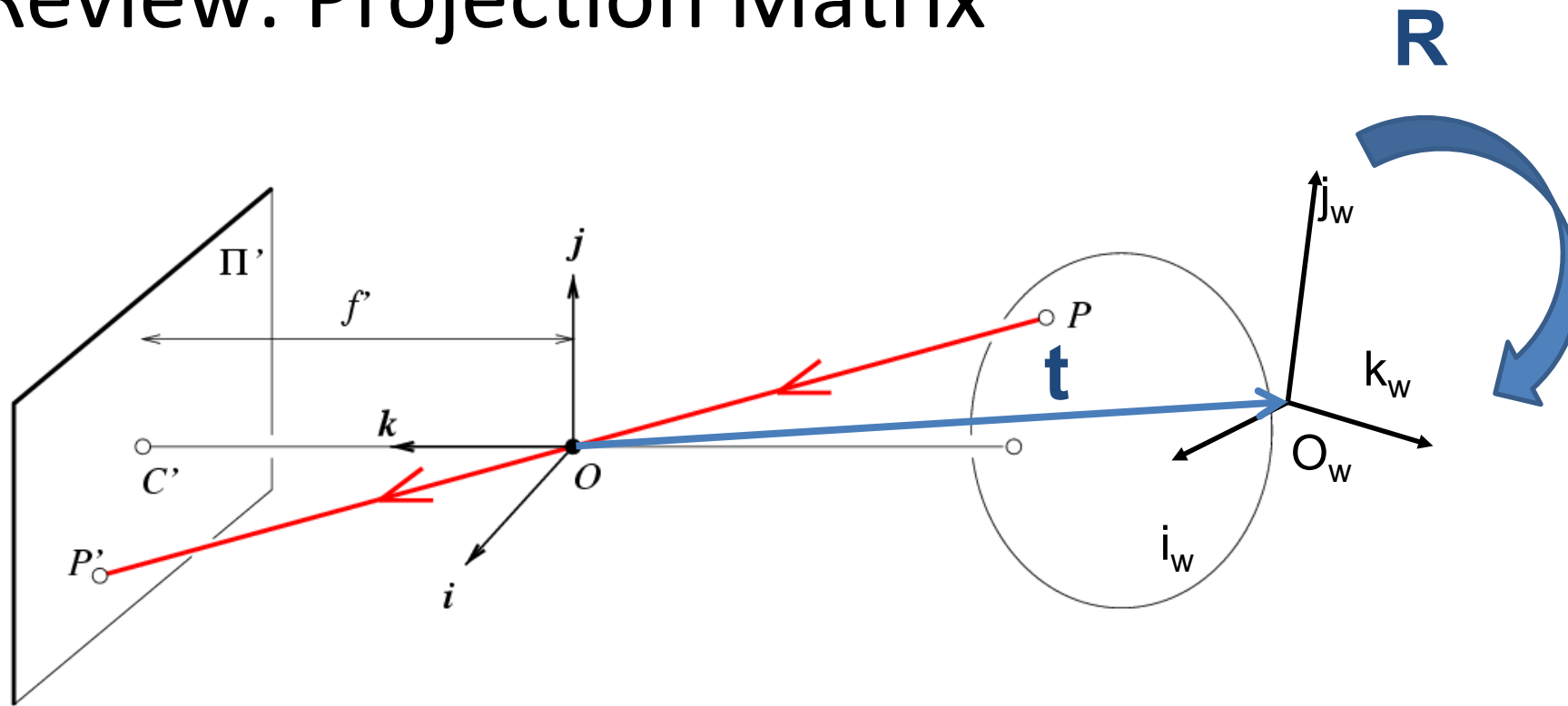
Filled



Review: Pinhole Camera



Review: Projection Matrix



$$\mathbf{x} = \mathbf{K}[\mathbf{R} \quad \mathbf{t}] \mathbf{X} \rightarrow_w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f & s & u_0 \\ 0 & \alpha f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Take-home question review

- Suppose the camera axis is in the direction of $(x=0, y=0, z=1)$ in its own coordinate system. What is the camera axis in world coordinates given the extrinsic parameters \mathbf{R}, \mathbf{t}
- Suppose a camera at height $y=h$ ($x=0, z=0$) observes a point at (u, v) known to be on the ground ($y=0$). Assume R is identity. What is the 3D position of the point in terms of f, u_0, v_0 ?

Take-home question review

Suppose we have two 3D cubes on the ground facing the viewer, one near, one far.

1. What would they look like in perspective?
2. What would they look like in weak perspective?



Review: Vanishing Points



This class

- How can we calibrate the camera?
- How can we measure the size of objects in the world from an image?
- What about other camera properties: focal length, field of view, depth of field, aperture, f-number?
- How to do “focus stacking” to get a sharp picture of a nearby object
- How the “vertigo effect” works

How to calibrate the camera?

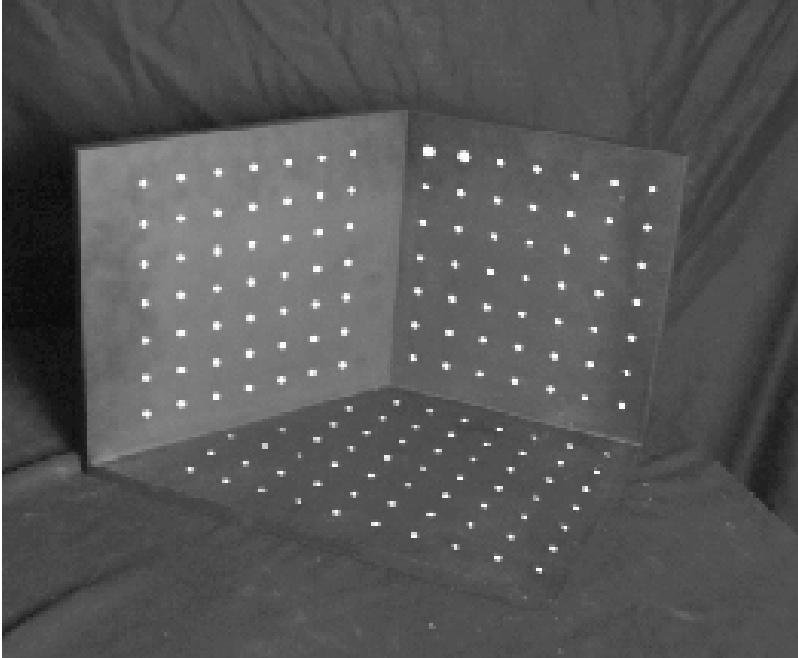
$$\mathbf{x} = \mathbf{K} \begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix} \mathbf{X}$$

$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Method 1: Use an object (calibration grid) with known geometry

- Correspond image points to 3d points
- Get least squares solution (or non-linear solution)

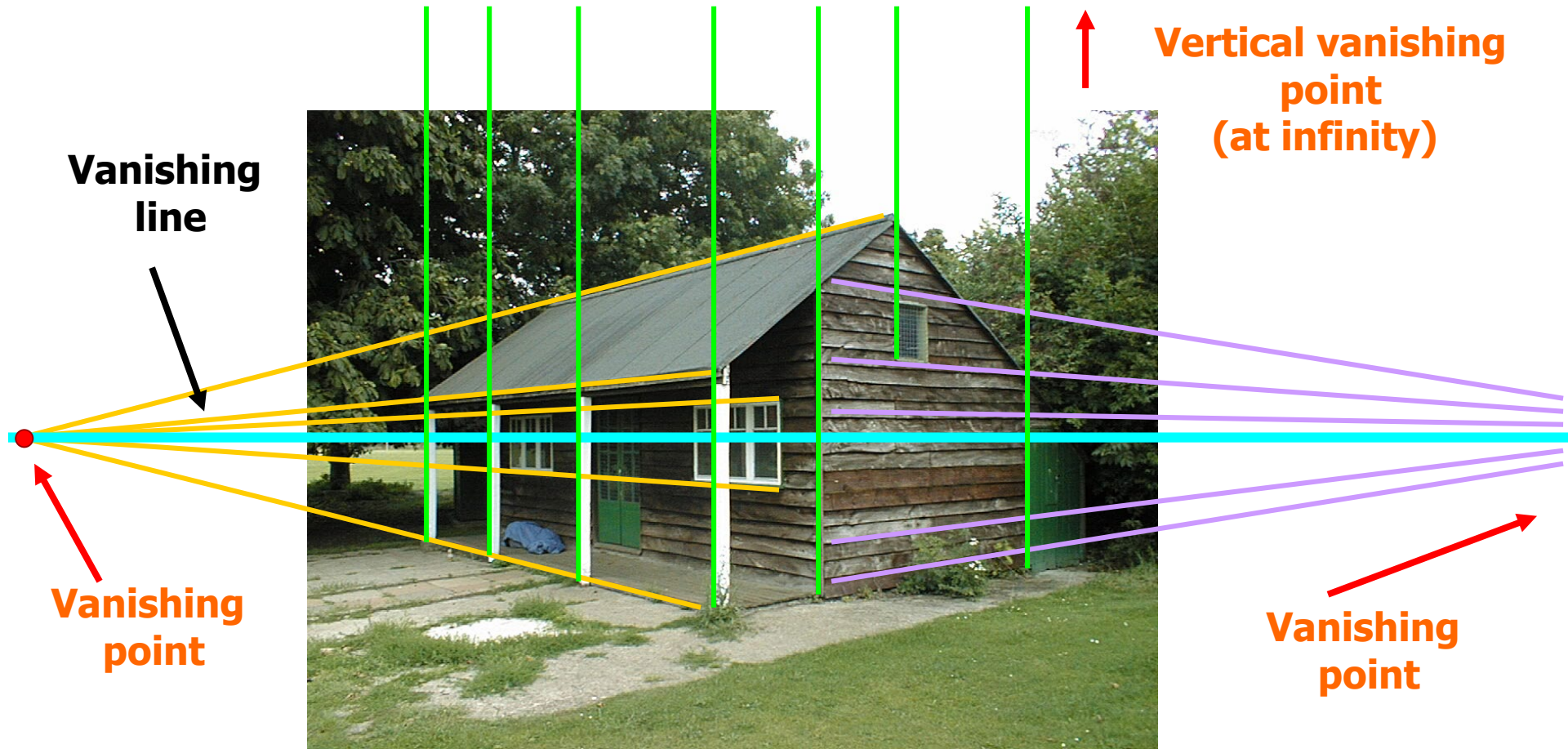


$$\begin{bmatrix} wu \\ wv \\ w \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Calibrating the Camera

Method 2: Use vanishing points

- Find vanishing points corresponding to orthogonal directions



Take-home question (for later)

Suppose you have estimated finite three vanishing points corresponding to orthogonal directions:

- 1) How to solve for intrinsic matrix? (assume K has three parameters)
 - The transpose of the rotation matrix is its inverse
 - Use the fact that the 3D directions are orthogonal
- 2) How to recover the rotation matrix that is aligned with the 3D axes defined by these points?
 - In homogeneous coordinates, 3d point at infinity is $(X, Y, Z, 0)$

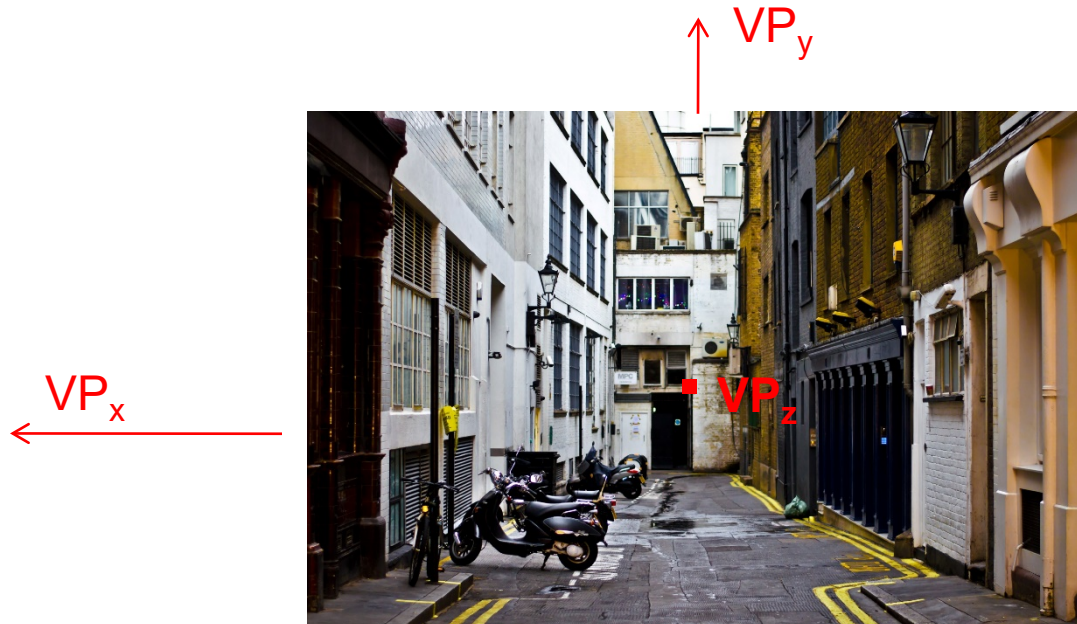
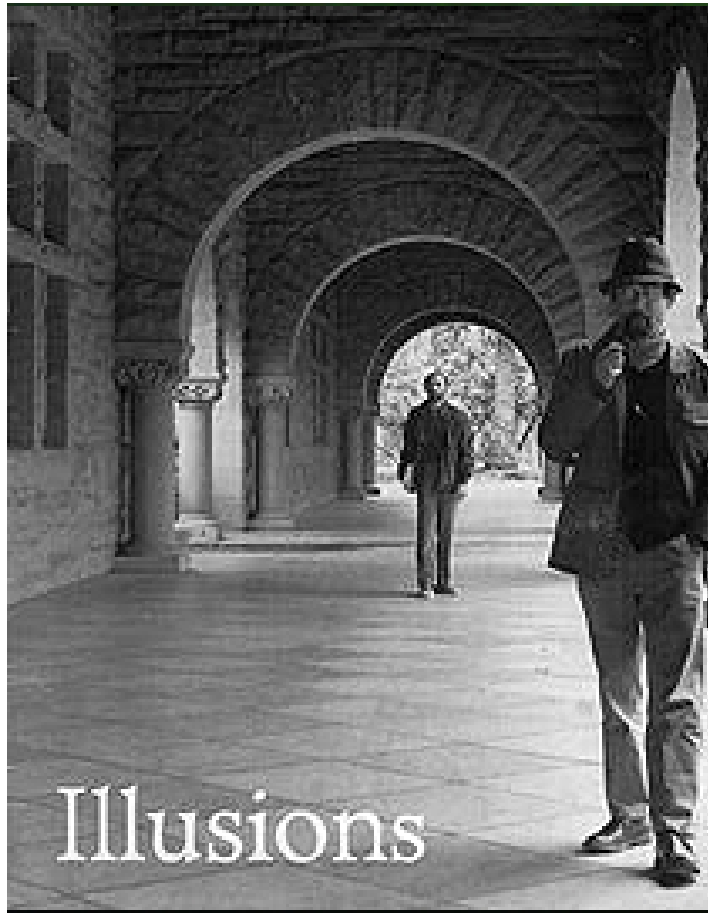
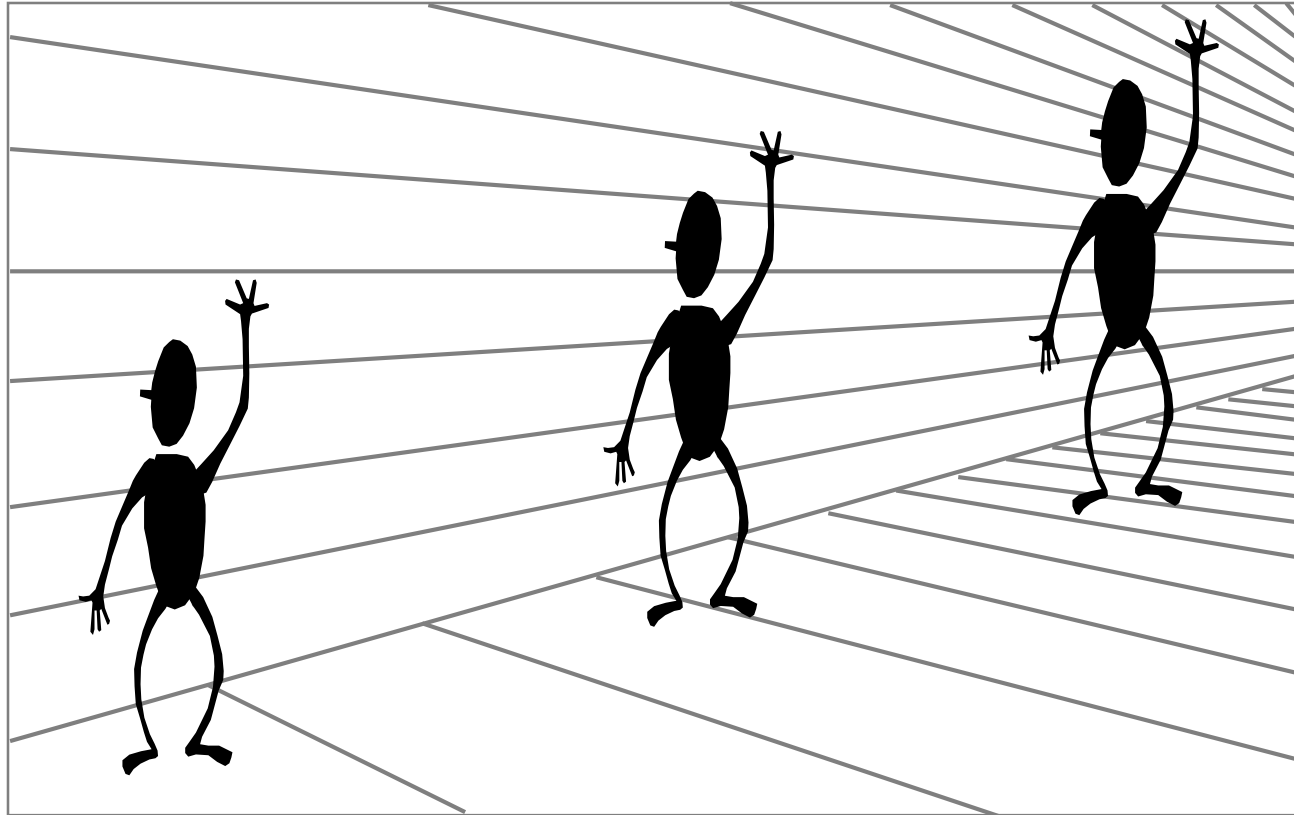


Photo from Garry Knight

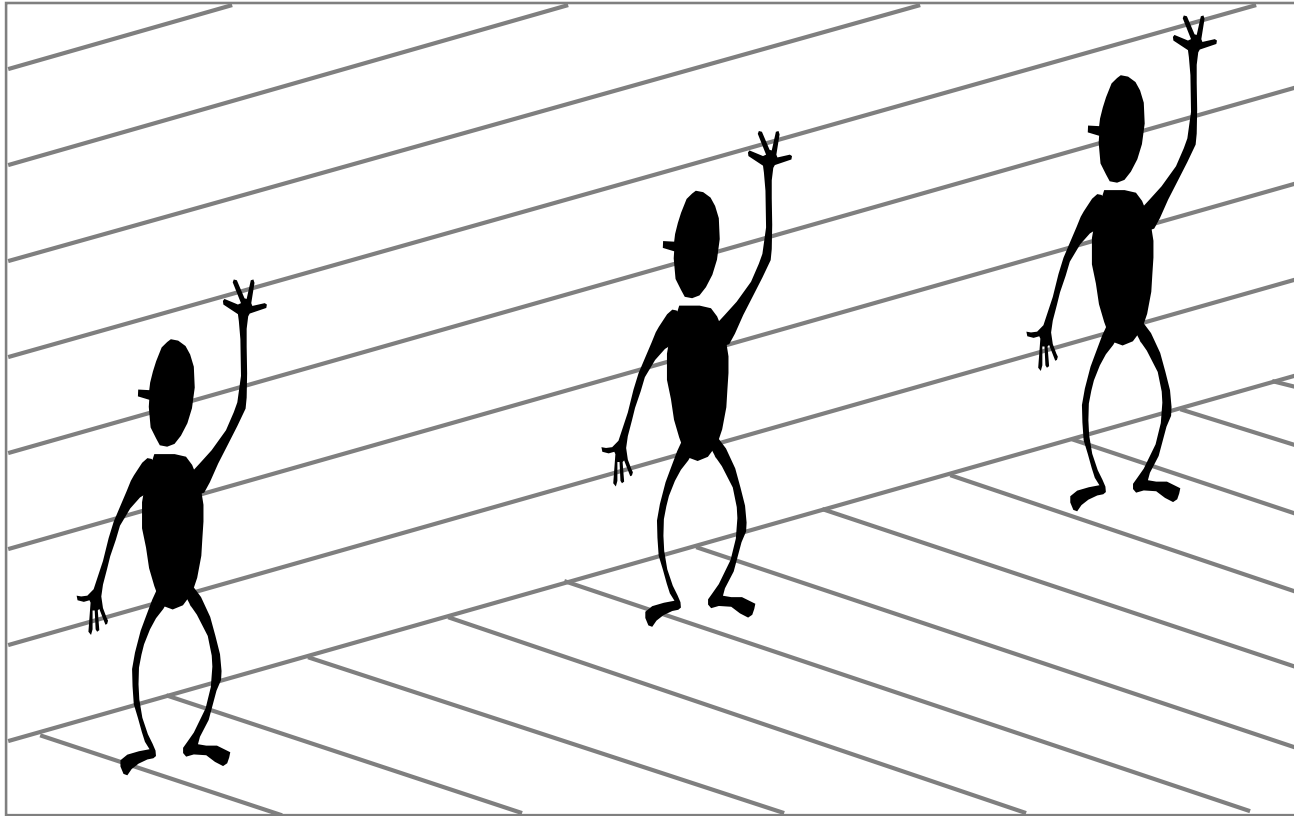
How can we measure the size of 3D objects from an image?



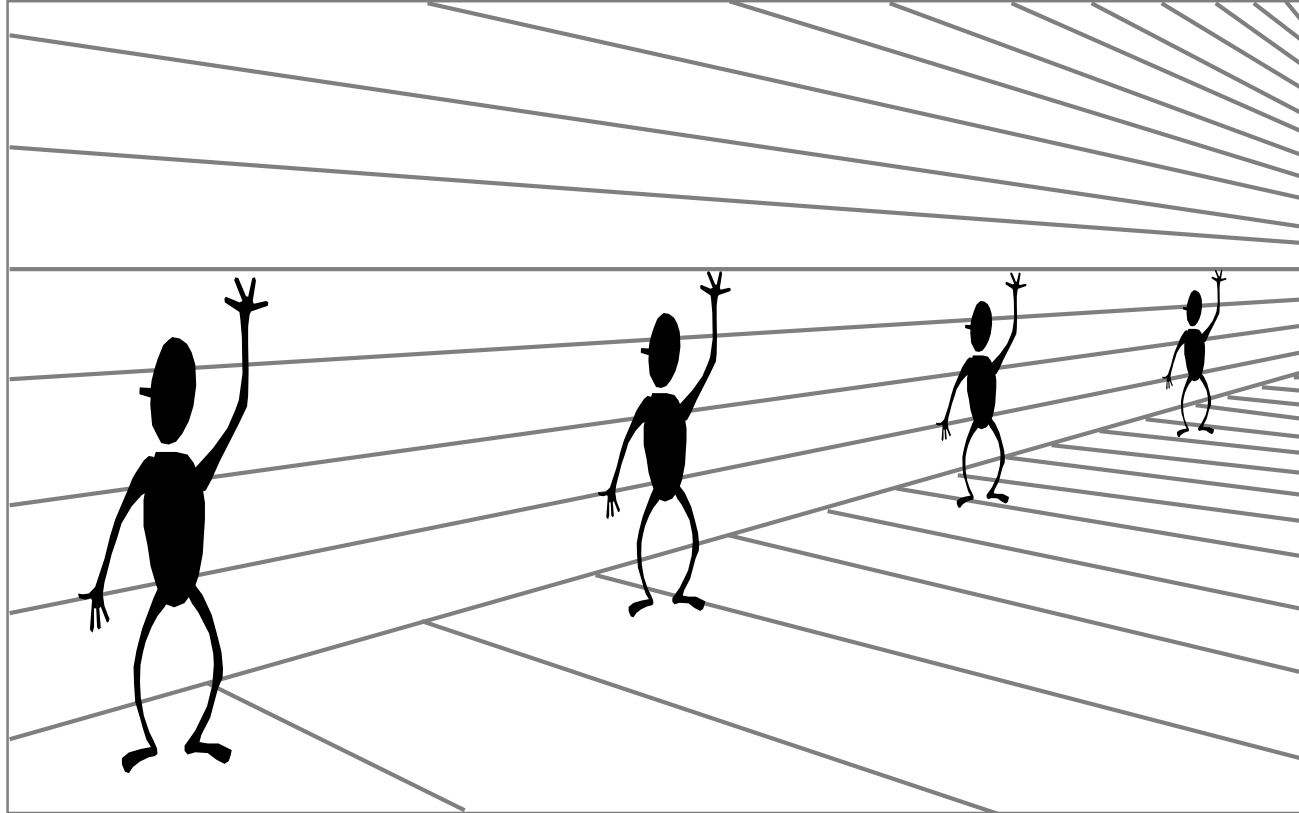
Perspective cues



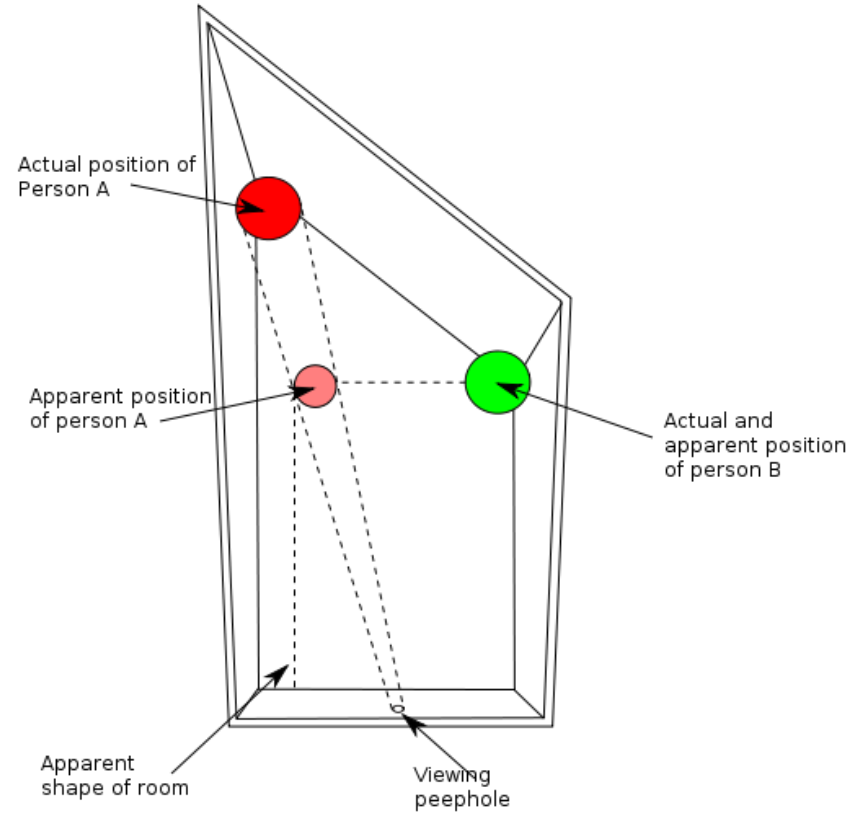
Perspective cues



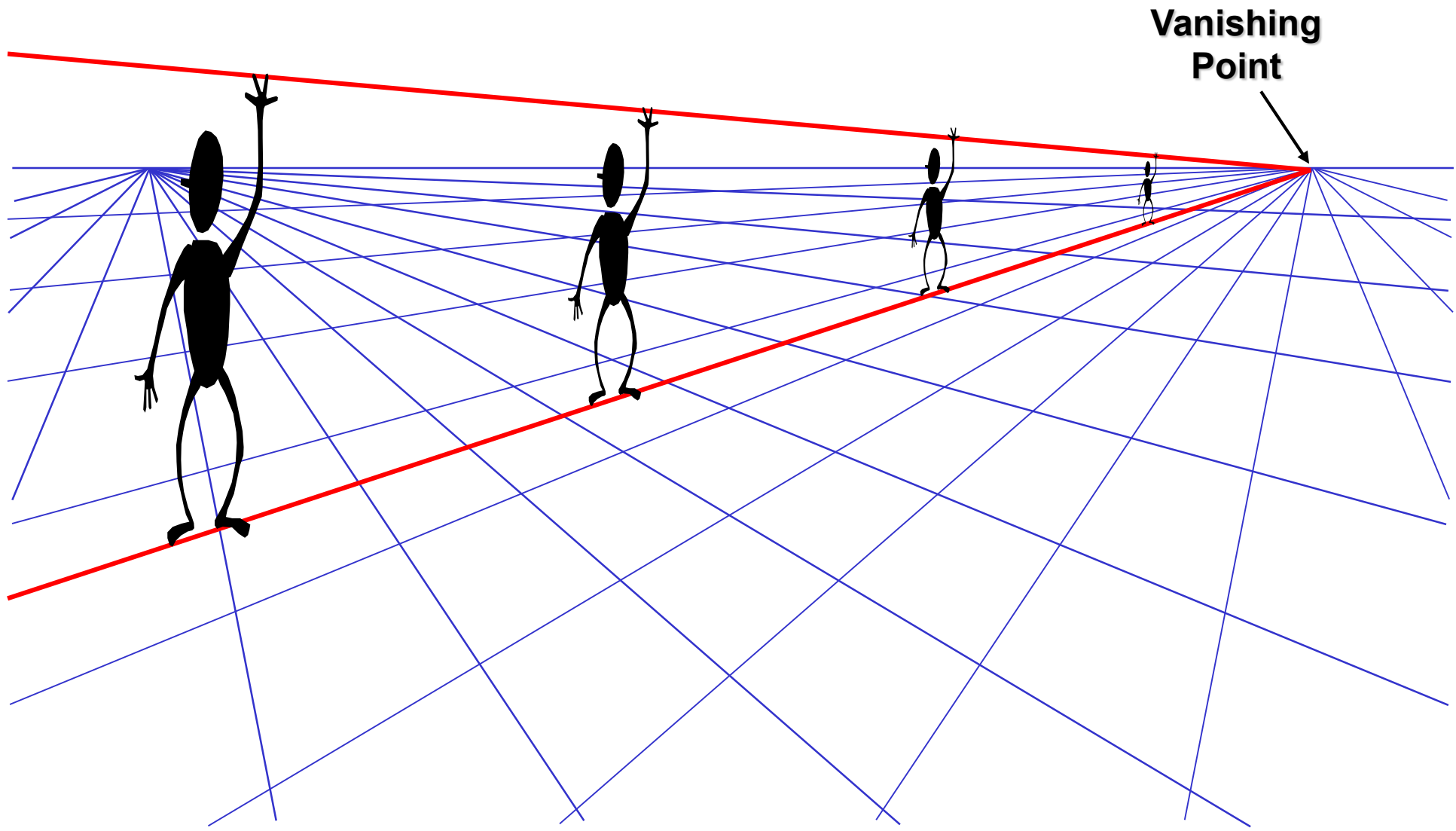
Perspective cues



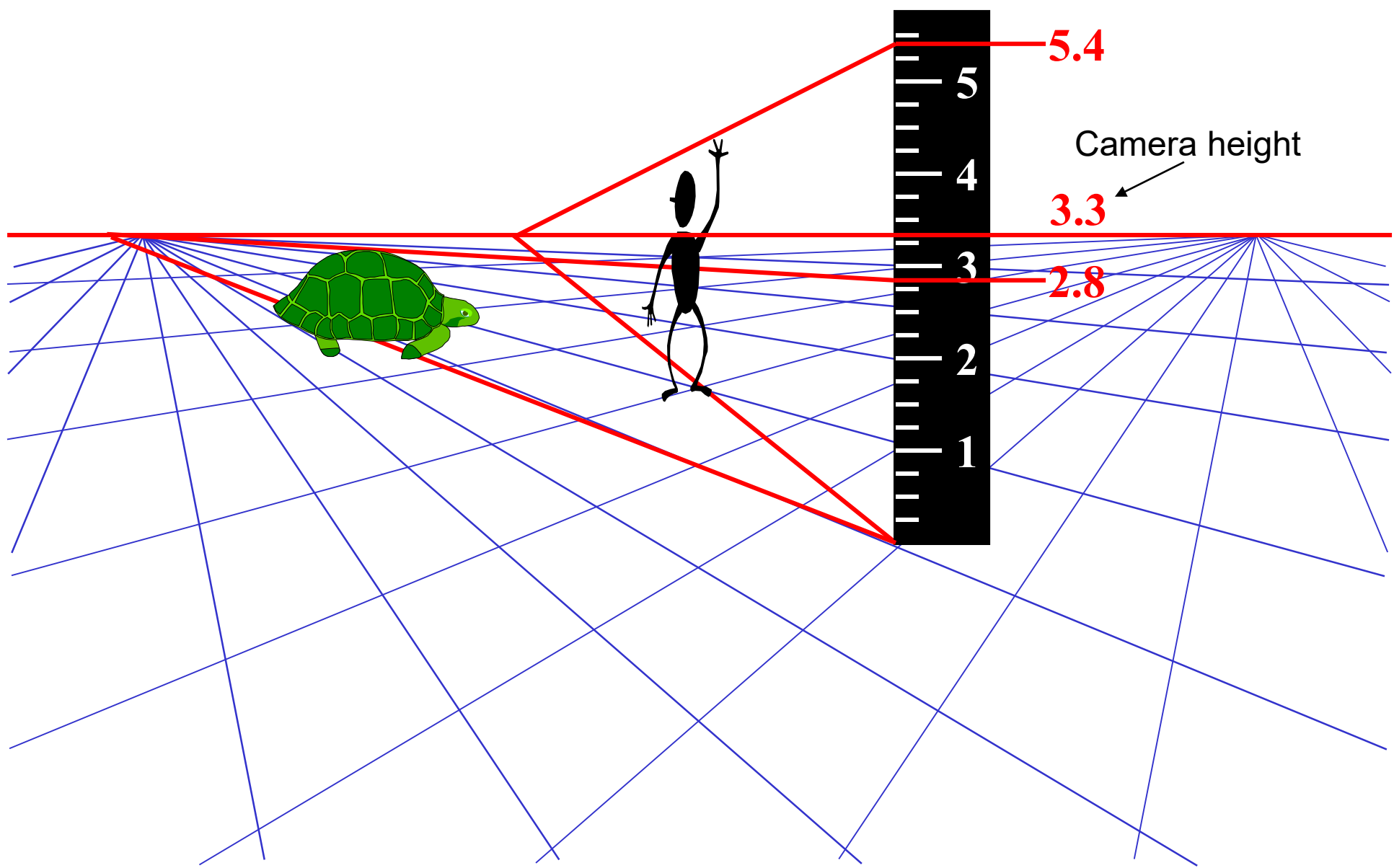
Ames Room



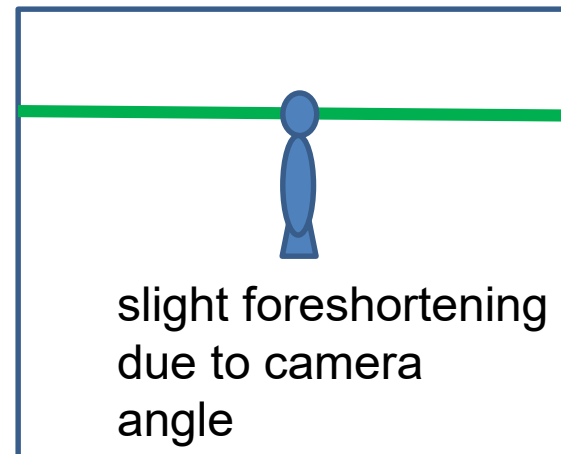
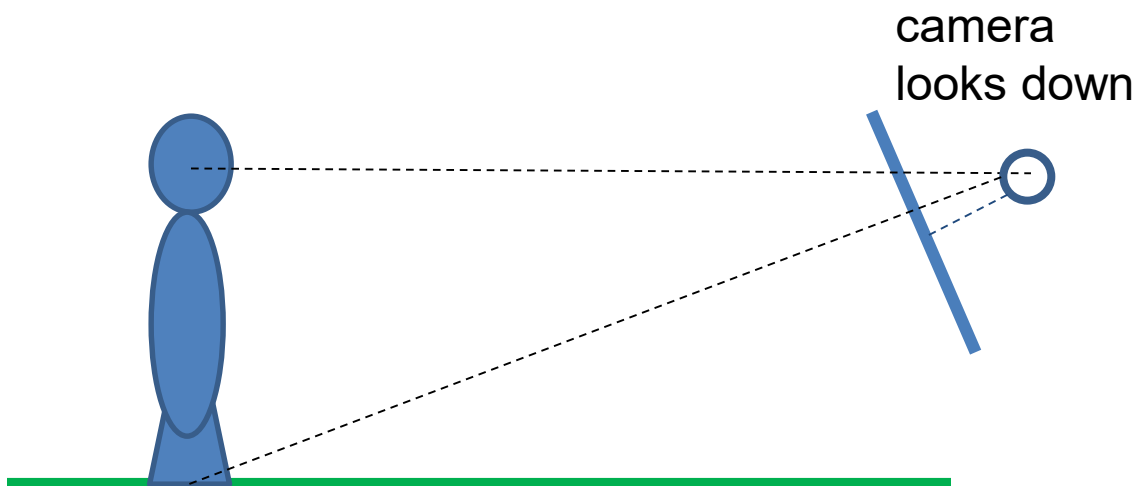
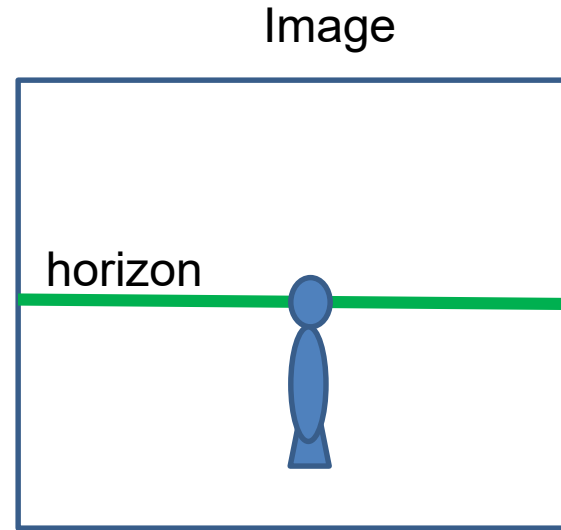
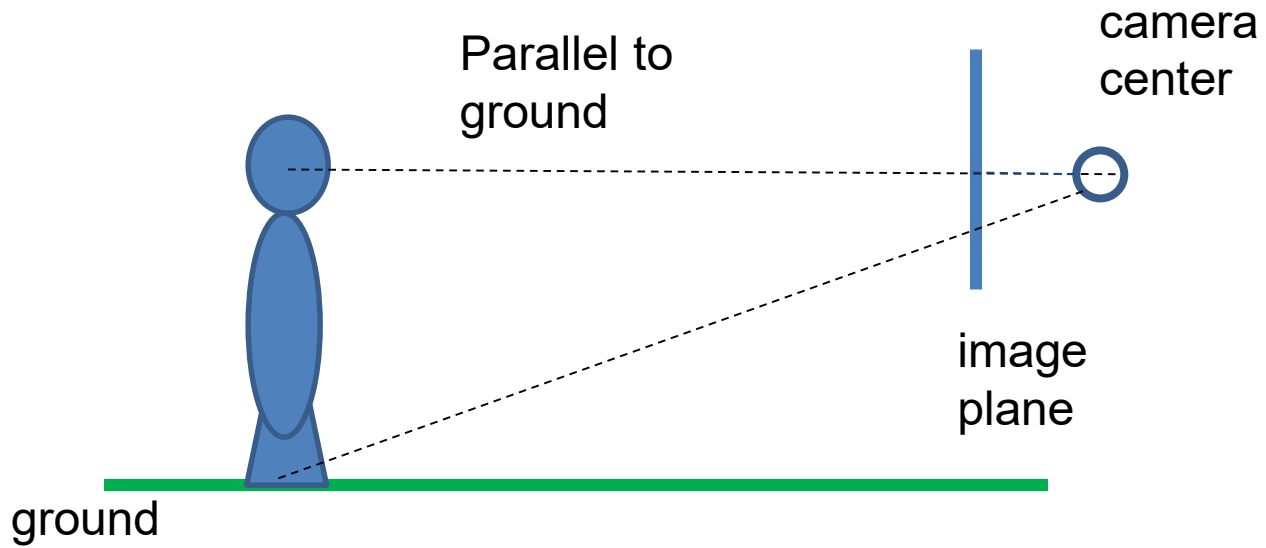
Comparing heights



Measuring height



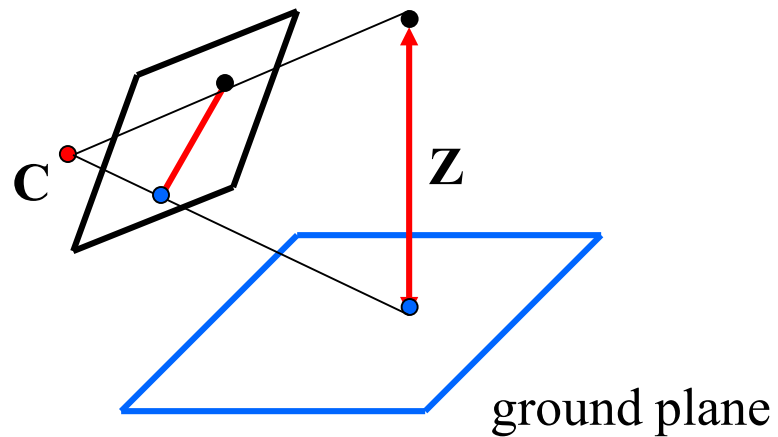
Two views of a scene



Which is higher – the camera or the parachute?



Measuring height without a giant ruler



Compute Z from image measurements

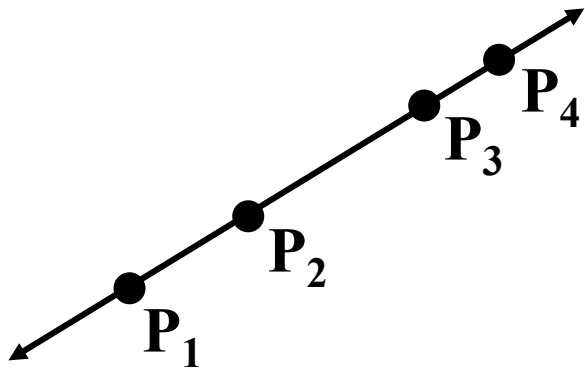
- Need a reference object

The cross ratio

A Projective Invariant

- Something that does not change under projective transformations (including perspective projection)

The cross-ratio of 4 collinear points



$$\frac{\| \mathbf{P}_3 - \mathbf{P}_1 \| \| \mathbf{P}_4 - \mathbf{P}_2 \|}{\| \mathbf{P}_3 - \mathbf{P}_2 \| \| \mathbf{P}_4 - \mathbf{P}_1 \|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

$$\frac{\| \mathbf{P}_1 - \mathbf{P}_3 \| \| \mathbf{P}_4 - \mathbf{P}_2 \|}{\| \mathbf{P}_1 - \mathbf{P}_2 \| \| \mathbf{P}_4 - \mathbf{P}_3 \|}$$

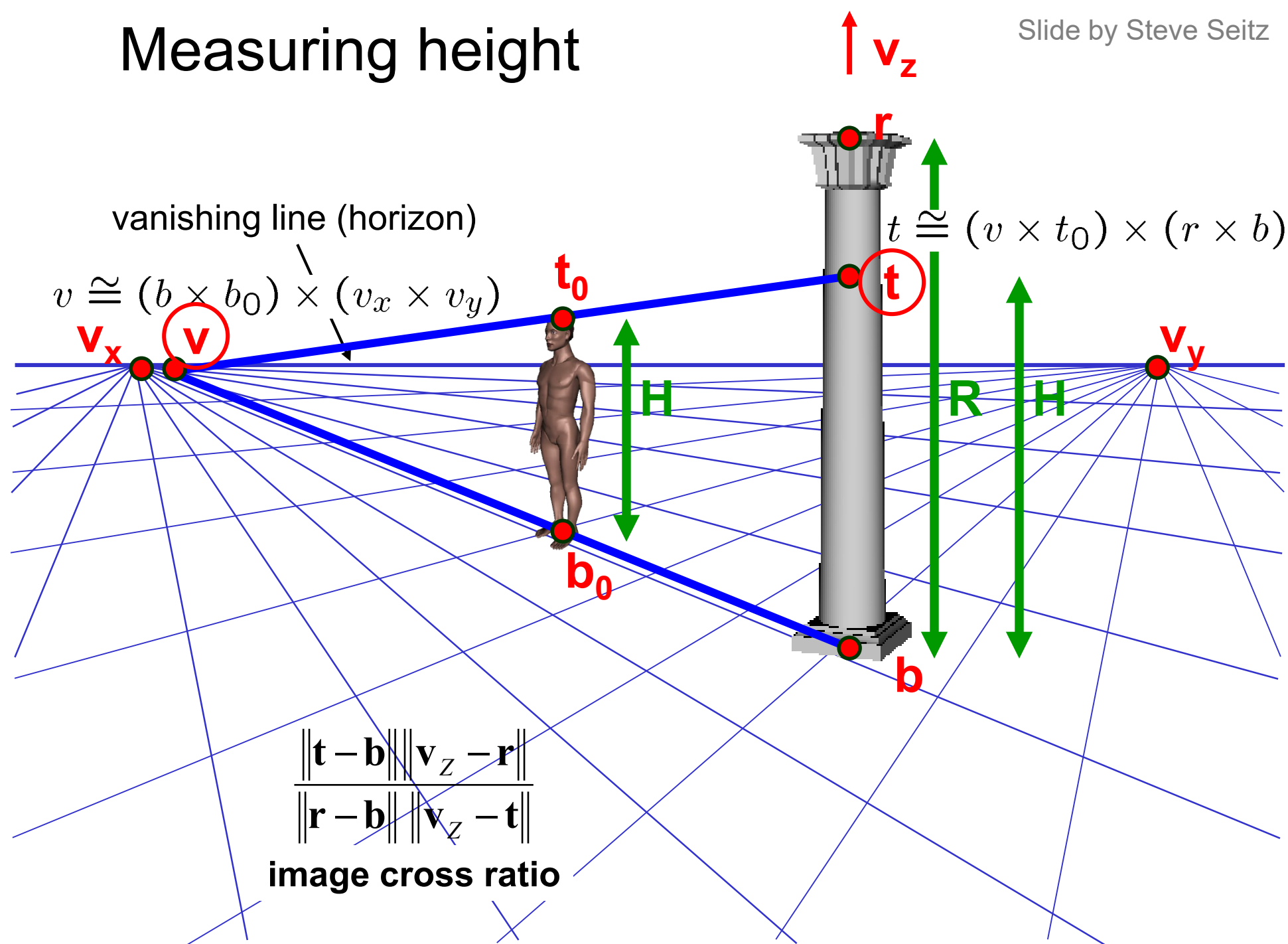
Can permute the point ordering

- $4! = 24$ different orders (but only 6 distinct values)

This is the fundamental invariant of projective geometry

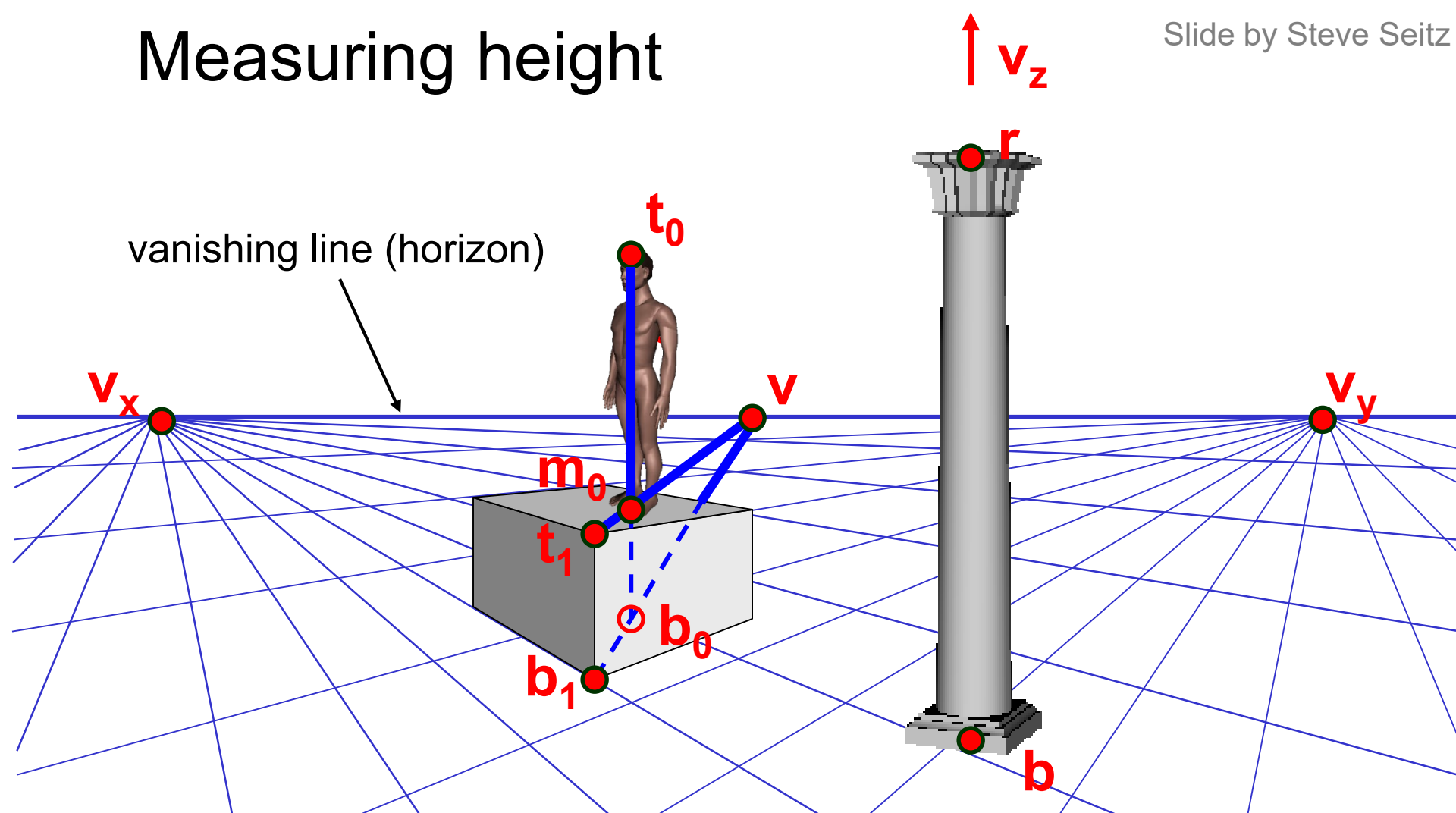
Measuring height

Slide by Steve Seitz



Measuring height

Slide by Steve Seitz



What if the point on the ground plane \mathbf{b}_0 is not known?

- Here the guy is standing on the box, height of box is known
- Use one side of the box to help find \mathbf{b}_0 as shown above

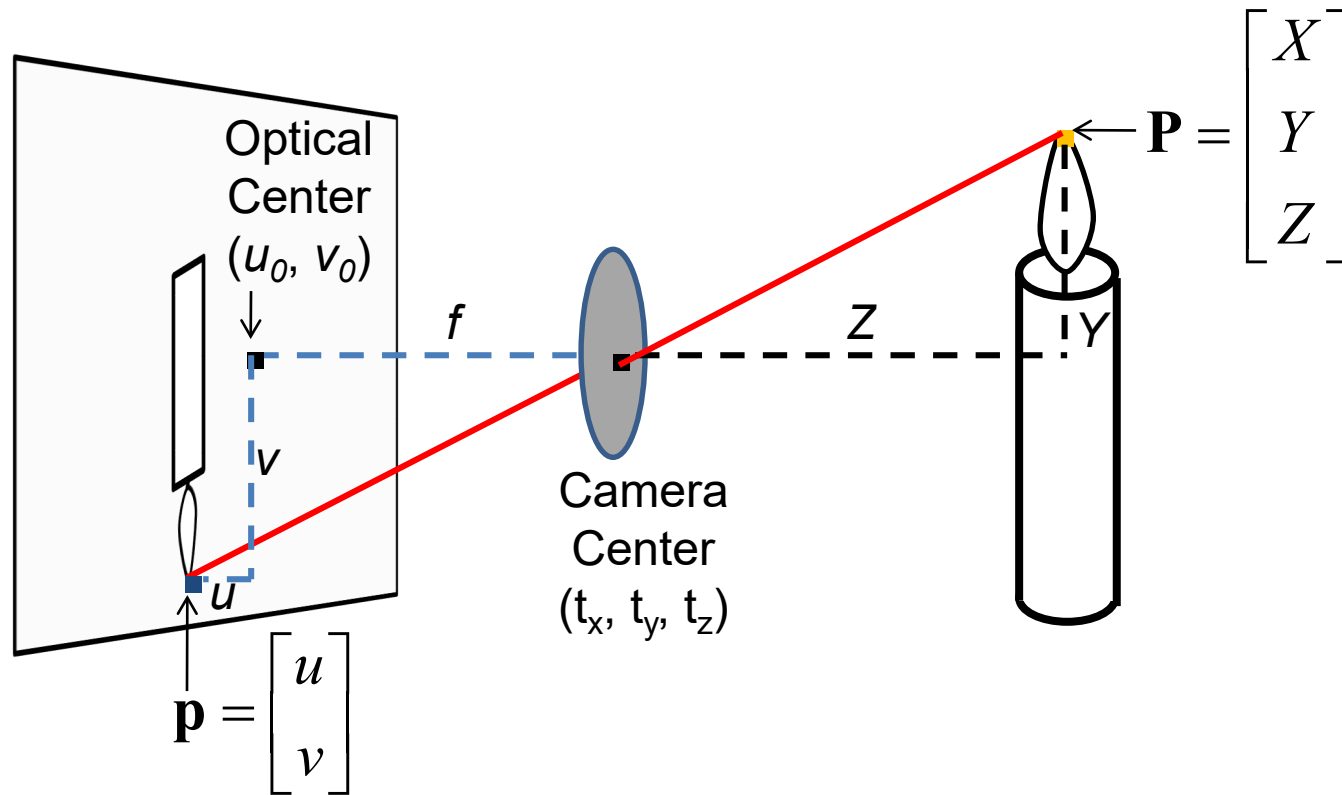
Take-home question

Assume that the man is 6 ft tall

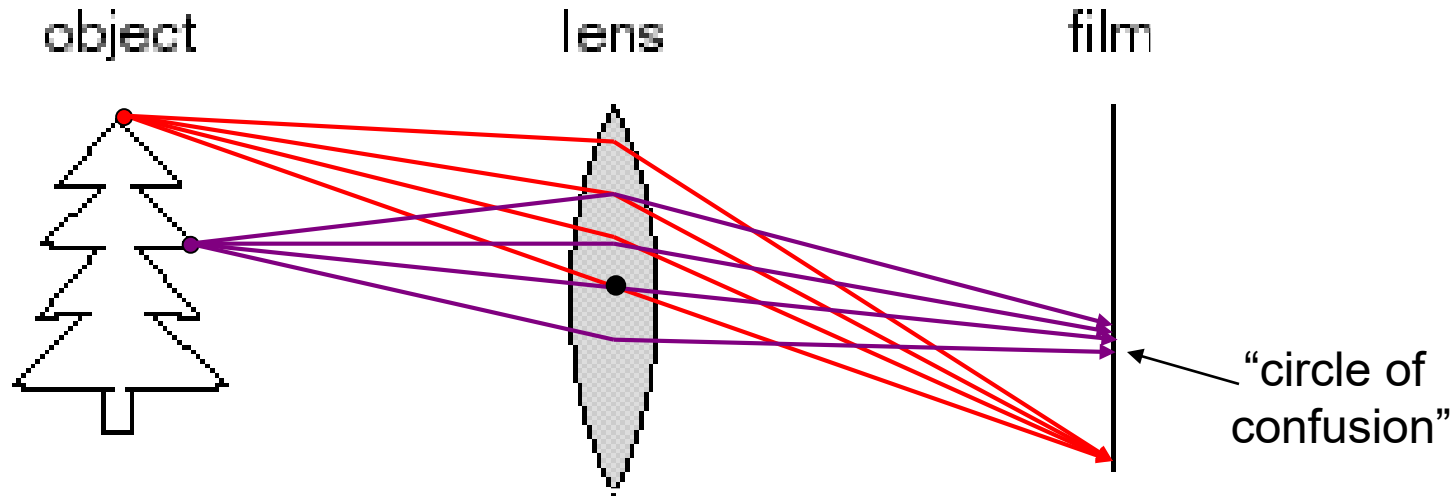
- What is the height of the front of the building?
- What is the height of the camera?



Beyond the pinhole: What about focus, aperture, DOF, FOV, etc?

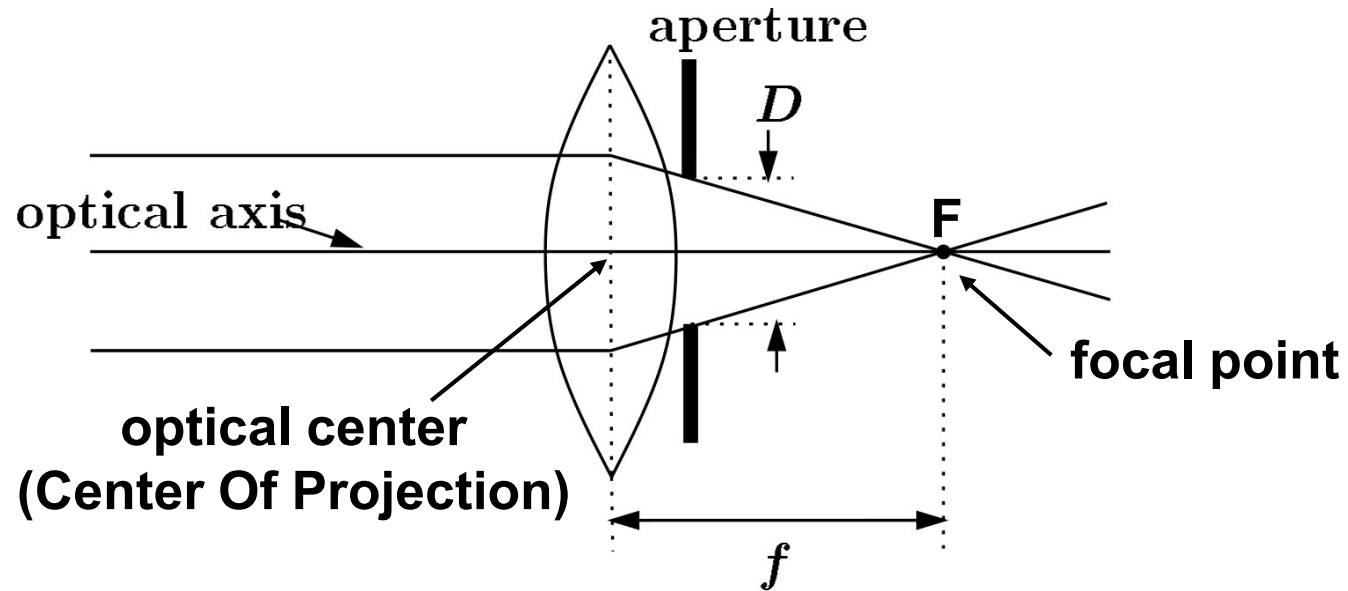


Adding a lens



- A lens focuses light onto the film
 - There is a specific distance at which objects are “in focus”
 - other points project to a “circle of confusion” in the image
 - Changing the shape of the lens changes this distance

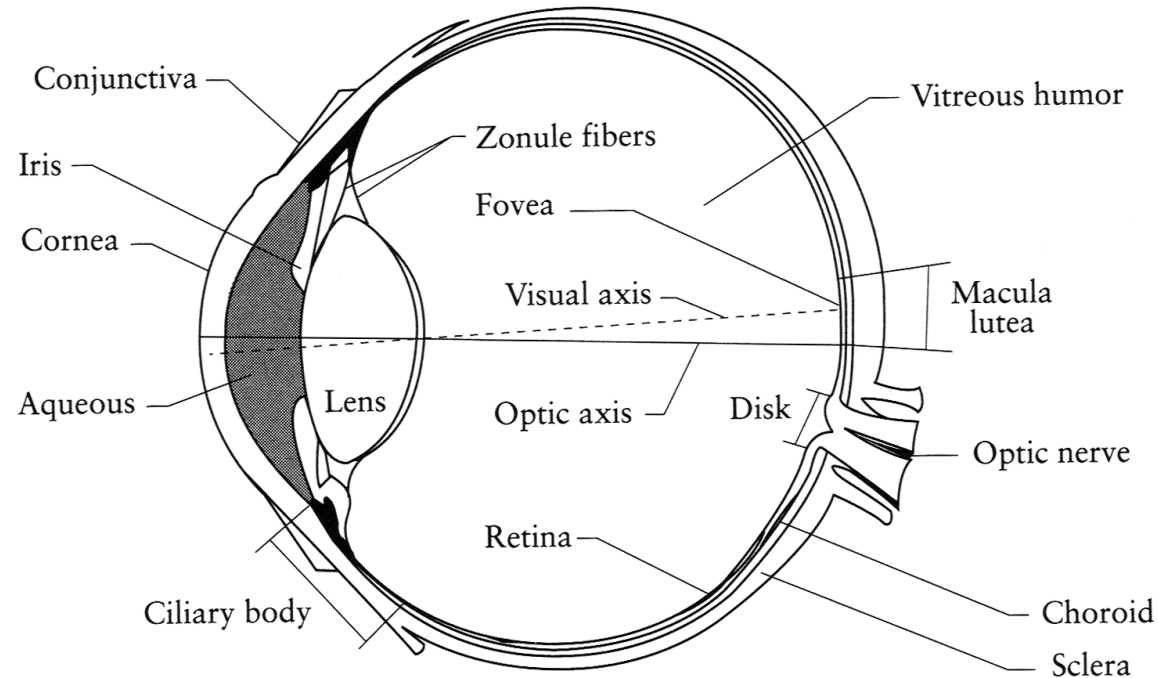
Focal length, aperture, depth of field



A lens focuses parallel rays onto a single focal point

- focal point at a distance f beyond the plane of the lens
- Aperture of diameter D restricts the range of rays

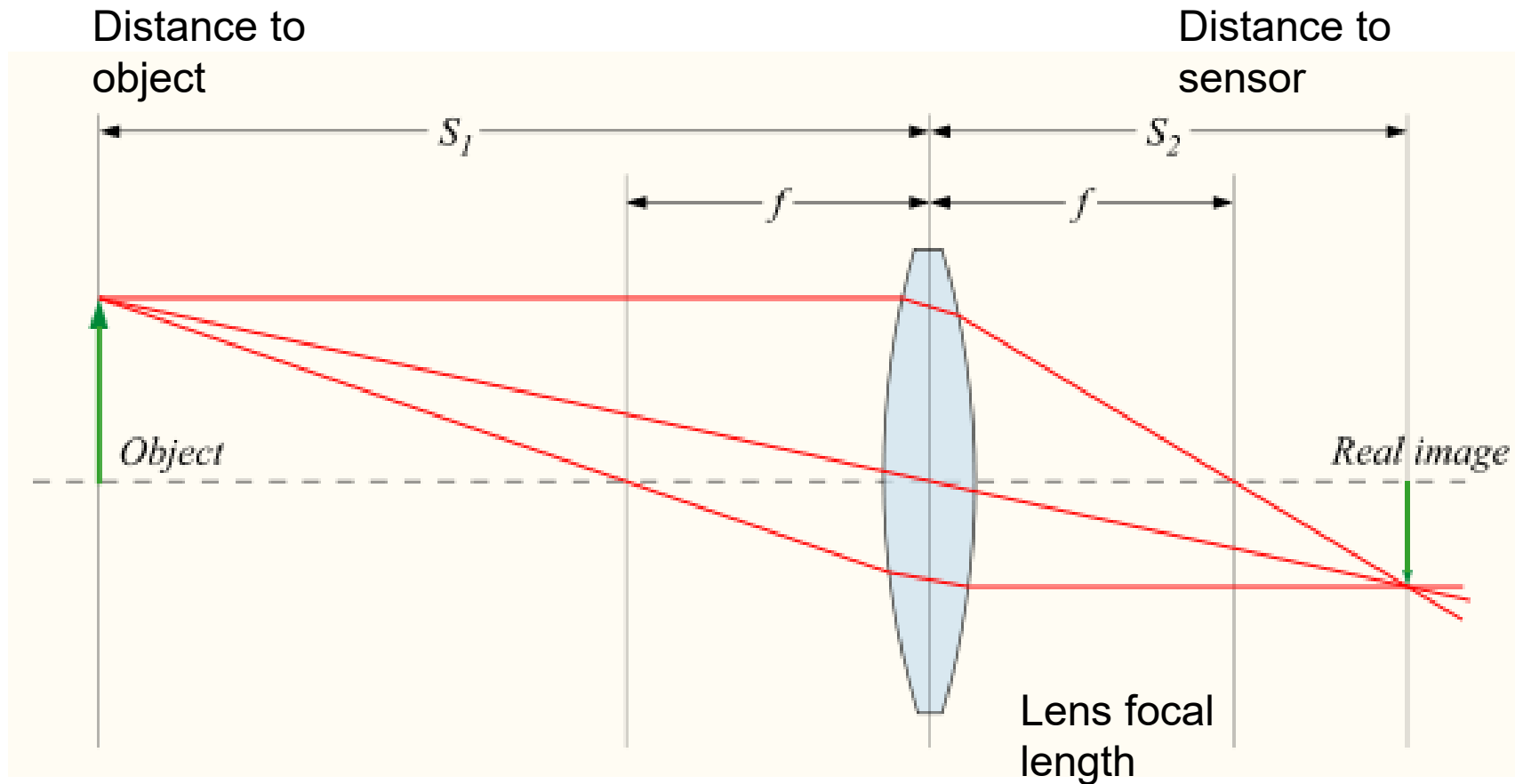
The eye



The human eye is a camera

- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris

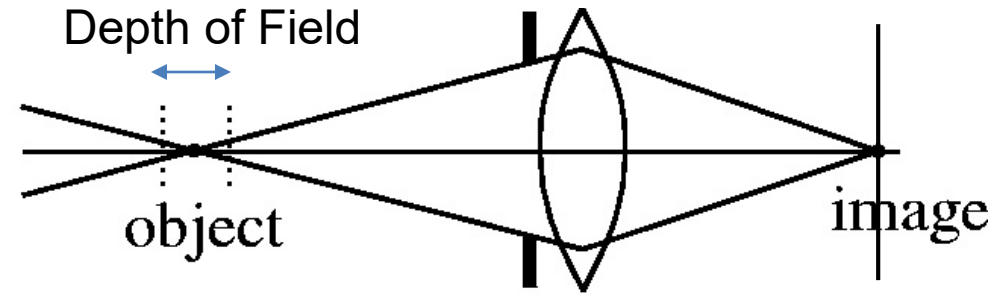
Focus with lenses



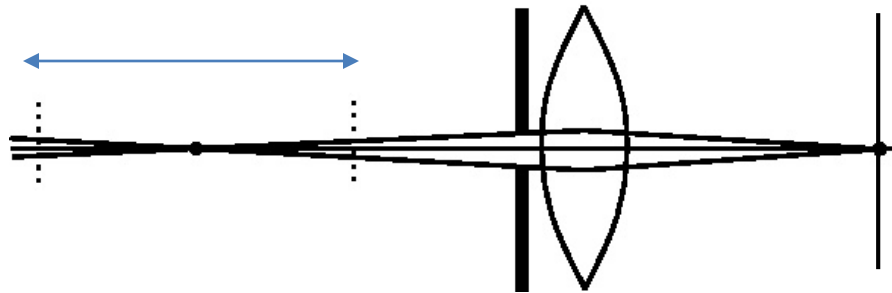
Equation for
objects in
focus

$$\frac{1}{S_1} + \frac{1}{S_2} = \frac{1}{f}$$

The aperture and depth of field



$f/5.6$



$f/32$

Changing the aperture size or focusing distance affects depth of field

f-number ($f/\#$) = focal_length / aperture_diameter (e.g., $f/16$ means that the focal length is 16 times the diameter)

When you change the f-number, you are changing the aperture

Depth of Field = range around focused distance that leads to smaller than threshold circle of confusion

Varying the aperture

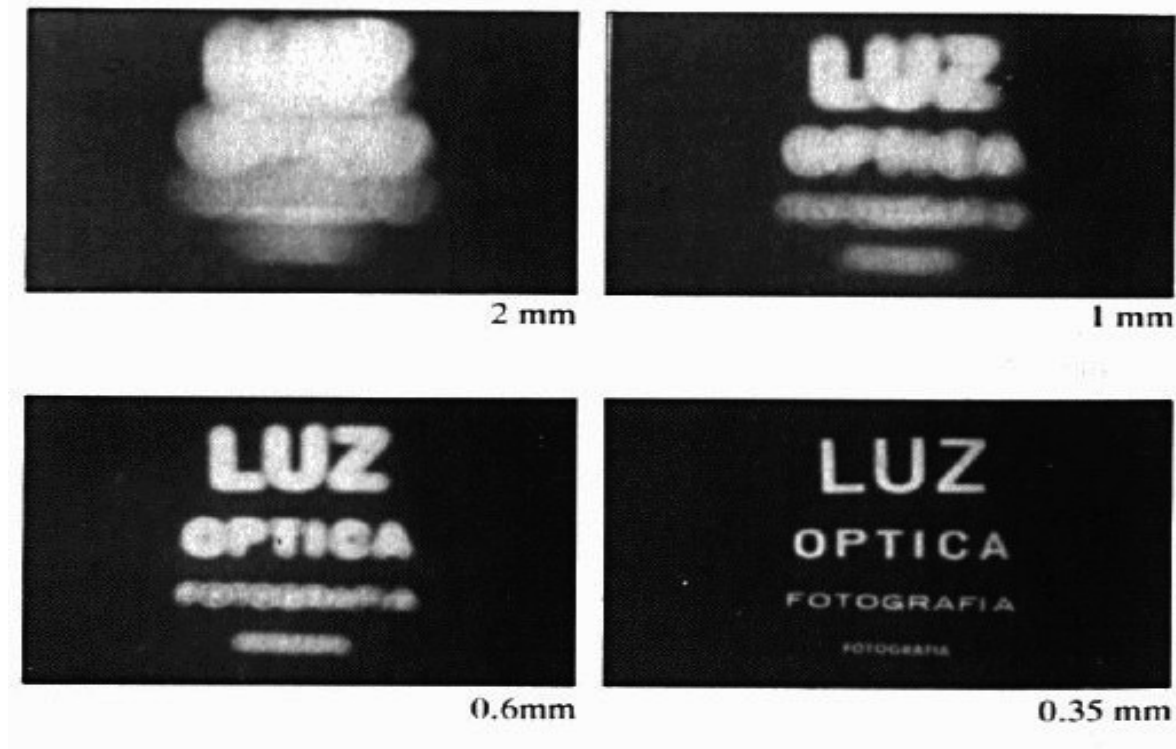


Large aperture = small DOF



Small aperture = large DOF

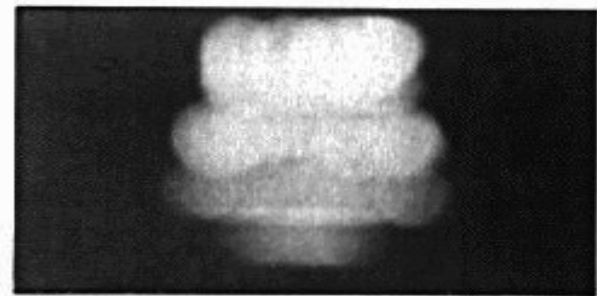
Shrinking the aperture



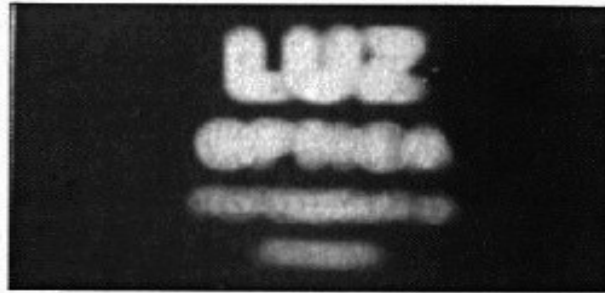
Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects

Shrinking the aperture



2 mm



1 mm



0.6mm



0.35 mm



0.15 mm



0.07 mm

Figure: *Optics*. Eugene Hecht

The Photographer's Great Compromise

What we want

How we get it

Cost

More spatial resolution

Increase focal length

Light, FOV

Decrease focal length

DOF

Broader field of view

Decrease aperture

Light

More depth of field

Increase aperture

DOF

More temporal resolution

Shorten exposure

Light

Lengthen exposure

Temporal Res

More light

Difficulty in macro (close-up) photography

- For close objects, we have a small relative DOF
- Can only shrink aperture so far

How to get both bugs in focus?



Solution: Focus stacking

1. Take pictures with varying focal length



Example from

http://www.wonderfulphotos.com/articles/macro/focus_stacking/

Solution: Focus stacking

1. Take pictures with varying focal length
2. Combine



Focus stacking



http://www.wonderfulphotos.com/articles/macro/focus_stacking/

Focus stacking

How to combine?

Web answer: With software (Photoshop, CombineZM)

How to do it automatically?

Focus stacking

How to combine?

1. Align images (e.g., using corresponding points)
2. Two ideas
 - a) Mask regions by hand and combine with pyramid blend
 - b) Gradient domain fusion (mixed gradient) without masking

Automatic solution would make an interesting final project

Recommended Reading:

<http://www.digital-photography-school.com/an-introduction-to-focus-stacking>

<http://www.zen20934.zen.co.uk/photography/Workflow.htm#Focus%20Stacking>

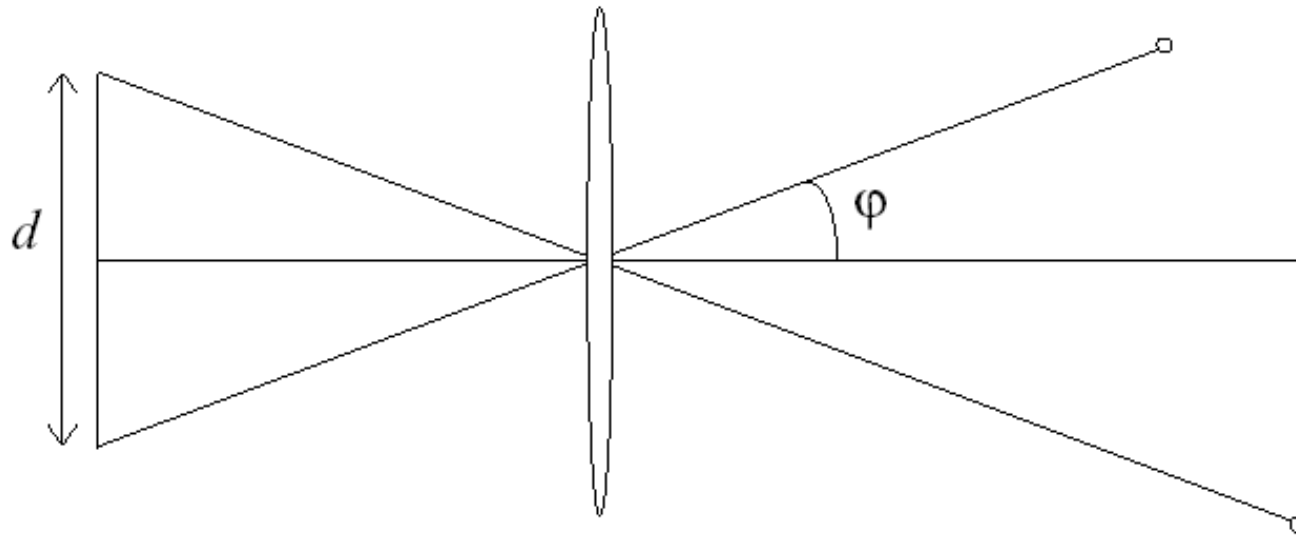
Relation between field of view and focal length

Field of view (angle width)

$$fov = 2 \tan^{-1} \frac{d}{2f}$$

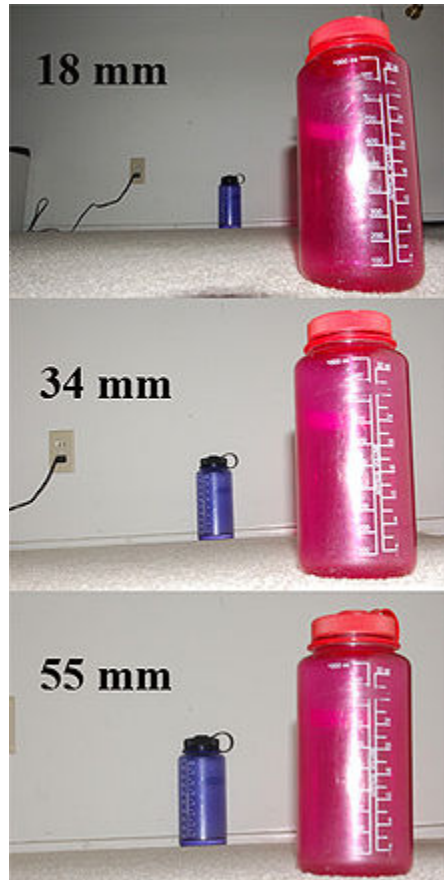
Film/Sensor Width

Focal length



Dolly Zoom or “Vertigo Effect”

<http://www.youtube.com/watch?v=NB4bikrNzMk>



How is this done?

Zoom in while
moving away

http://en.wikipedia.org/wiki/Focal_length

Dolly zoom (or “Vertigo effect”)

Field of view (angle width)

$$fov = 2 \tan^{-1} \frac{d}{2f}$$

Film/Sensor Width

Focal length

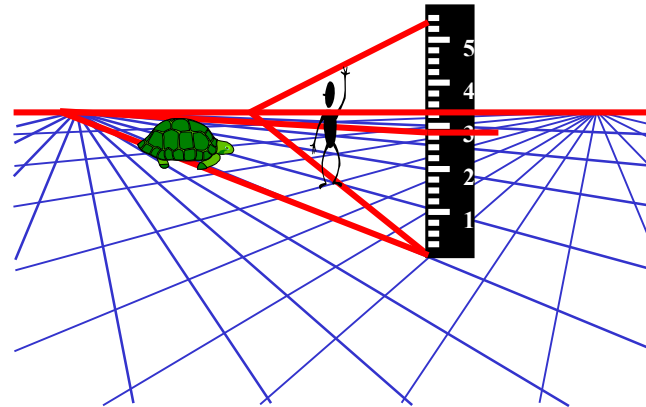
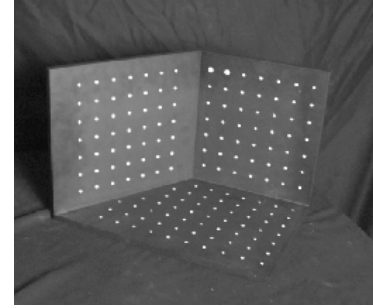
$$2 \tan \frac{fov}{2} = \frac{width}{distance}$$

width of object

Distance between object and camera

Things to remember

- Can calibrate using grid or VP
- Can measure relative sizes using VP
- Effects of focal length, aperture + tricks



Next class

- Go over take-home questions from today
- Single-view 3D Reconstruction