Reminder: Proj 2 due monday

• Much more difficult than project 1 – get started asap if not already
• Must compute SSD cost for every pixel (slow but not horribly slow using filtering method; see tips at end of project page)
• Learn how to debug visual algorithms: imshow, plot, dbstop if error, keyboard and break points are your friends
  – Debugging suggestion: For “quilt_simple”, first set upper-left patch to be upper-left patch in source and iteratively find minimum cost patch and overlay --- should reproduce original source image, at least for part of the output
Review from last class: Gradient Domain Editing

General concept: Solve for pixels of new image that satisfy constraints on the gradient and the intensity

– Constraints can be from one image (for filtering) or more (for blending)
Project 3: Reconstruction from Gradients

1. Preserve x-y gradients
2. Preserve intensity of one pixel

Source pixels: s
Variable pixels: v
1. minimize \((v(x+1,y)-v(x,y) - (s(x+1,y)-s(x,y)))^2\)
2. minimize \((v(x,y+1)-v(x,y) - (s(x,y+1)-s(x,y)))^2\)
3. minimize \((v(1,1)-s(1,1))^2\)
Project 3 (extra): NPR

• Preserve gradients on edges
  – e.g., get canny edges with edge(im, ‘canny’)
• Reduce gradients not on edges
• Preserve original intensity
Colorization using optimization

• Solve for uv channels (in Luv space) such that similar intensities have similar colors

• Minimize squared color difference, weighted by intensity similarity

\[ J(U) = \sum_r \left( U(r) - \sum_{s \in N(r)} w_{rs} U(s) \right)^2 \]

• Solve with sparse linear system of equations

http://www.cs.huji.ac.il/~yweiss/Colorization/
Gradient-domain editing

Many image processing applications can be thought of as trying to manipulate gradients or intensities:

– Contrast enhancement
– Denoising
– Poisson blending
– HDR to RGB
– Color to Gray
– Recoloring
– Texture transfer

See Perez et al. 2003 for many examples
Gradient-domain processing

Saliency-based Sharpening
Gradient-domain processing

Non-photorealistic rendering
Project 3: Gradient Domain Editing

General concept: Solve for pixels of new image that satisfy constraints on the gradient and the intensity

– Constraints can be from one image (for filtering) or more (for blending)
Project 3: Reconstruction from Gradients

1. Preserve x-y gradients
2. Preserve intensity of one pixel

Source pixels: s
Variable pixels: v

1. minimize \((v(x+1,y) - v(x,y) - (s(x+1,y) - s(x,y)))^2\)
2. minimize \((v(x,y+1) - v(x,y) - (s(x,y+1) - s(x,y)))^2\)
3. minimize \((v(1,1) - s(1,1))^2\)
Project 3 (extra): NPR

- Preserve gradients on edges
  - e.g., get canny edges with edge(im, ‘canny’)
- Reduce gradients not on edges
- Preserve original intensity

Perez et al. 2003
Take-home questions

1) I am trying to blend this bear into this pool. What problems will I have if I use:
   a) Alpha compositing with feathering
   b) Laplacian pyramid blending
   c) Poisson editing?

Lap. Pyramid

Poisson Editing
Take-home questions

2) How would you make a sharpening filter using gradient domain processing? What are the constraints on the gradients and the intensities?
Next two classes: warping and morphing

• Today
  – Global coordinate transformations
  – Image alignment

• Tuesday
  – Interpolation and texture mapping
  – Meshes and triangulation
  – Shape morphing
Photo stitching: projective alignment

Find corresponding points in two images

Solve for transformation that aligns the images
Capturing light fields

Estimate light via projection from spherical surface onto image
Morphing

Blend from one object to other with a series of local transformations
Image Transformations

image filtering: change **range** of image

\[ g(x) = T(f(x)) \]

image warping: change **domain** of image

\[ g(x) = f(T(x)) \]
Image Transformations

image filtering: change **range** of image

\[ g(x) = T(f(x)) \]

image warping: change **domain** of image

\[ g(x) = f(T(x)) \]
Parametric (global) warping

Transformation $T$ is a coordinate-changing machine:

$$p' = T(p)$$

What does it mean that $T$ is global?

- Is the same for any point $p$
- Can be described by just a few numbers (parameters)

For linear transformations, we can represent $T$ as a matrix

$$p' = Mp$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} M$$
Parametric (global) warping

Examples of parametric warps:

- translation
- rotation
- aspect
- affine
- perspective
- cylindrical
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar.
- *Uniform scaling* means this scalar is the same for all components:
Scaling

- **Non-uniform scaling**: different scalars per component:

\[ X \times 2, \quad Y \times 0.5 \]
Scaling

• Scaling operation: 
  \[ x' = ax \]
  \[ y' = by \]

• Or, in matrix form: 
  \[
  \begin{bmatrix}
  x' \\
  y'
  \end{bmatrix}
  =
  \begin{bmatrix}
  a & 0 \\
  0 & b
  \end{bmatrix}
  \begin{bmatrix}
  x \\
  y
  \end{bmatrix}
  \]

What is the transformation from \((x', y')\) to \((x, y)\)?
2-D Rotation

\[ x' = x \cos(\theta) - y \sin(\theta) \]

\[ y' = x \sin(\theta) + y \cos(\theta) \]
2-D Rotation

Polar coordinates…

\[
x = r \cos (\phi) \\
y = r \sin (\phi) \\
x' = r \cos (\phi + \theta) \\
y' = r \sin (\phi + \theta)
\]

Trig Identity…

\[
x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta) \\
y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta)
\]

Substitute…

\[
x' = x \cos(\theta) - y \sin(\theta) \\
y' = x \sin(\theta) + y \cos(\theta)
\]
2-D Rotation

This is easy to capture in matrix form:

\[
\begin{bmatrix}
    x' \\
    y'
\end{bmatrix} =
\begin{bmatrix}
    \cos(\theta) & -\sin(\theta) \\
    \sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
    x \\
    y
\end{bmatrix}
\]

Even though \(\sin(\theta)\) and \(\cos(\theta)\) are nonlinear functions of \(\theta\),

\(x'\) is a linear combination of \(x\) and \(y\)

\(y'\) is a linear combination of \(x\) and \(y\)

What is the inverse transformation?

\(\text{Rotation by } -\theta\)

For rotation matrices

\[R^{-1} = R^T\]
2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Identity?

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]

2D Scale around (0,0)?

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}
\]
2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

\[
\begin{align*}
x' &= \cos \Theta \cdot x - \sin \Theta \cdot y \\
y' &= \sin \Theta \cdot x + \cos \Theta \cdot y
\end{align*}
\]

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
\cos \Theta & -\sin \Theta \\
\sin \Theta & \cos \Theta
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]

2D Shear?

\[
\begin{align*}
x' &= x + k_x \cdot y \\
y' &= k_y \cdot x + y
\end{align*}
\]

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} =
\begin{bmatrix}
1 & k_x \\
k_y & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
\]
2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

\[ x' = -x \]
\[ y' = y \]

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  -1 & 0 \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

2D Mirror over (0,0)?

\[ x' = -x \]
\[ y' = -y \]

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  -1 & 0 \\
  0 & -1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]
2x2 Matrices

What types of transformations can be represented with a 2x2 matrix?

2D Translation?

\[ x' = x + t_x \]
\[ y' = y + t_y \]

NO!
All 2D Linear Transformations

- Linear transformations are combinations of ...
  - Scale,
  - Rotation,
  - Shear, and
  - Mirror

- Properties of linear transformations:
  - Origin maps to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

\[
\begin{bmatrix}
  x' \\
y'
\end{bmatrix} = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
a & b & e & f & i & j \\
c & d & g & h & k & l
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]
Homogeneous Coordinates

Q: How can we represent translation in matrix form?

\[ x' = x + t_x \]
\[ y' = y + t_y \]
Homogeneous Coordinates

Homogeneous coordinates

- represent coordinates in 2 dimensions with a 3-vector

\[
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\rightarrow
\begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\text{homogeneous coords}
\]
Homogeneous Coordinates

2D Points $\rightarrow$ Homogeneous Coordinates
• Append 1 to every 2D point: $(x \ y) \rightarrow (x \ y \ 1)$

Homogeneous coordinates $\rightarrow$ 2D Points
• Divide by third coordinate $(x \ y \ w) \rightarrow (x/w \ y/w)$

Special properties
• Scale invariant: $(x \ y \ w) = k \cdot (x \ y \ w)$
• $(x, y, 0)$ represents a point at infinity
• $(0, 0, 0)$ is not allowed

![Graph showing scale invariance with points (2,1,1) or (4,2,2) or (6,3,3) plotted on a 2D coordinate system.](image-url)
Homogeneous Coordinates

Q: How can we represent translation in matrix form?

\[ x' = x + t_x \]
\[ y' = y + t_y \]

A: Using the rightmost column:

\[
\text{Translation} = \begin{bmatrix}
1 & 0 & t_x \\
0 & 1 & t_y \\
0 & 0 & 1
\end{bmatrix}
\]
Translation Example

\[
\begin{bmatrix}
x' \\
y' \\
1
\end{bmatrix} = \begin{bmatrix}
1 & 0 & t_x \\
0 & 1 & t_y \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix} = \begin{bmatrix}
x + t_x \\
y + t_y \\
1
\end{bmatrix}
\]

\[t_x = 2\]
\[t_y = 1\]
Basic 2D transformations as 3x3 matrices

Translate

\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix} = \begin{bmatrix}
  1 & 0 & t_x \\
  0 & 1 & t_y \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]

Scale

\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix} = \begin{bmatrix}
  s_x & 0 & 0 \\
  0 & s_y & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]

Rotate

\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix} = \begin{bmatrix}
  \cos \Theta & -\sin \Theta & 0 \\
  \sin \Theta & \cos \Theta & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]

Shear

\[
\begin{bmatrix}
  x' \\
  y' \\
  1
\end{bmatrix} = \begin{bmatrix}
  1 & \beta_x & 0 \\
  \beta_y & 1 & 0 \\
  0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  1
\end{bmatrix}
\]
Matrix Composition

Transformations can be combined by matrix multiplication

\[
\begin{bmatrix}
x' \\
y' \\
w'
\end{bmatrix} = 
\left( 
\begin{bmatrix}
1 & 0 & tx \\
0 & 1 & ty \\
0 & 0 & 1
\end{bmatrix} 
\begin{bmatrix}
\cos \Theta & -\sin \Theta & 0 \\
\sin \Theta & \cos \Theta & 0 \\
0 & 0 & 1
\end{bmatrix} 
\begin{bmatrix}
sx & 0 & 0 \\
0 & sy & 0 \\
0 & 0 & 1
\end{bmatrix}
\right) 
\begin{bmatrix}
x \\
y \\
w
\end{bmatrix}
\]

\[
p' = T(t_x, t_y) \cdot R(\Theta) \cdot S(s_x, s_y) \cdot p
\]

Does the order of multiplication matter?
Affine Transformations

Affine transformations are combinations of
• Linear transformations, and
• Translations

Properties of affine transformations:
• Origin does not necessarily map to origin
• Lines map to lines
• Parallel lines remain parallel
• Ratios are preserved
• Closed under composition

\[
\begin{bmatrix}
    x' \\
    y' \\
    1
\end{bmatrix} = \begin{bmatrix}
    a & b & c \\
    d & e & f \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x \\
    y \\
    1
\end{bmatrix}
\]
Projective Transformations

Projective transformations are combos of
- Affine transformations, and
- Projective warps

Properties of projective transformations:
- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis
- Projective matrix is defined up to a scale (8 DOF)
2D image transformations

These transformations are a nested set of groups

- Closed under composition and inverse is a member
Recovering Transformations

- What if we know $f$ and $g$ and want to recover the transform $T$?
  - willing to let user provide correspondences
    - How many do we need?

\[ T(x,y) \]
Translation: # correspondences?

• How many Degrees of Freedom?
• How many correspondences needed for translation?
• What is the transformation matrix?

\[
M = \begin{bmatrix}
1 & 0 & p'_x - p_x \\
0 & 1 & p'_y - p_y \\
0 & 0 & 1
\end{bmatrix}
\]
Euclidean: # correspondences?

• How many DOF?
• How many correspondences needed for translation+rotation?
Affine: # correspondences?

- How many DOF?
- How many correspondences needed for affine?
Affine transformation estimation

- Math
- Matlab demo
Projective: # correspondences?

- How many DOF?
- How many correspondences needed for projective?
Take-home Question

1) Suppose we have two triangles: ABC and A’B’C’. What transformation will map A to A’, B to B’, and C to C’? How can we get the parameters?
Take-home Question

2) Show that distance ratios along a line are preserved under 2d linear transformations.

\[
\begin{vmatrix}
 p_1 &= (x_1, y_1) \\
 p_2 &= (x_2, y_2) \\
 p_3 &= (x_3, y_3)
\end{vmatrix}
\]

\[
\frac{\| p_2 - p_1 \|}{\| p_3 - p_1 \|} = \frac{\| p'_2 - p'_1 \|}{\| p'_3 - p'_1 \|}
\]

Hint: Write down \( x_2 \) in terms of \( x_1 \) and \( x_3 \), given that the three points are co-linear.
Next class: texture mapping and morphing