Pixels and Image Filtering

Computational Photography

Derek Hoiem

Administrative stuff

• Any questions?

• Tutorial:
  • Looks like Sept 6 at 5pm (lasting 1.5-2 hrs), but will finalize later today --- vote soon
    https://beta.doodle.com/poll/xz799uch67x6sxsd

• Please wait to pack up until I’m done talking
Today’s Class: Pixels and Linear Filters

• What is a pixel? How is an image represented?

• What is image filtering and how do we do it?

• Introduce Project 1: Hybrid Images
Next three classes

• Image filters in spatial domain
  – Smoothing, sharpening, measuring texture

• Image filters in the frequency domain
  – Denoising, sampling, image compression

• Templates and Image Pyramids
  – Detection, coarse-to-fine registration
Image Formation
Digital camera

A digital camera replaces film with a sensor array

• Each cell in the array is light-sensitive diode that converts photons to electrons

• Two common types:
  – Charge Coupled Device (CCD): larger yet slower, better quality
  – Complementary Metal Oxide Semiconductor (CMOS): high bandwidth, lower quality

Sensor Array

FIGURE 2.17 (a) Continuous image projected onto a sensor array. (b) Result of image sampling and quantization.
The raster image (pixel matrix)
The raster image (pixel matrix)

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Perception of Intensity

from Ted Adelson
Perception of Intensity

from Ted Adelson
Digital Color Images

Bayer filter

© 2000 How Stuff Works
Color Image
Images in Matlab

- Images represented as a matrix
- Suppose we have a NxM RGB image called “im”
  - \( \text{im}(1,1,1) = \) top-left pixel value in R-channel
  - \( \text{im}(y, x, b) = y \) pixels down, \( x \) pixels to right in the \( b^{th} \) channel
  - \( \text{im}(N, M, 3) = \) bottom-right pixel in B-channel
- \text{imread(filename)} returns a uint8 image (values 0 to 255)
  - Convert to double format (values 0 to 1) with \text{im2double}
Image filtering

- Image filtering: compute function of local neighborhood at each position

- Really important!
  - Enhance images
    - Denoise, resize, increase contrast, etc.
  - Extract information from images
    - Texture, edges, distinctive points, etc.
  - Detect patterns
    - Template matching
Example: box filter

\[ g[\cdot, \cdot] \]

\[
\begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{array}
\]

\[ \frac{1}{9} \]
Image filtering

\[ \begin{align*}
  f[\cdot, \cdot] & \\
  h[\cdot, \cdot] & = \sum_{k,l} g[k,l] f[m+k, n+l]
\end{align*} \]

Credit: S. Seitz
Image filtering

$f[\ldots]$

$g[\cdot, \cdot] \frac{1}{9}$

$h[\ldots]$

$$h[m,n] = \sum_{k,l} g[k,l] \cdot f[m+k,n+l]$$

Credit: S. Seitz
Image filtering

\[ f[\cdot, \cdot] \]

\[ h[\cdot, \cdot] \]

\[ h[m,n] = \sum_{k,l} g[k,l] f[m+k,n+l] \]

Credit: S. Seitz
Image filtering

\[ f[\ldots] \]

\[ h[\ldots] \]

\[ h[m,n] = \sum_{k,l} g[k,l] f[m+k,n+l] \]

Credit: S. Seitz
Image filtering

\[ f[\cdot, \cdot] \]

\[ h[\cdot, \cdot] \]

\[ h[m, n] = \sum_{k, l} g[k, l] f[m + k, n + l] \]

Credit: S. Seitz
Image filtering

\[
f[\ldots] \quad h[\ldots]
\]

\[
h[m,n] = \sum_{k,l} g[k,l] f[m+k,n+l]
\]

Credit: S. Seitz
Image filtering

\[ f[\cdot, \cdot] \]

\[ h[\cdot, \cdot] \]

\[
\begin{array}{cccccc}
0 & 10 & 20 & 30 & 30 & \\
50 & \end{array}
\]

\[
h[m,n] = \sum_{k,l} g[k,l] \ f[m+k,n+l]
\]

Credit: S. Seitz
Image filtering

\[ f[\ldots] \]

\[ h[\ldots] \]

\[ h[m,n] = \sum_{k,l} g[k,l] f[m+k,n+l] \]

Credit: S. Seitz
Informally, what does the filter do?

\[ h[m,n] = \sum_{k,l} g[k,l] f[m+k,n+l] \]
Box Filter

What does it do?

• Replaces each pixel with an average of its neighborhood

• Achieve smoothing effect (remove sharp features)

Slide credit: David Lowe (UBC)
Smoothing with box filter
One more on board...
Practice with linear filters

Original

0 0 0
0 1 0
0 0 0

Source: D. Lowe
Practice with linear filters

Original

Filtered (no change)

Source: D. Lowe
Practice with linear filters

Original

Source: D. Lowe
Practice with linear filters

Original

Shifted left
By 1 pixel

Source: D. Lowe
Practice with linear filters

Original

(Note that filter sums to 1)
Practice with linear filters

Original

Sharpening filter
- Accentuates differences with local average

Source: D. Lowe
Sharpening

before

after

Source: D. Lowe
Other filters

Sobel

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Vertical Edge (absolute value)
Other filters

Sobel

Horizontal Edge (absolute value)
How could we synthesize motion blur?

```matlab
theta = 30; len = 20;
fil = imrotate(ones(1, len), theta, 'bilinear');
fil = fil / sum(fil(:));
figure(2), imshow(imfilter(im, fil));
```
Filtering vs. Convolution

• 2d filtering

- \( h = \text{filter2}(g,f); \) or
- \( h = \text{imfilter}(f,g); \)

\[
h[m,n] = \sum_{k,l} g[k,l] f[m+k,n+l]
\]

• 2d convolution

- \( h = \text{conv2}(g,f); \)

\[
h[m,n] = \sum_{k,l} g[k,l] f[m-k,n-l]
\]
Key properties of linear filters

Linearity:
\[ \text{filter}(f_1 + f_2) = \text{filter}(f_1) + \text{filter}(f_2) \]

Shift invariance: same behavior regardless of pixel location
\[ \text{filter}(\text{shift}(f)) = \text{shift}(\text{filter}(f)) \]

Any linear, shift-invariant operator can be represented as a convolution

Source: S. Lazebnik
More properties

• Commutative: \( a * b = b * a \)
  – Conceptually no difference between filter and signal (image)

• Associative: \( a * (b * c) = (a * b) * c \)
  – Often apply several filters one after another: \(((a * b_1) * b_2) * b_3)\)
  – This is equivalent to applying one filter: \( a * (b_1 * b_2 * b_3) \)

• Distributes over addition: \( a * (b + c) = (a * b) + (a * c) \)

• Scalars factor out: \( k a * b = a * k b = k (a * b) \)

• Identity: unit impulse \( e = [0, 0, 1, 0, 0] \), \( a * e = a \)

Source: S. Lazebnik
Important filter: Gaussian

- Weight contributions of neighboring pixels by nearness

\[
G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}
\]

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5 x 5, \( \sigma = 1 \)

Slide credit: Christopher Rasmussen
Smoothing with Gaussian filter
Smoothing with box filter
Gaussian filters

• Remove “high-frequency” components from the image (low-pass filter)
  – Images become more smooth

• Convolution with self is another Gaussian
  – So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  – Convolving two times with Gaussian kernel of width $\sigma$
    is same as convolving once with kernel of width $\sigma \sqrt{2}$

• Separable kernel
  – Factors into product of two 1D Gaussians

Source: K. Grauman
Separability of the Gaussian filter

\[ G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \]

\[ = \left(\frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{x^2}{2\sigma^2}\right)\right) \left(\frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{y^2}{2\sigma^2}\right)\right) \]

The 2D Gaussian can be expressed as the product of two functions, one a function of \( x \) and the other a function of \( y \).

In this case, the two functions are the (identical) 1D Gaussian.

Source: D. Lowe
Separability example

2D filtering (center location only)

\[
\begin{bmatrix}
1 & 2 & 1 \\
2 & 4 & 2 \\
1 & 2 & 1 \\
\end{bmatrix} \times 
\begin{bmatrix}
2 & 3 & 3 \\
3 & 5 & 5 \\
4 & 4 & 6 \\
\end{bmatrix}
\]

The filter factors into a product of 1D filters:

\[
\begin{bmatrix}
1 & 2 & 1 \\
2 & 4 & 2 \\
1 & 2 & 1 \\
\end{bmatrix} = 
\begin{bmatrix}
1 \\
2 \\
1 \\
\end{bmatrix} \times 
\begin{bmatrix}
1 & 2 & 1 \\
\end{bmatrix}
\]

Perform filtering along rows:

\[
\begin{bmatrix}
1 & 2 & 1 \\
\end{bmatrix} \times 
\begin{bmatrix}
2 & 3 & 3 \\
3 & 5 & 5 \\
4 & 4 & 6 \\
\end{bmatrix} = 
\begin{bmatrix}
11 \\
18 \\
18 \\
\end{bmatrix}
\]

Followed by filtering along the remaining column:

Source: K. Grauman
Separability

• Why is separability useful in practice?
Some practical matters
Practical matters

How big should the filter be?

• Values at edges should be near zero
• Rule of thumb for Gaussian: set filter half-width to about 3 $\sigma$
Practical matters

• What about near the edge?
  – the filter window falls off the edge of the image
  – need to extrapolate
  – methods:
    • clip filter (black)
    • wrap around
    • copy edge
    • reflect across edge

Source: S. Marschner
Practical matters

– methods (MATLAB):
  • clip filter (black): \text{imfilter}(f, g, 0)
  • wrap around: \text{imfilter}(f, g, \text{‘circular’})
  • copy edge: \text{imfilter}(f, g, \text{‘replicate’})
  • reflect across edge: \text{imfilter}(f, g, \text{‘symmetric’})

Source: S. Marschner
Practical matters

• What is the size of the output?
• MATLAB: filter2(g, f, shape)
  – shape = ‘full’: output size is sum of sizes of f and g
  – shape = ‘same’: output size is same as f
  – shape = ‘valid’: output size is difference of sizes of f and g

Source: S. Lazebnik
Application: Representing Texture

Source: Forsyth
Texture and Material

http://www-cvr.ai.uiuc.edu/ponce_grp/data/texture_database/samples/
Texture and Orientation

http://www-cvr.ai.uiuc.edu/ponce_grp/data/texture_database/samples/
Texture and Scale

http://www-cvr.ai.uiuc.edu/ponce_grp/data(texture_database/samples/)
What is texture?

Regular or stochastic patterns caused by bumps, grooves, and/or markings
How can we represent texture?

• Compute responses of blobs and edges at various orientations and scales
Overcomplete representation: filter banks

Code for filter banks: www.robots.ox.ac.uk/~vgg/research/texclass/filters.html
Filter banks

• Process image with each filter and keep responses (or squared/abs responses)
How can we represent texture?

• Measure responses of blobs and edges at various orientations and scales

• Record simple statistics (e.g., mean, std.) of absolute filter responses
Can you match the texture to the response?

Filters

1

2

3

Mean abs responses

A

B

C
Representing texture by mean abs response
Project 1: Hybrid Images


Project Instructions:
http://courses.engr.illinois.edu/cs498dh3/projects/hybrid/ComputationalPhotography_ProjectHybrid.html
Take-home messages

• Image is a matrix of numbers

• Linear filtering is a dot product at each position
  – Can smooth, sharpen, translate (among many other uses)

• Be aware of details for filter size, extrapolation, cropping

• Start thinking about project (read the paper, create a test project page)
Take-home questions

1. Write down a 3x3 filter that returns a positive value if the average value of the 4-adjacent neighbors is less than the center and a negative value otherwise.

2. Write down a filter that will compute the gradient in the x-direction:

   \[ \text{grad}_x(y,x) = \text{im}(y,x+1) - \text{im}(y,x) \] for each \( x, y \)
Take-home questions

3. Fill in the blanks:
   a) _ = D * B
   b) A = _ * _
   c) F = D * _
   d) _ = D * D

Filtering Operator
Next class: Thinking in Frequency