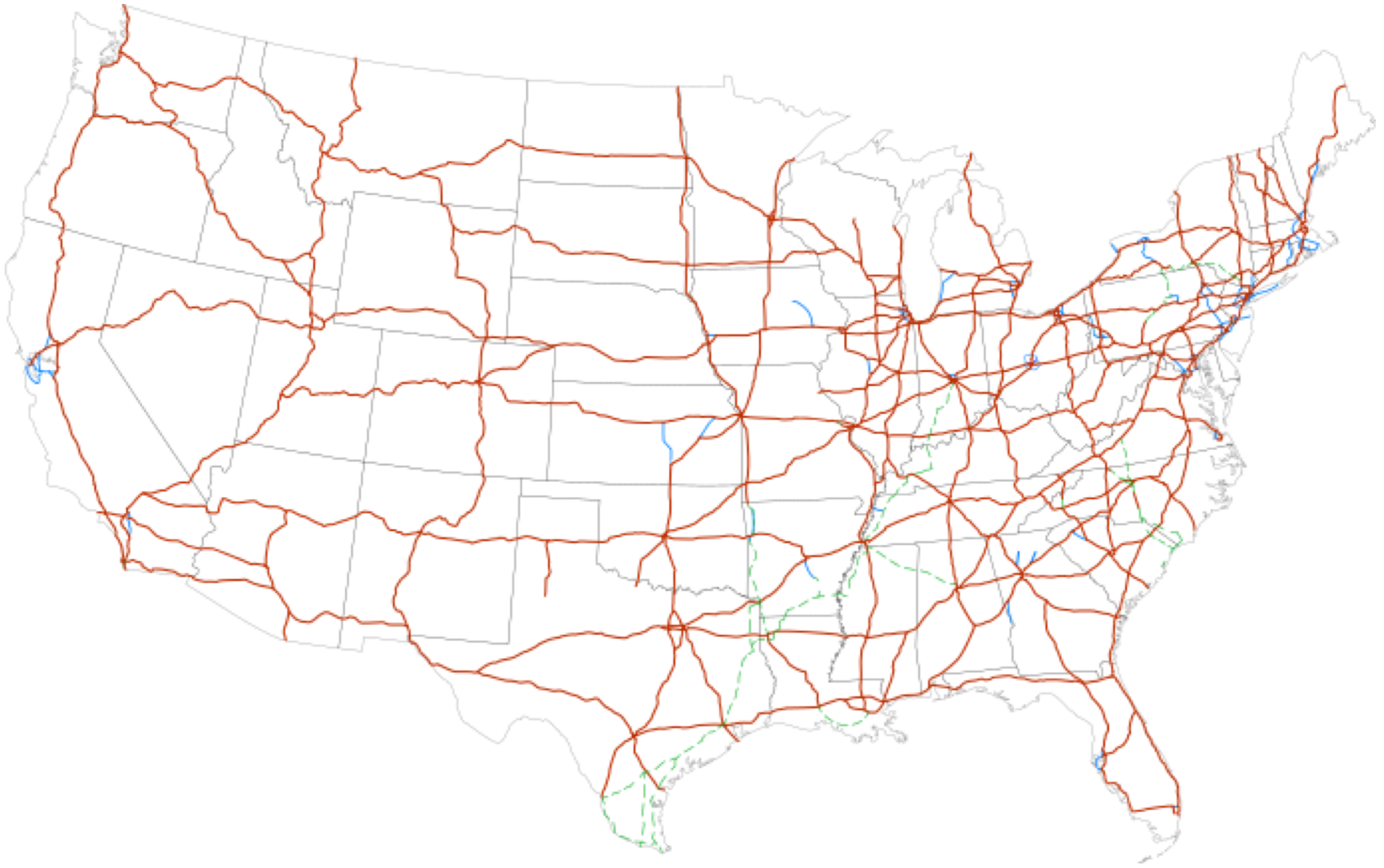
CS440/ECE 448 Lecture 4: Search Intro

Slides by Svetlana Lazebnik, 9/2016

Modified by Mark Hasegawa-Johnson, 1/2019



Types of agents

Reflex agent

- Consider how the world IS
- Choose action based on current percept
- Do not consider the future consequences of actions

Goal-directed agent

- Consider how the world WOULD BE
- Decisions based on (hypothesized) consequences of actions
- Must have a model of how the world evolves in response to actions
- Must formulate a goal

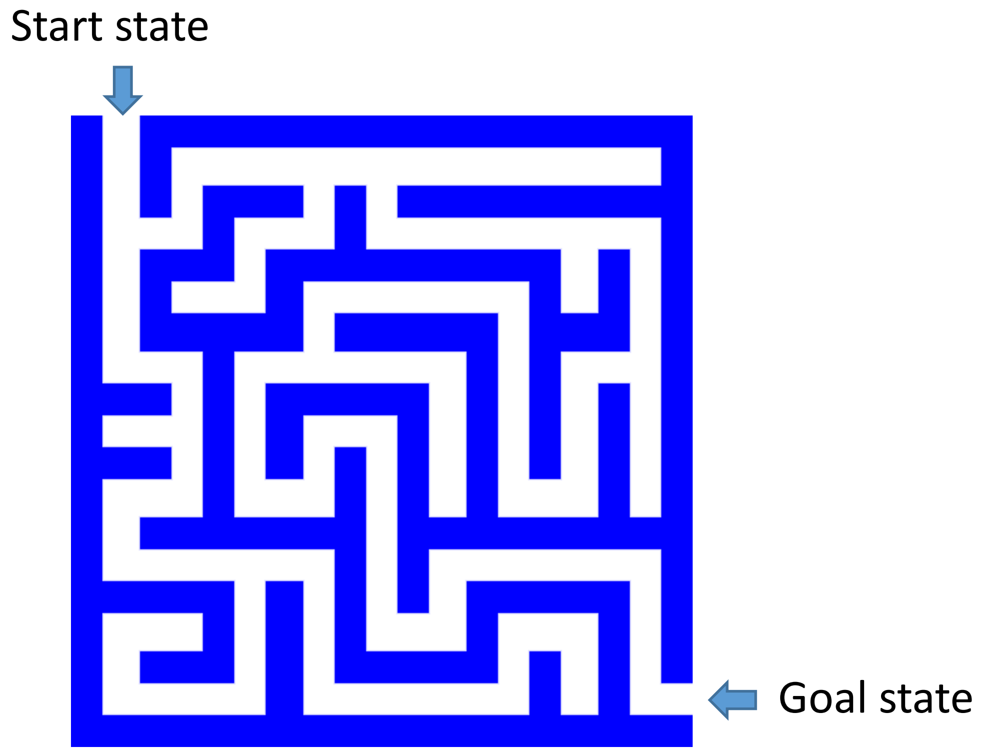
Source: D. Klein, P. Abbeel

Outline of today's lecture

1. How to define search problems:
    1. Initial state, goal state, transition model
    2. Actions, path cost
2. General algorithm for solving search problems
    1. First data structure: a frontier list
    2. Second data structure: a search tree
    3. Third data structure: a "visited states" list
3. Depth-first search: very fast, but not guaranteed
4. Breadth-first search: guaranteed optimal
5. Uniform cost search = Dijkstra's algorithm = BFS with variable costs

Search

We will consider the problem of designing **goal-based agents** in **fully observable, deterministic, discrete, static, known** environments



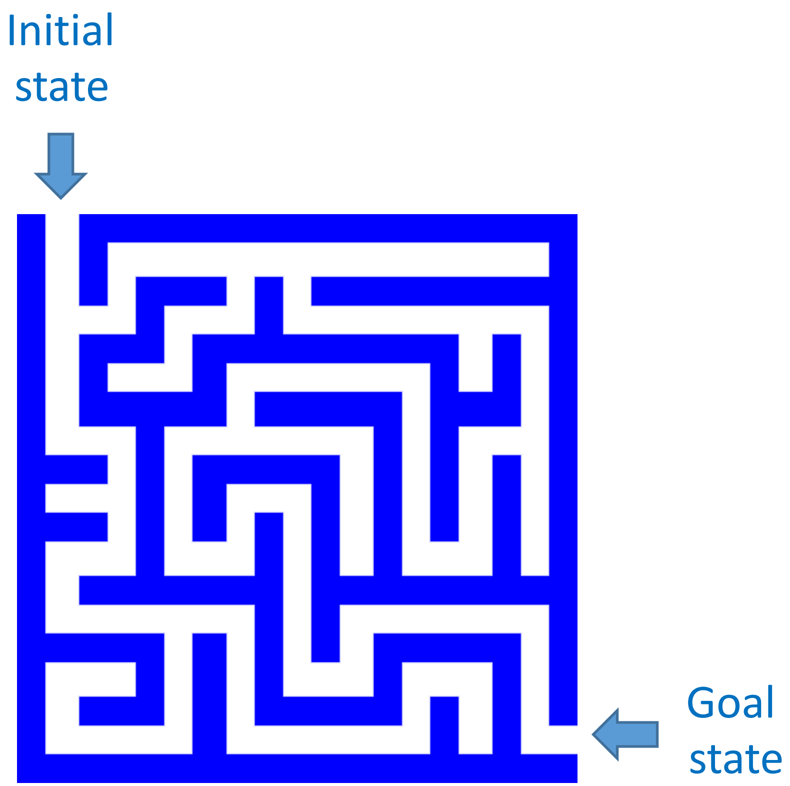
Search

We will consider the problem of designing **goal-based agents** in **fully observable, deterministic, discrete, static, known** environments

- The agent must find a ***sequence of actions*** that reaches the goal
- The **performance measure** is defined by (a) **reaching the goal** and (b) **how "expensive" the path** to the goal is
  - The **agent doesn't know** the performance measure. This is a goal-directed agent, not a utility-directed agent
  - The **programmer (you) DOES know** the performance measure. So you design **a goal-seeking strategy** that minimizes cost.
- We are focused on the process of finding the solution; we assume that the agent can safely ignore its percepts while executing the solution (**static environment, open-loop system**)

Search problem components

- **Initial state**
- **Actions**
- **Transition model**
  - What state results from performing a given action in a given state?
- **Goal state**
- **Path cost**
  - Assume that this is a sum of nonnegative *step costs*



- The **optimal solution** is the sequence of actions that gives the *lowest* path cost for reaching the goal

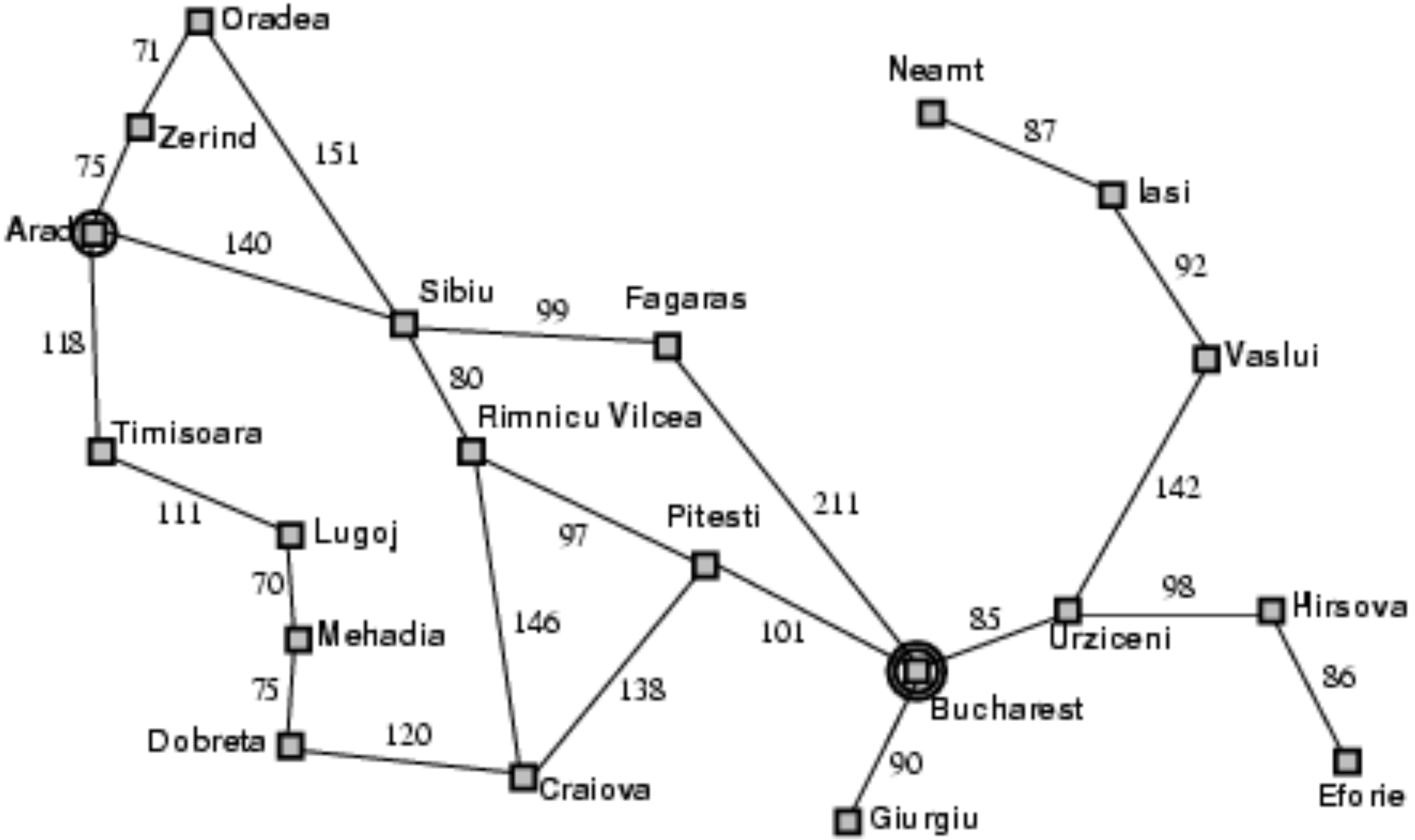
Knowledge Representation: State

- State = description of the world
  - Must have enough detail to decide whether or not you're currently in the **initial state**
  - Must have enough detail to decide whether or not you've reached the **goal state**
  - Often but not always: "defining the state" and "defining the transition model" are the same thing

Example: Romania

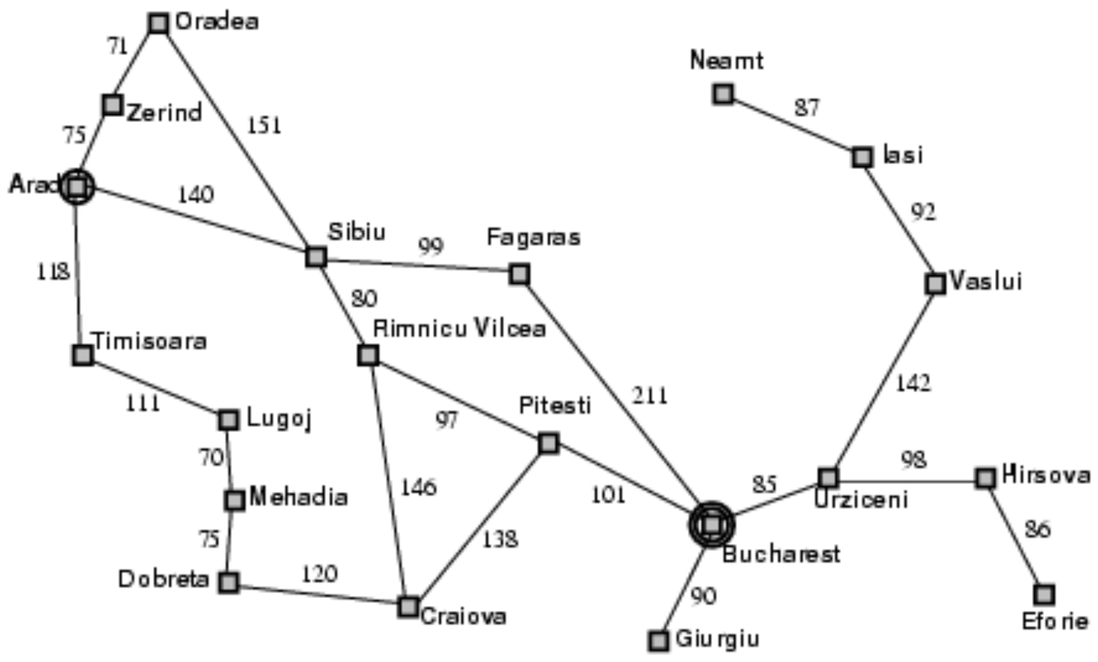
- On vacation in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

- **Initial state**
  - Arad
- **Actions**
  - Go from one city to another
- **Transition model**
  - If you go from city A to city B, you end up in city B
- **Goal state**
  - Bucharest
- **Path cost**
  - Sum of edge costs (total distance traveled)

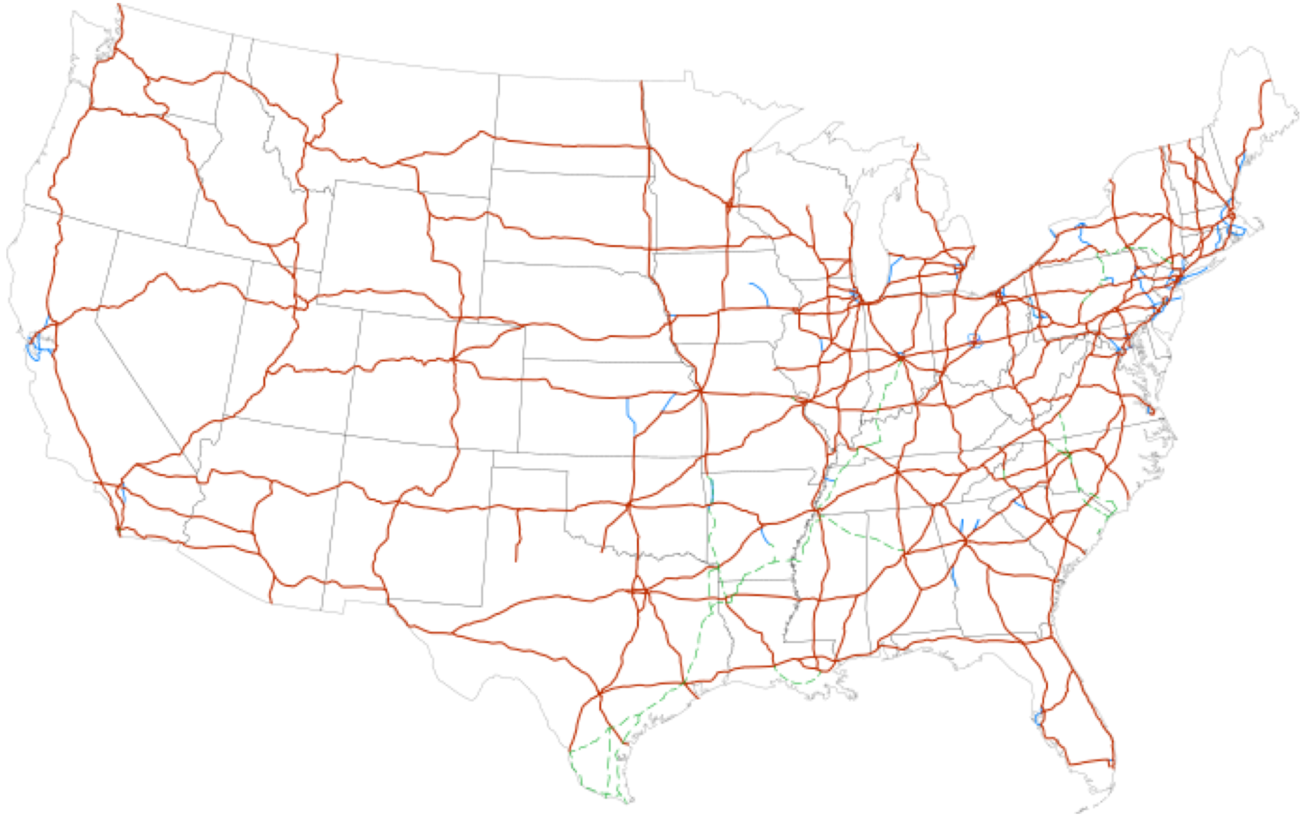


State space

- The initial state, actions, and transition model define the **state space** of the problem
  - The set of all states reachable from the initial state by any sequence of actions
  - Can be represented as a **directed graph** where the nodes are states and links between nodes are actions
- What is the state space for the Romania problem?
  - State Space = O{# cities}

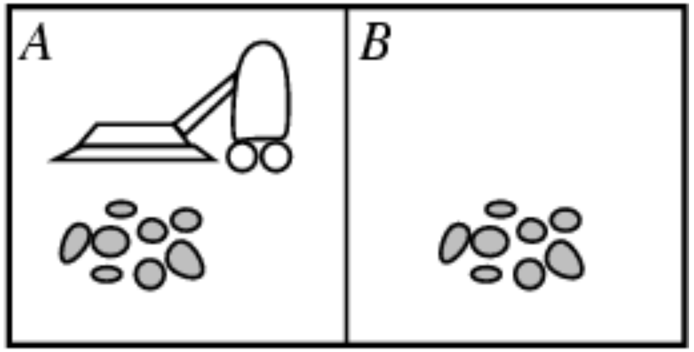


Traveling Salesman Problem



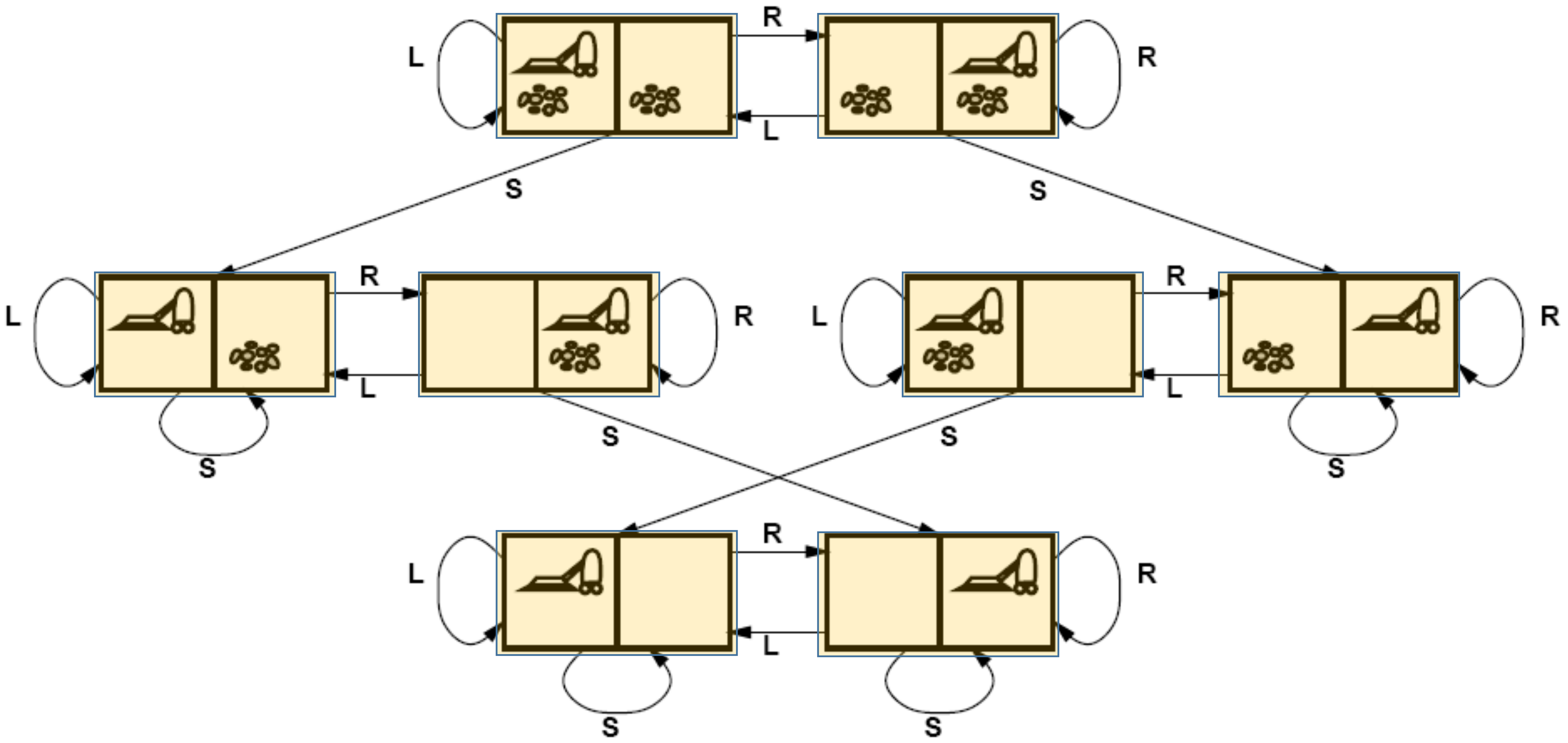
- **Goal**:
  Visit every city in US
- **Path cost**:
  Total miles traveled
- **Initial state**:
  Champaign, IL
- **Actions:**
  Travel from one city to another
- **Transition model:**
  When you visit a city, mark it as "visited."
  - State Space = $O(2^{\#cities})$

Example: Vacuum world



- **States**
  - Agent location and dirt location
  - How many possible states?
  - What if there are *n* possible locations?
    - The size of the state space grows exponentially with the “size” of the world!
- **Actions**
  - Left, right, suck
- **Transition model**

Vacuum world state space graph



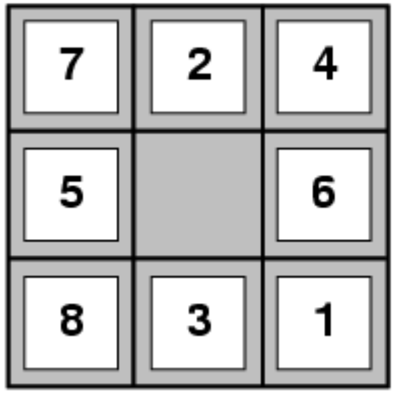
Complexity of the State Space

- Many “video game” style problems can be subdivided:
  - If there are ***M*** **different things your character needs to pick up**:
    2^M different world states (one for each subset of things that you’ve picked up)
  - If there are ***N*** **different locations you can be in while carrying any subset of those M objects:**
    Total number of world states = $O(2^M N)$

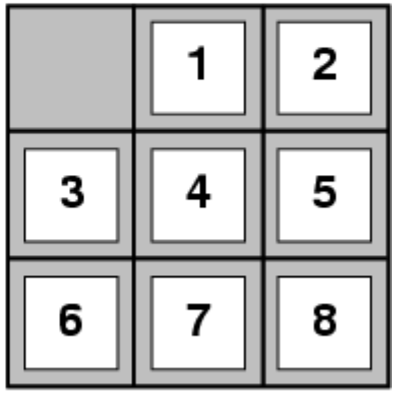
- Why a maze is nice: you don’t need to pick anything up
  - Only N different world states to consider

Example: The 8-puzzle

- **States**
  - Locations of tiles
    - 8-puzzle: 181,440 states (9!/2)
    - 15-puzzle: ~10 trillion states
    - 24-puzzle: ~10^{25} states
- **Actions**
  - Move blank left, right, up, down
- **Path cost**
  - 1 per move
- Finding the optimal solution of n-Puzzle is NP-hard

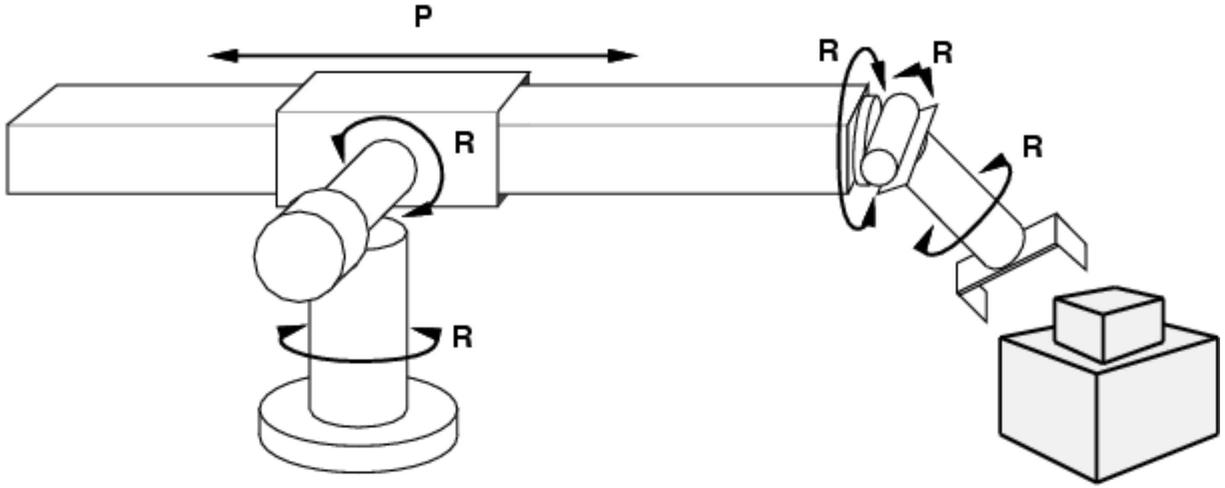


Start State



Goal State

Example: Robot motion planning



- **States**
  - Real-valued joint parameters (angles, displacements)
- **Actions**
  - Continuous motions of robot joints
- **Goal state**
  - Configuration in which object is grasped
- **Path cost**
  - Time to execute, smoothness of path, etc.

Outline of today's lecture

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2. General algorithm for solving search problems
    1. First data structure: a frontier list
    2. Second data structure: a search tree
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3. Depth-first search: very fast, but not guaranteed
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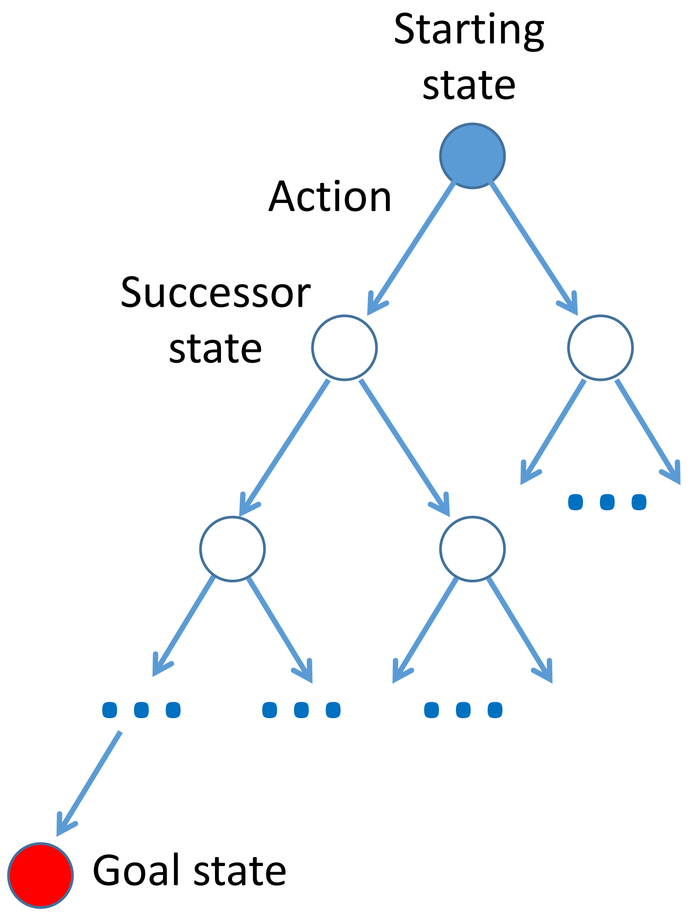
First data structure: a frontier list

- Let's begin at the **start state** and **expand** it
  by making a **list of all possible** (immediate) **successor states**
- Maintain a **frontier**, i.e. a list of unexpanded states
- At each step, **pick a state from the frontier to expand**:
  - Check to see if it's a goal state
  - If not, find the other states that can be reached from this state,
    and add them to the frontier, if they're not already there
- Keep going until you reach a goal state

Second data structure: a search tree

- "What if" tree of sequences of actions and outcomes
- The **root node** corresponds to the starting state
- The **children** of a node correspond to the successor states of that node's state
- A **path** through the tree corresponds to a **sequence of actions**
  - A solution is a path ending in the goal state

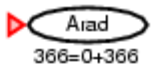


Knowledge Representation: States and Nodes

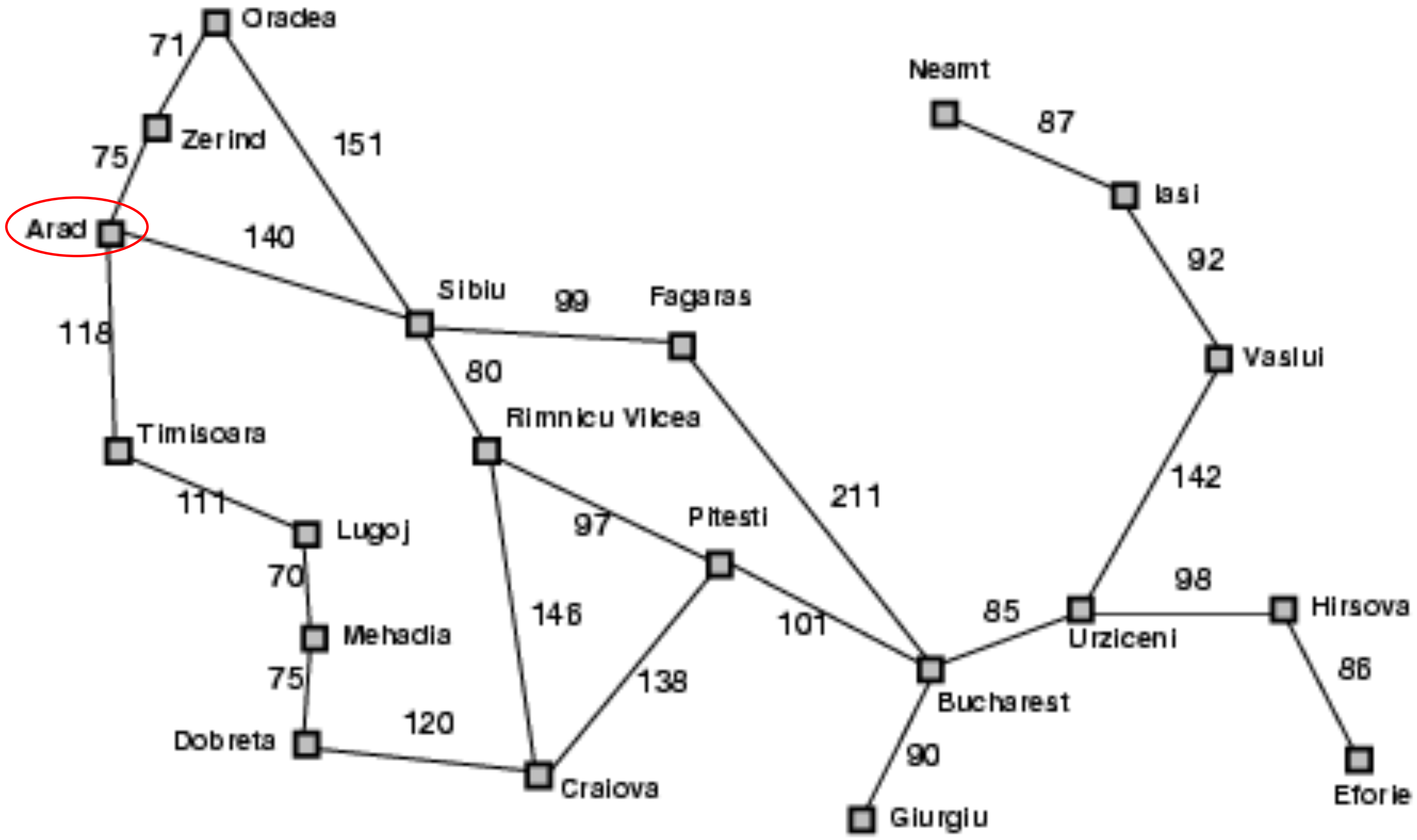
- **State** = description of the world
  - Must have enough detail to decide whether or not you're currently in the **initial state**
  - Must have enough detail to decide whether or not you've reached the **goal state**
  - Often but not always: "defining the state" and "defining the transition model" are the same thing
- **Node** = a point in the search tree
  - Private data: ID of the state reached by this node
  - Private data: the ID of the parent node
  - NB: each state may occur multiple times in the same search tree

Tree Search Algorithm Outline

- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - Choose a frontier node according to **search strategy** and take it off the frontier
  - If the node contains the **goal state**, return solution
  - Else **expand** the node and add its children to the frontier
- **Search strategy** determines
  - Is this process guaranteed to return an OPTIMAL solution?
  - Is this process guaranteed to return ANY solution?
  - Time complexity: How much time does it take?
  - Space complexity: How much RAM is consumed by the frontier?
- For now: assume that search strategy = random



Tree search example



Straight–line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
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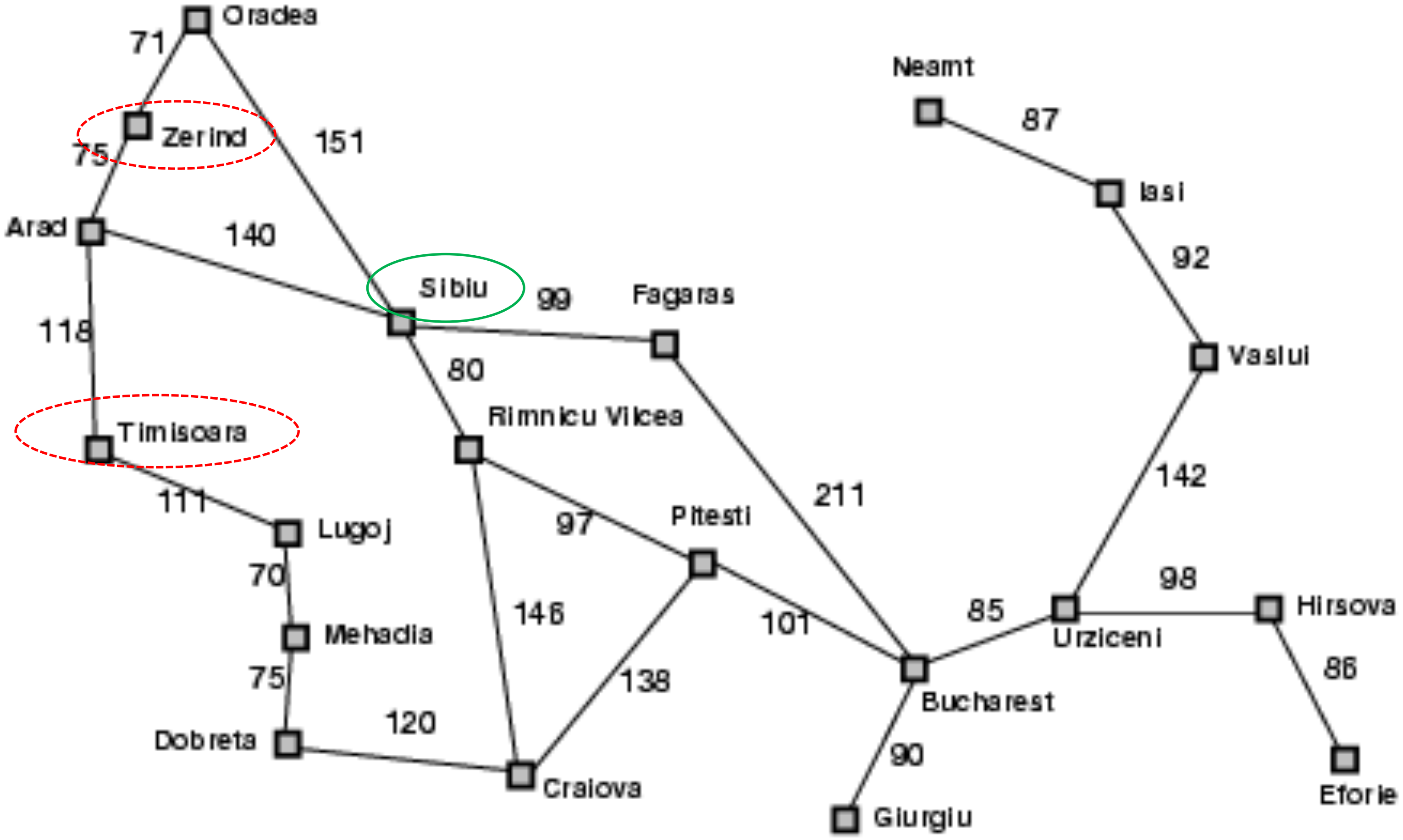
Start: Arad
Goal: Bucharest

Tree search example



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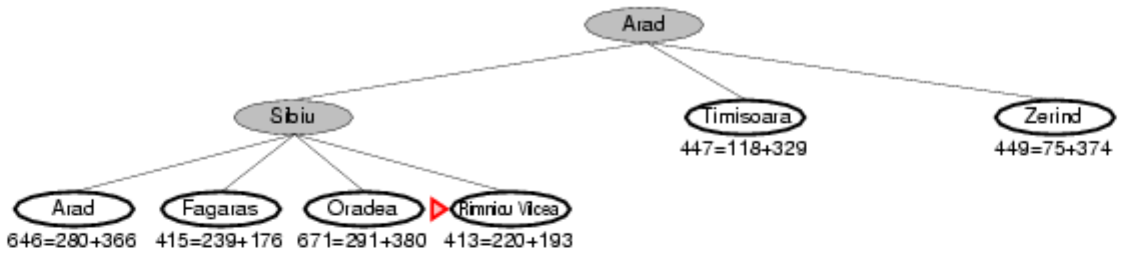
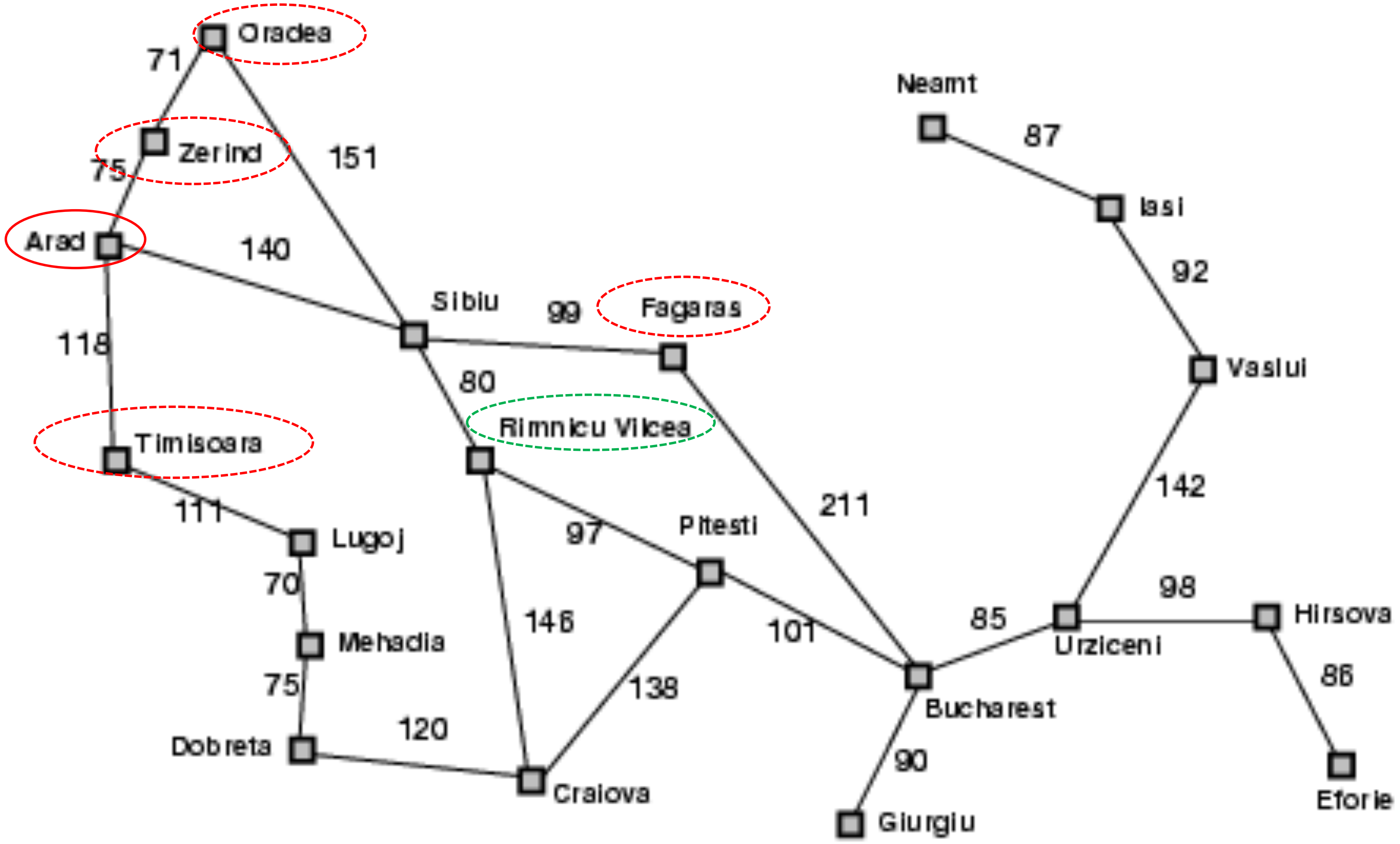
Tree search example





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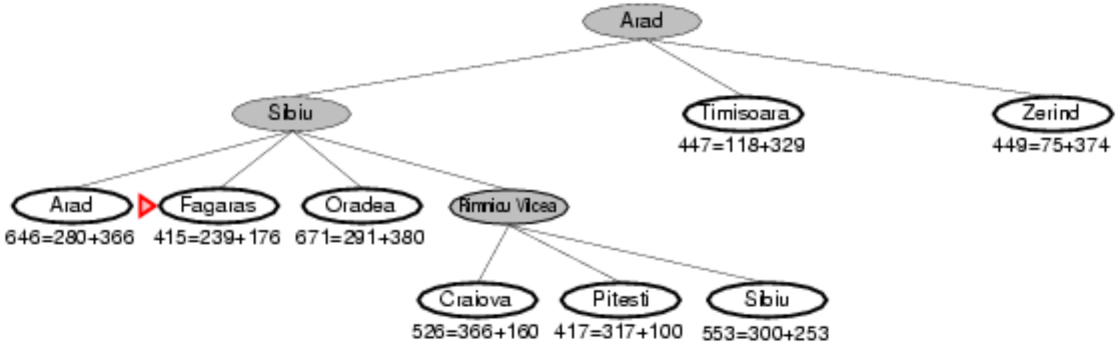

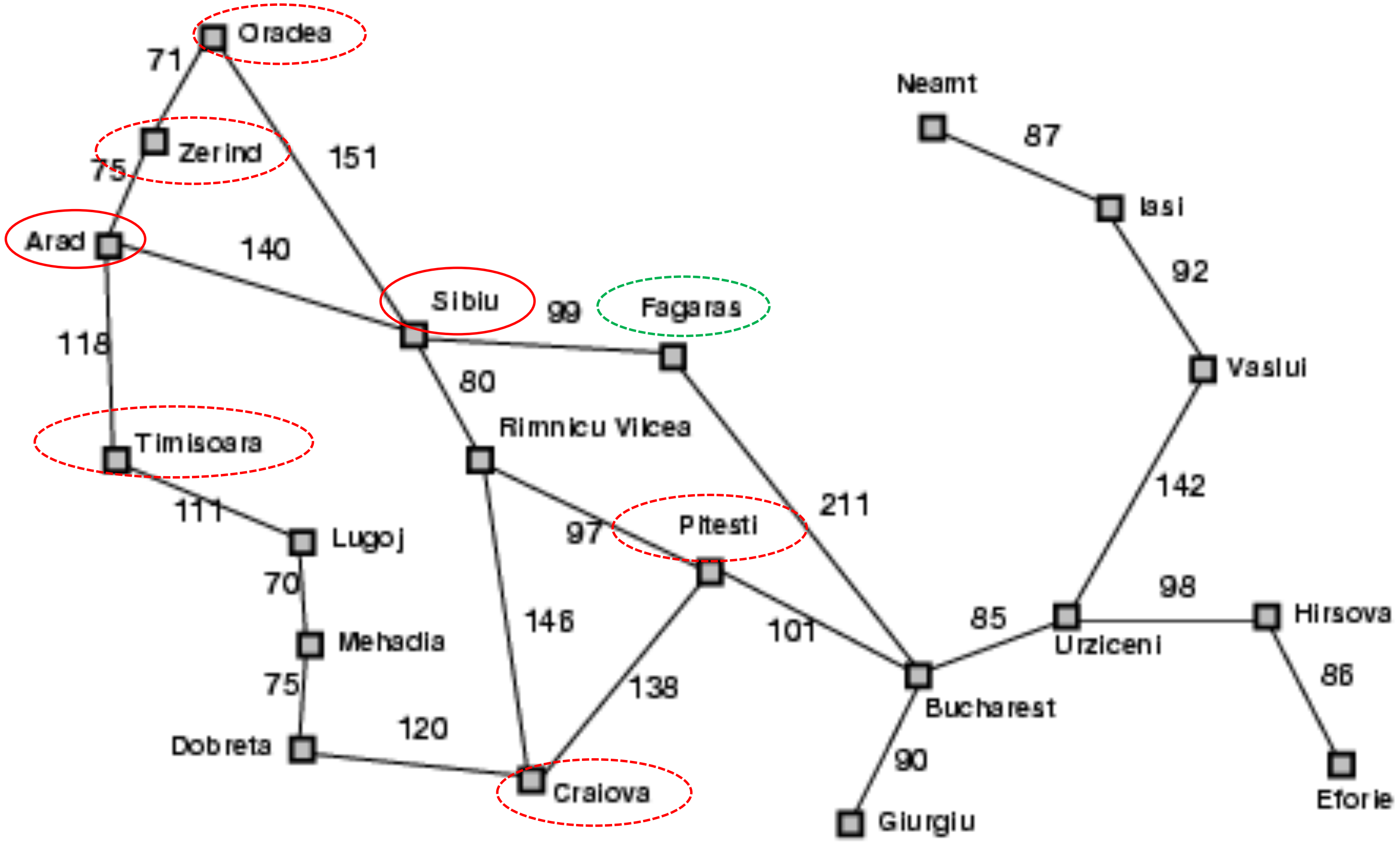

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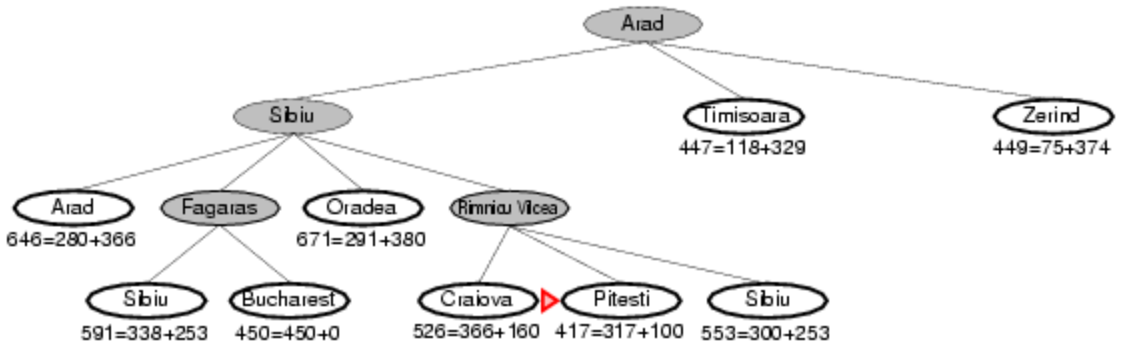
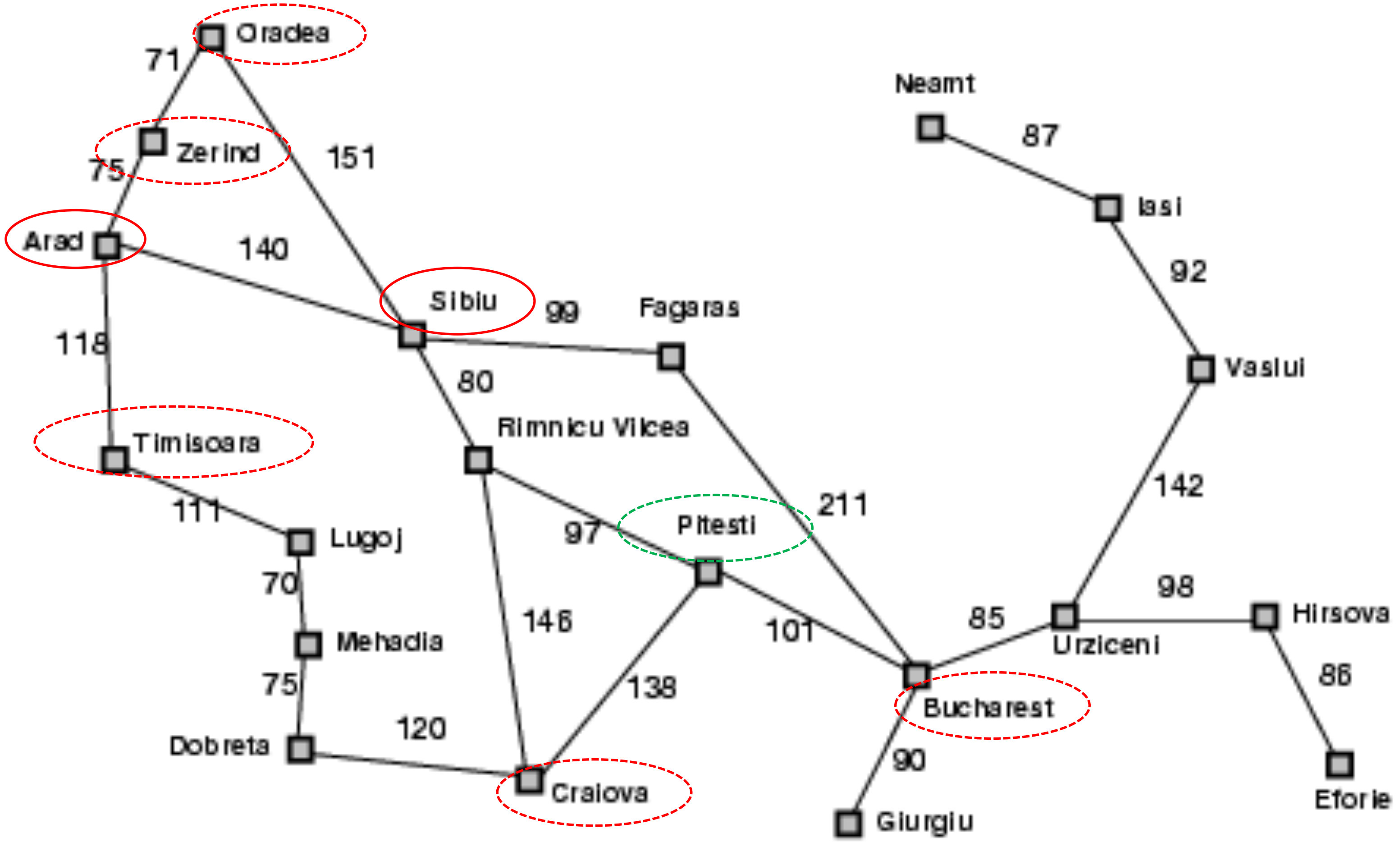
Tree search example





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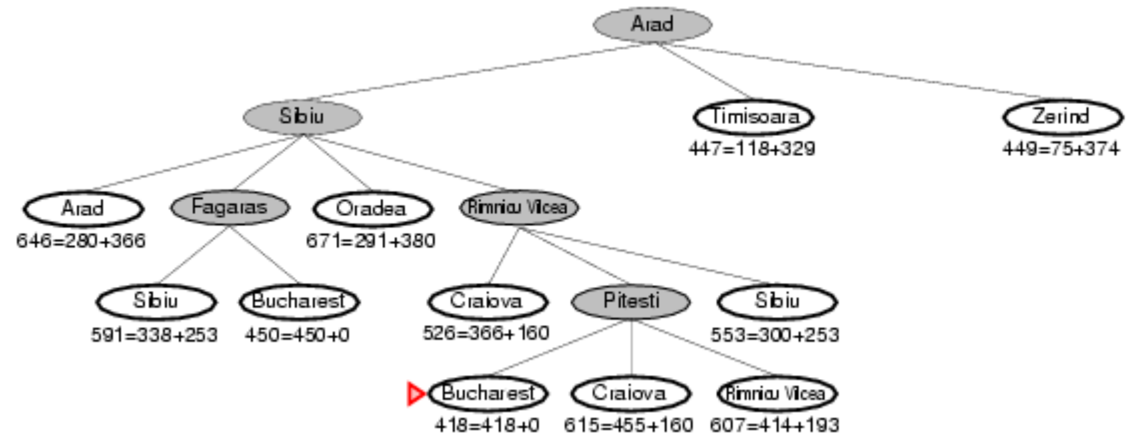
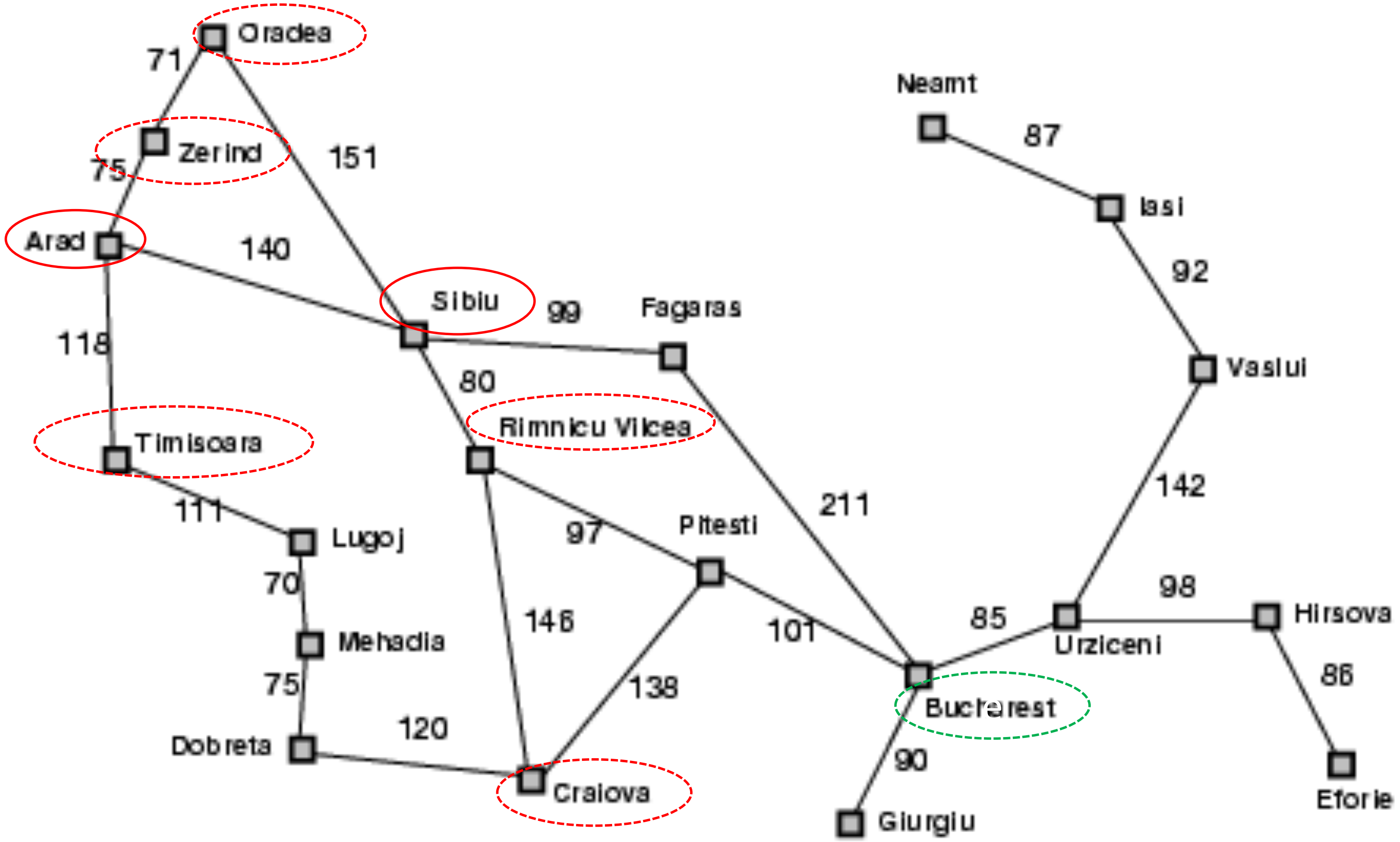
Tree search example





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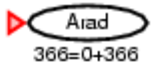
Goal: Bucharest

Handling repeated states

- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - Choose a frontier node according to **search strategy** and take it off the frontier
  - If the node contains the **goal state**, return solution
  - Else **expand** the node and add its children to the frontier
- To handle repeated states:
  - Every time you expand a node, add that state to the **explored set**
  - When adding nodes to the frontier, CHECK FIRST to see if they've already been explored

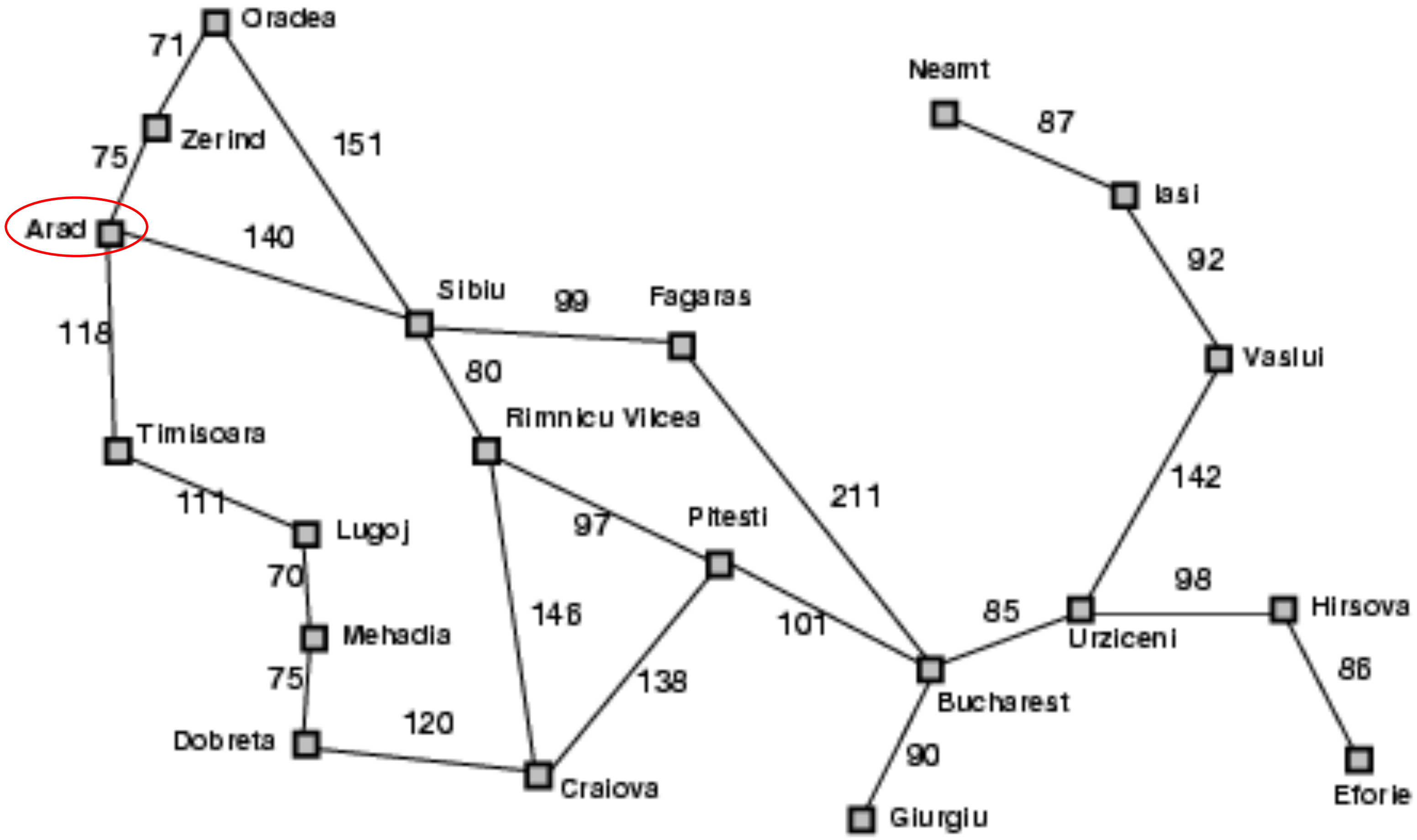
Time Complexity

- Without **explored set** :
  - $O\{1\}$/node
  - $O\{b^m\}$ = # nodes expanded
    - b = branching factor (number of children each node might have)
    - m = length of the longest possible path
- With **explored set** :
  - $O\{1\}$/node using a hash table to see if node is already in **explored set**
  - $O\{|S|\}$ = # nodes expanded
- Usually, $O\{|S|\} < O\{b^m\}$. I'll continue to talk about $O\{b^m\}$, but remember that it's upper-bounded by $O\{|S|\}$.



Tree search w/o repeats

Start: Arad
Goal: Bucharest



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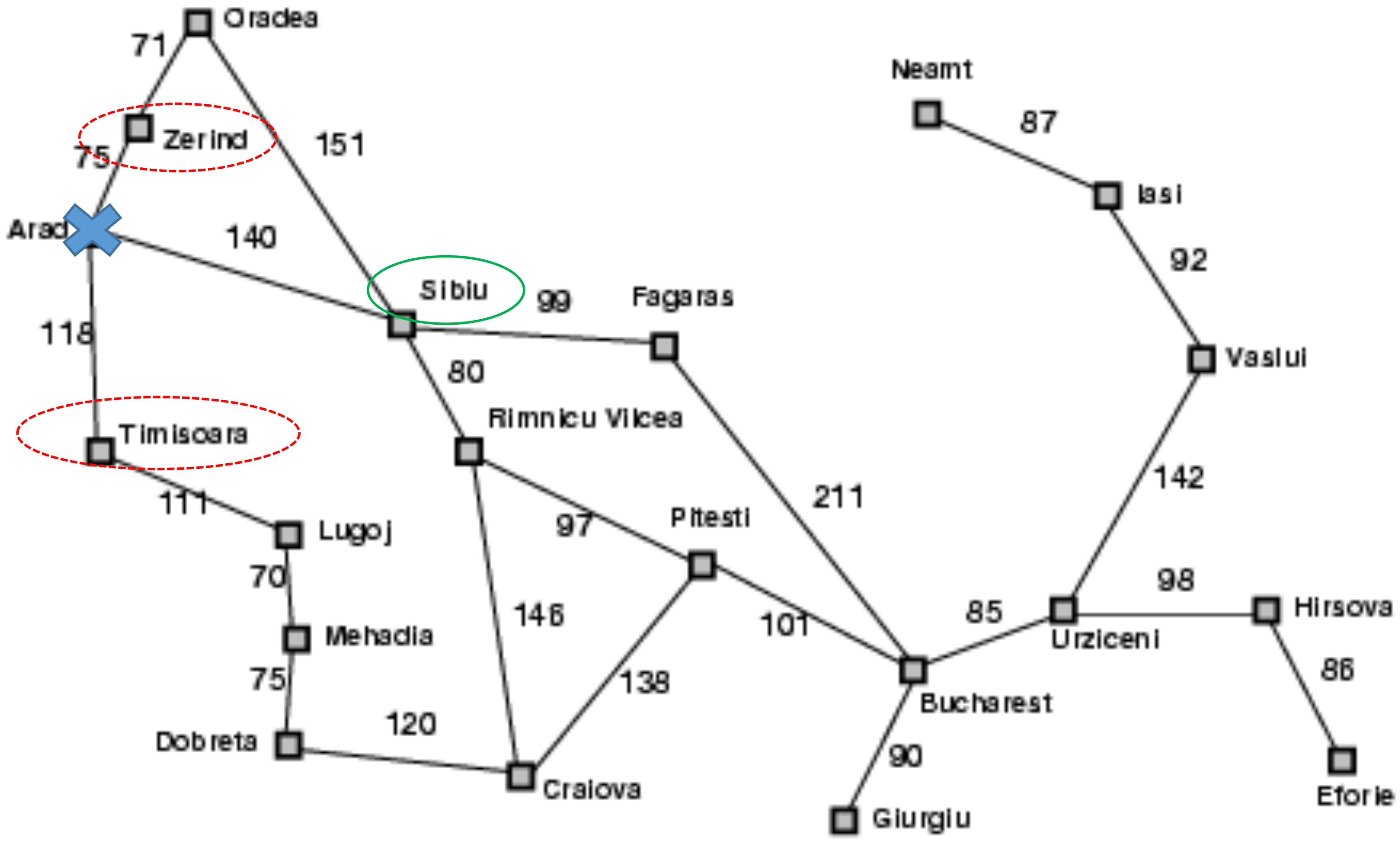
Tree search w/o repeats

Explored:
Arad



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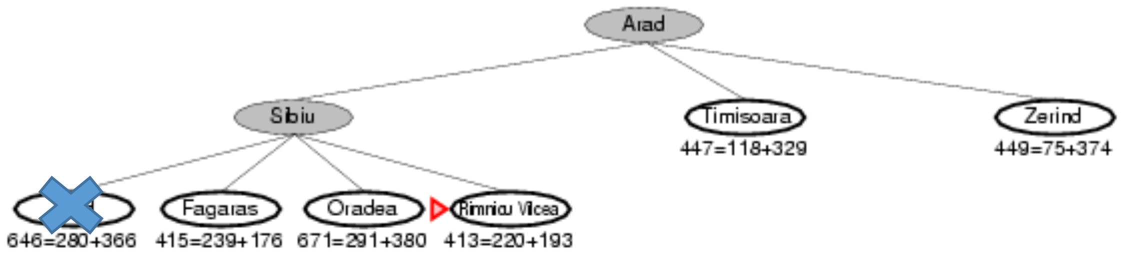
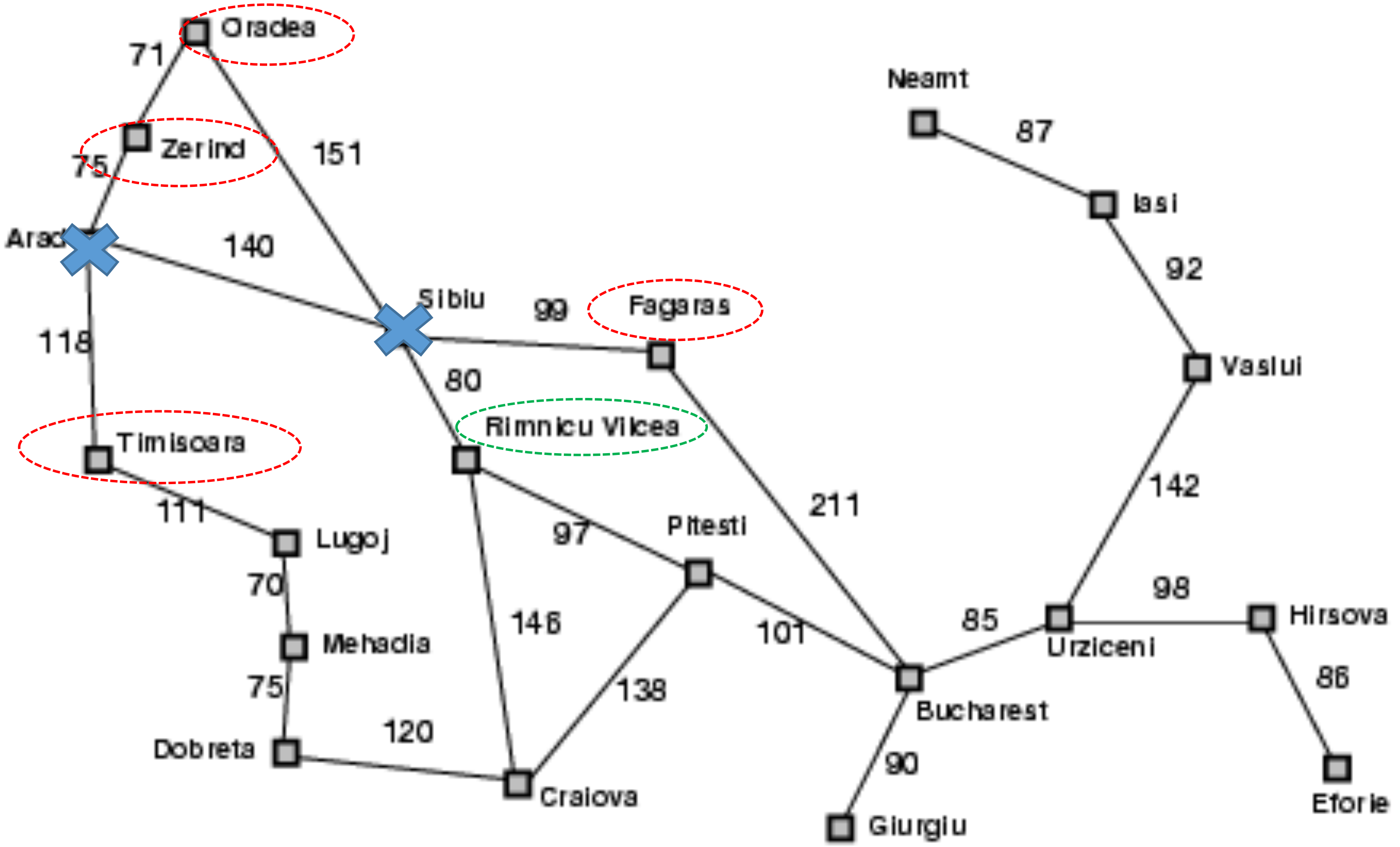
Tree search example



Explored:
Arad
Sibiu



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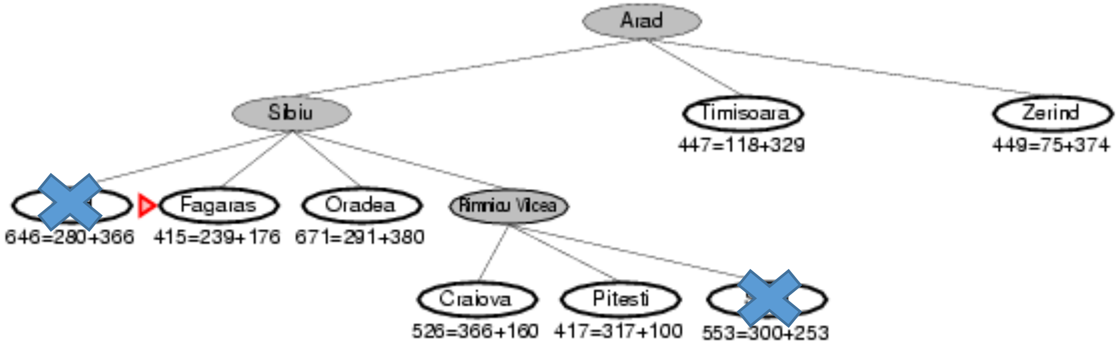
Tree search example



Explored:
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Sibiu
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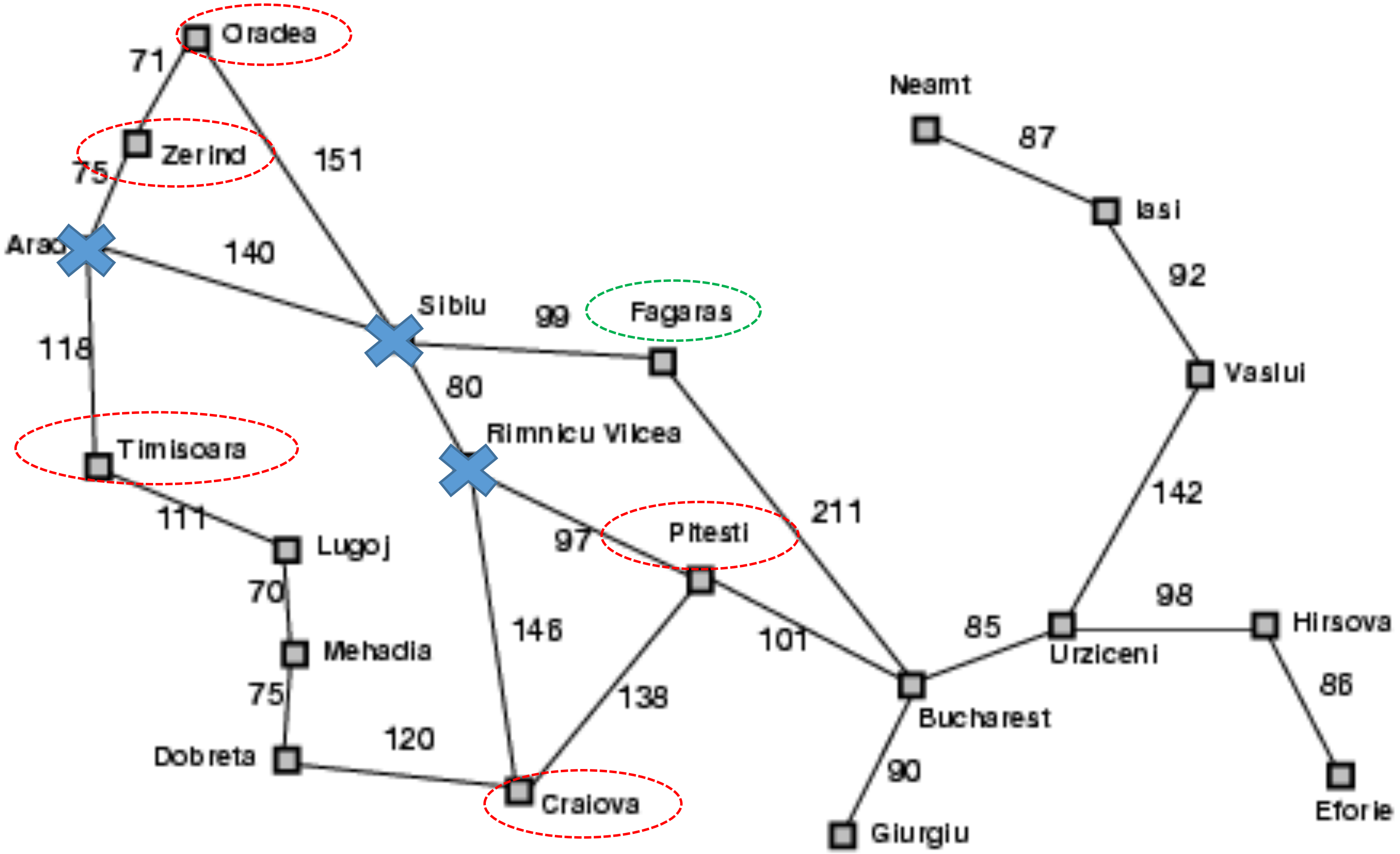

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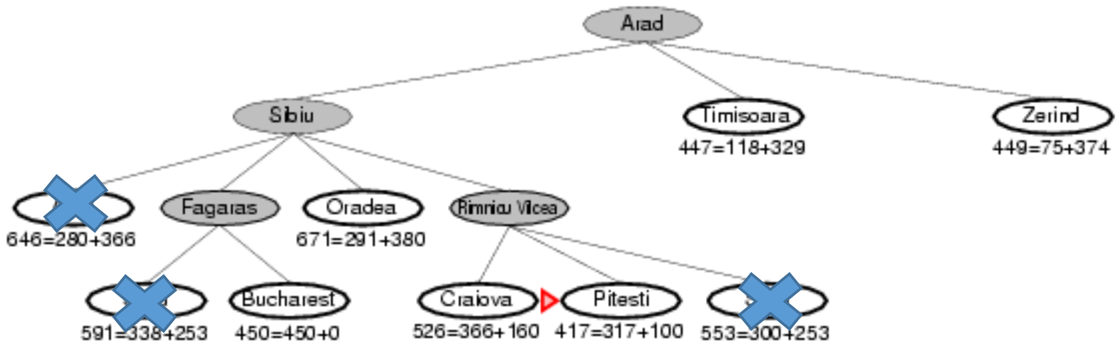

Explored:
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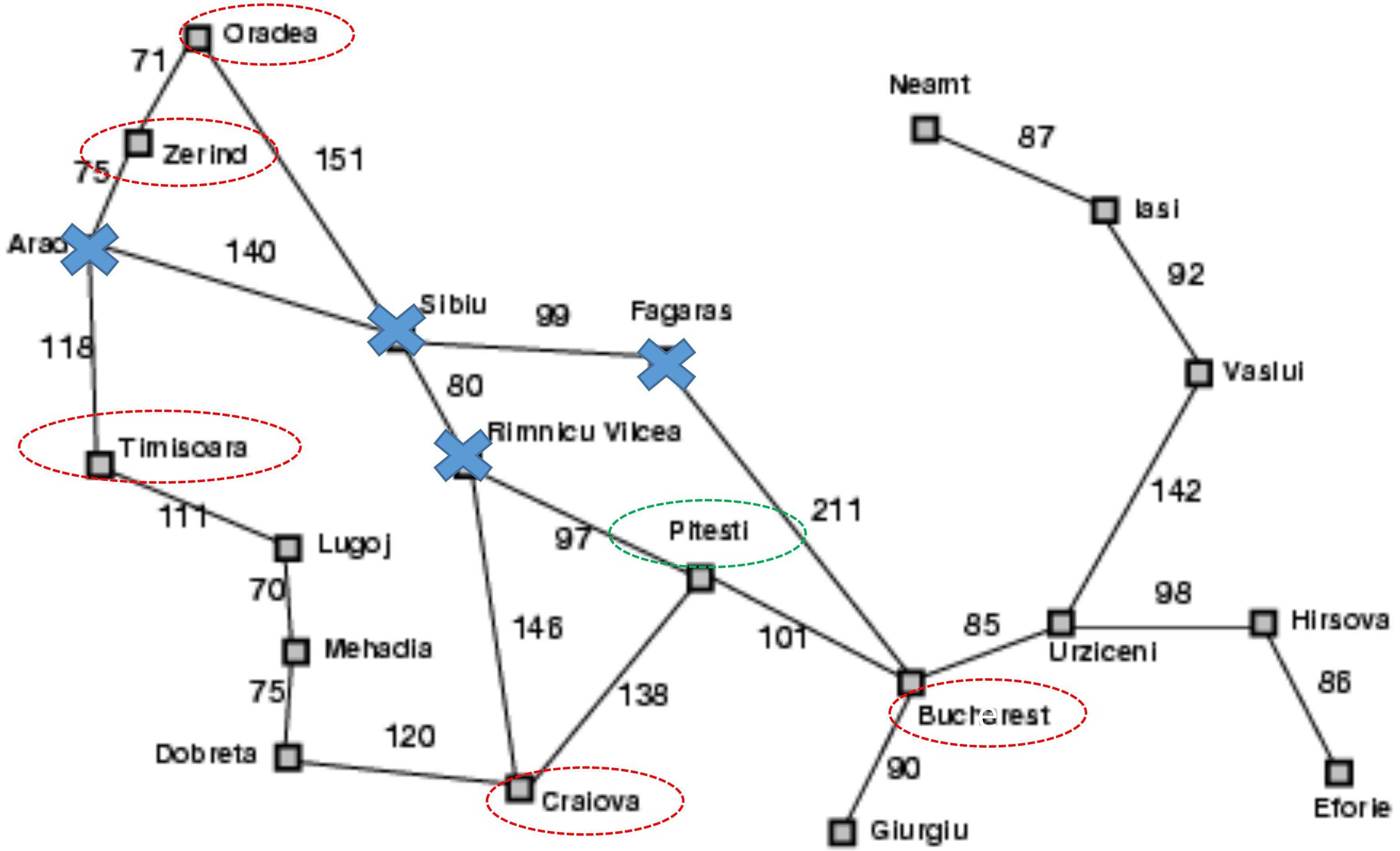
Sibiu
Rimnicu Vilces
Fagaras



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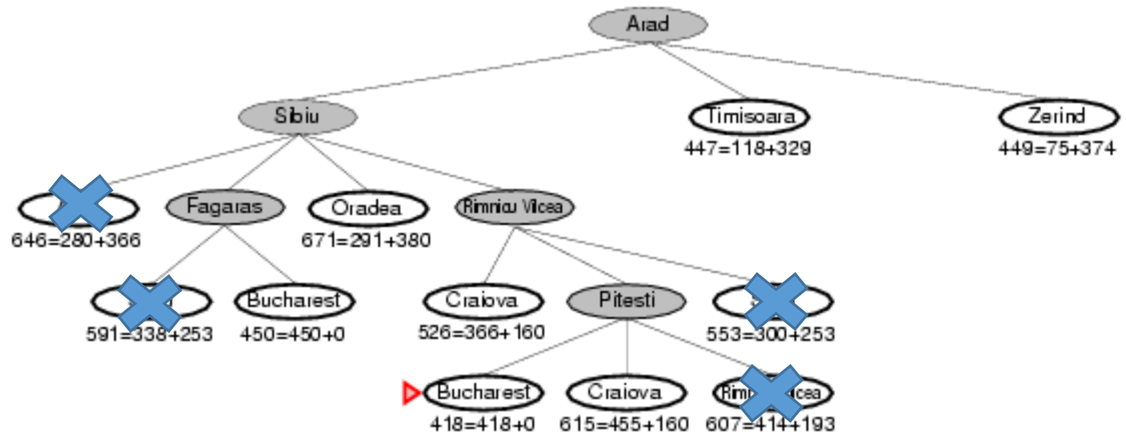

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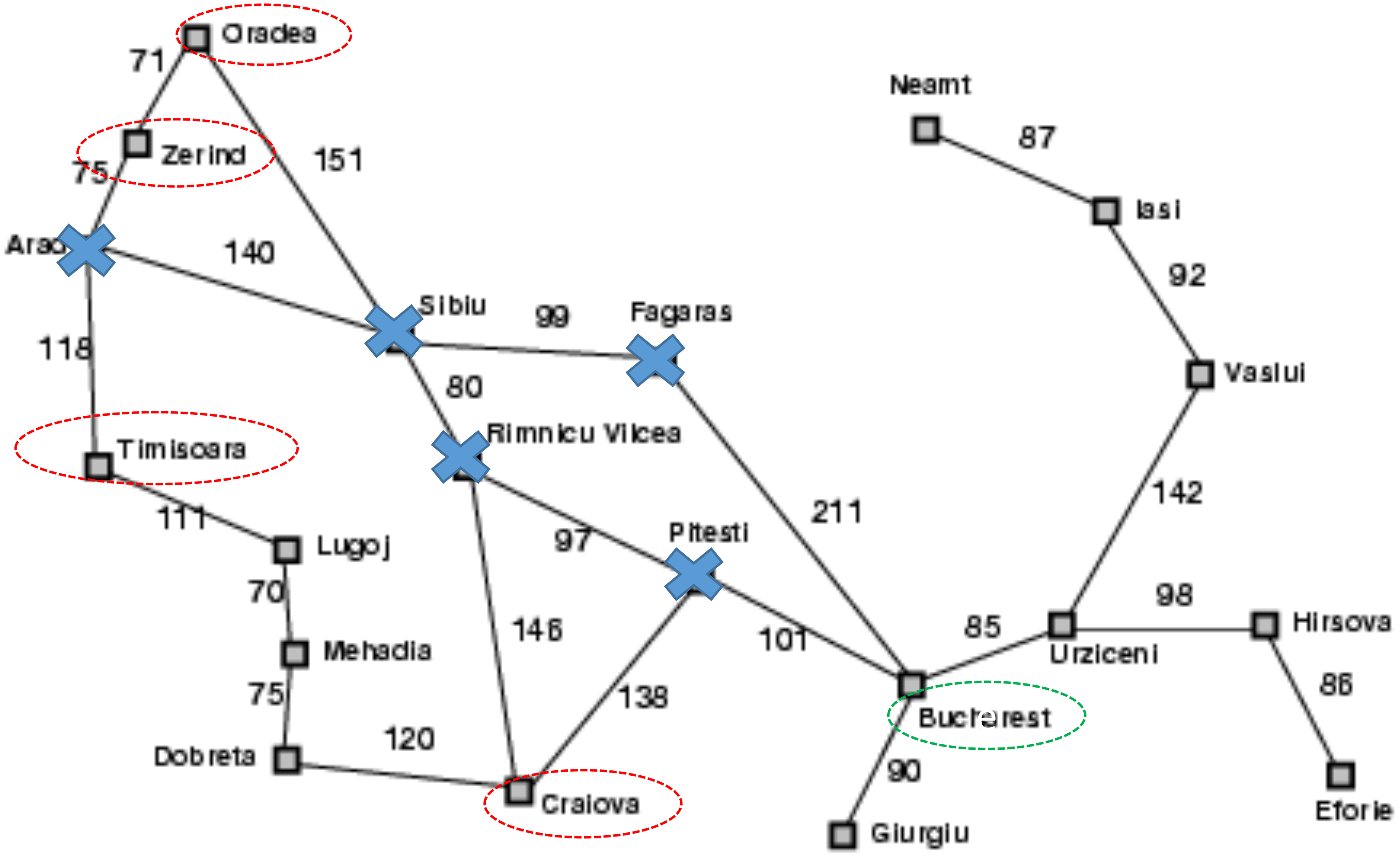
Sibiu
Rinnicu Vilces
Fagaras
Pitesti



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Outline of today's lecture

1. How to define search problems:
   1. Initial state, goal state, transition model
   2. Actions, path cost
2. General algorithm for solving search problems
   1. First data structure: a frontier list
   2. Second data structure: a search tree
   3. Third data structure: a "visited states" list
3. Depth-first search: very fast, but not guaranteed
4. Breadth-first search: guaranteed optimal
5. Uniform cost search = Dijkstra's algorithm = BFS with variable costs

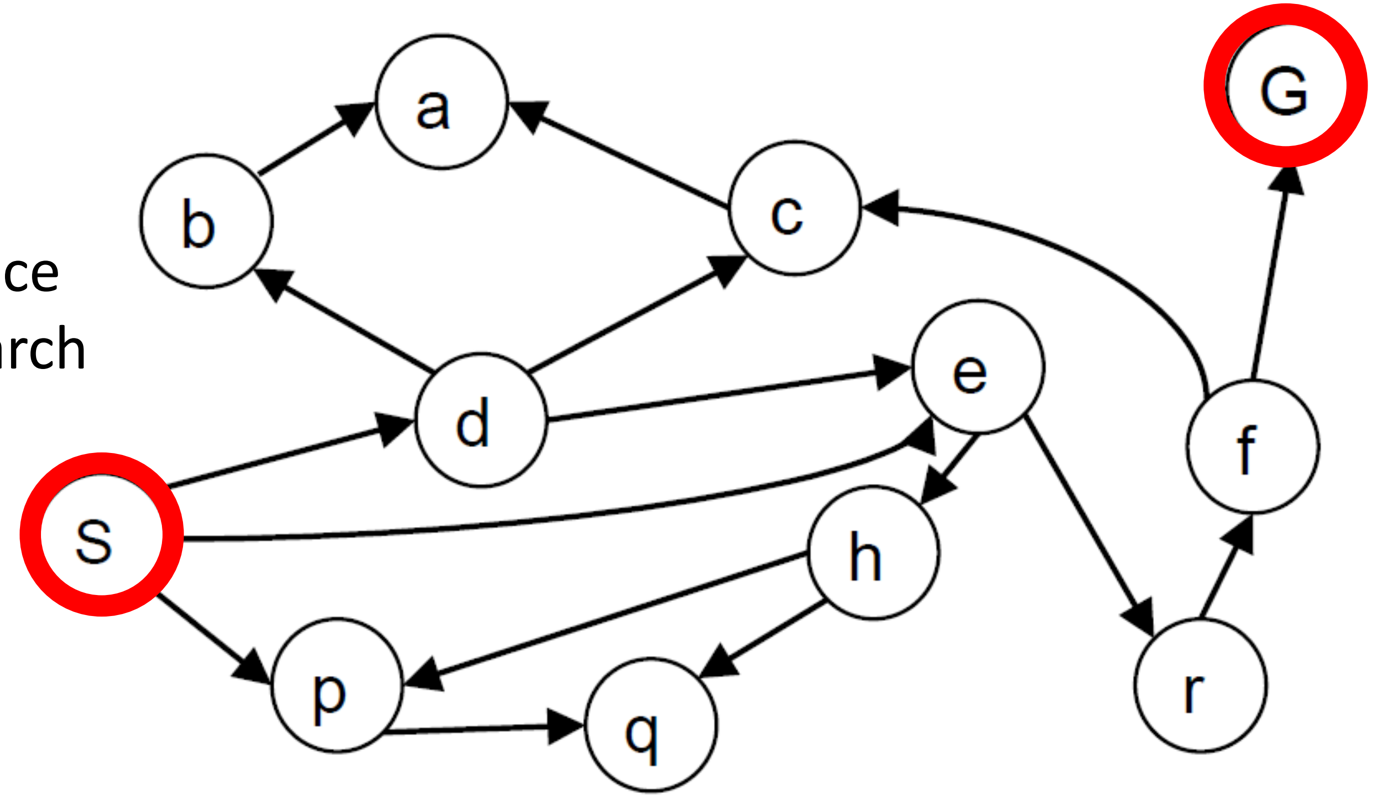
Depth-First Search

- Basic idea:
  - Try to find a solution as fast as possible
- How:
  - From the frontier, always choose a node which is AS FAR FROM THE STARTING POINT AS POSSIBLE
- How:
  - Frontier is a LIFO (last-in, first-out) stack.
  - The node you expand = whichever node has been most recently placed on the queue.

Depth-first search

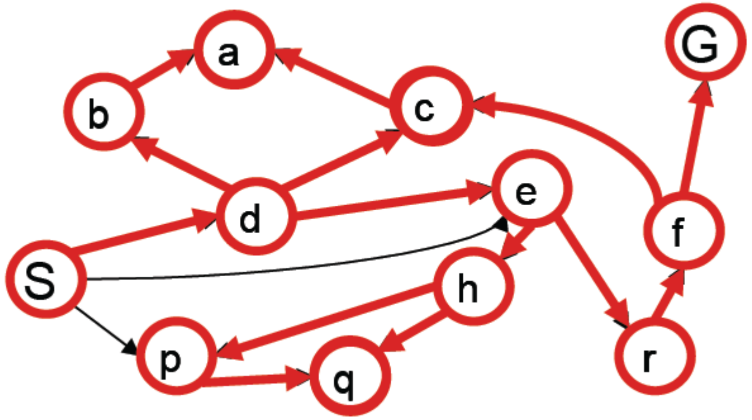
- Expand deepest unexpanded node
- Implementation: *frontier* is LIFO (a stack)

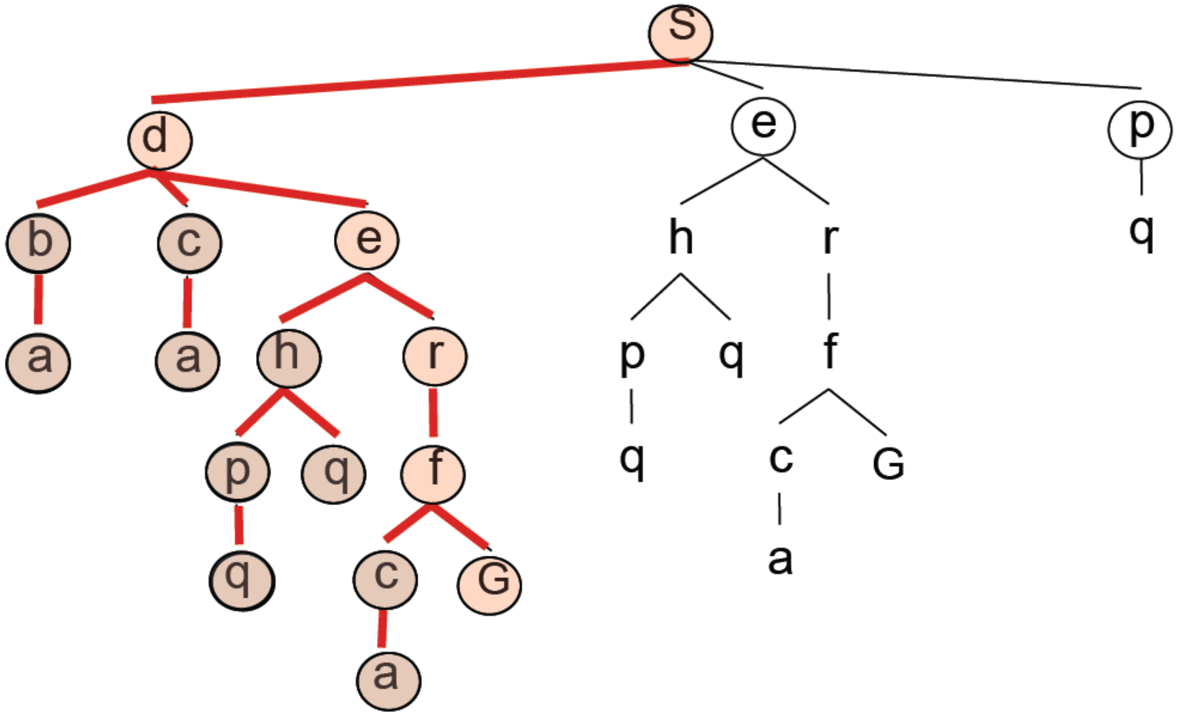
Example state space graph for a tiny search problem

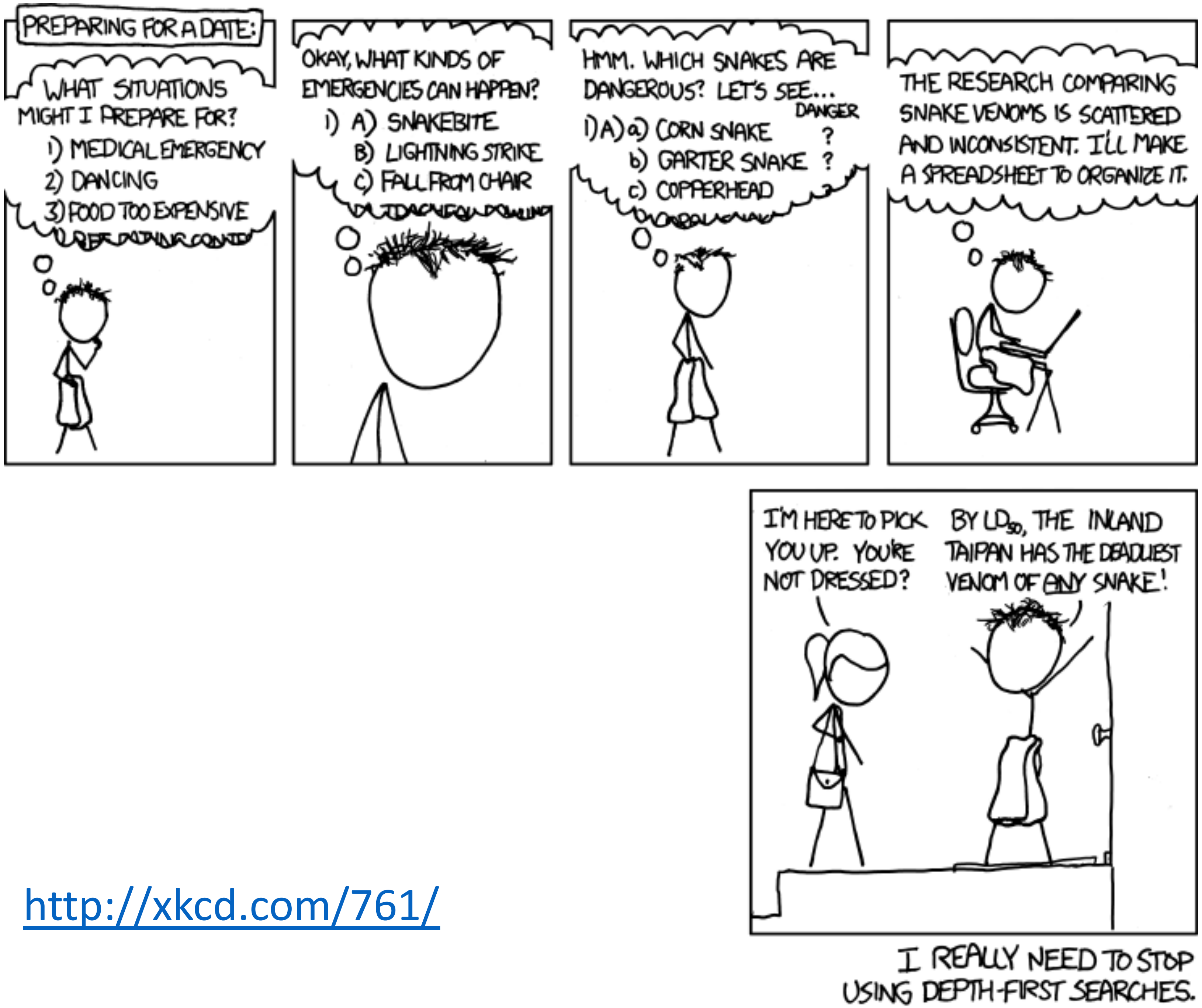


Depth-first search

Expansion order:
(s,d,b,a,
c,a,
e,h,p,q,
q,
r,f,c,a,
G)







http://xkcd.com/761/

Analysis of search strategies

- **Strategies are evaluated** along the following criteria:
  - **Completeness:** does it always find a solution if one exists?
  - **Optimality:** does it always find a least-cost solution?
  - **Time complexity:** number of nodes generated
  - **Space complexity:** maximum number of nodes in memory
- **Time and space complexity** are measured in terms of
  - b: maximum **branching factor** of the search tree
  - d: **depth of the optimal solution**
  - m: **maximum length of any path** in the state space (may be infinite)
  - $|S|$: number of distinct states

Properties of depth-first search

- **Complete? (always finds a solution if one exists?)**
  - Fails in infinite-depth spaces, spaces with loops
  - Modify to avoid repeated states along path
    - → complete in finite spaces
- **Optimal? (always finds an optimal solution?)**
  - No – returns the first solution it finds
- **Time? (how long does it take, in terms of b, d, m?)**
  - Could be the time to reach a solution at maximum depth *m*: $O(b^m)$
  - Terrible if *m* is much larger than *d*
  - But VERY FAST if there are LOTS of solutions
- **Space? (how much storage space, in terms of b, d, m?)**
  - $O(bm)$, i.e., linear space!

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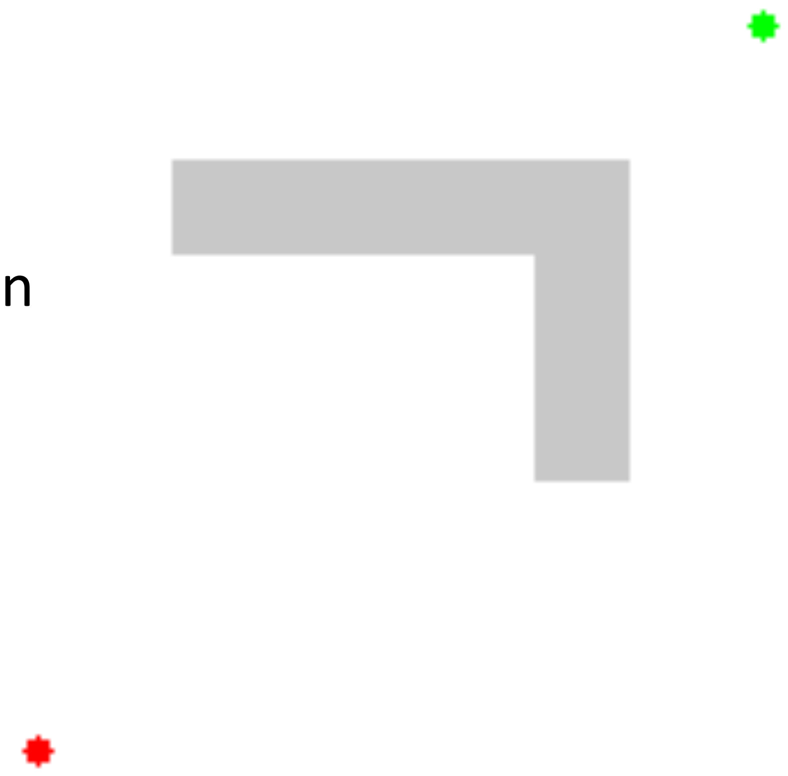
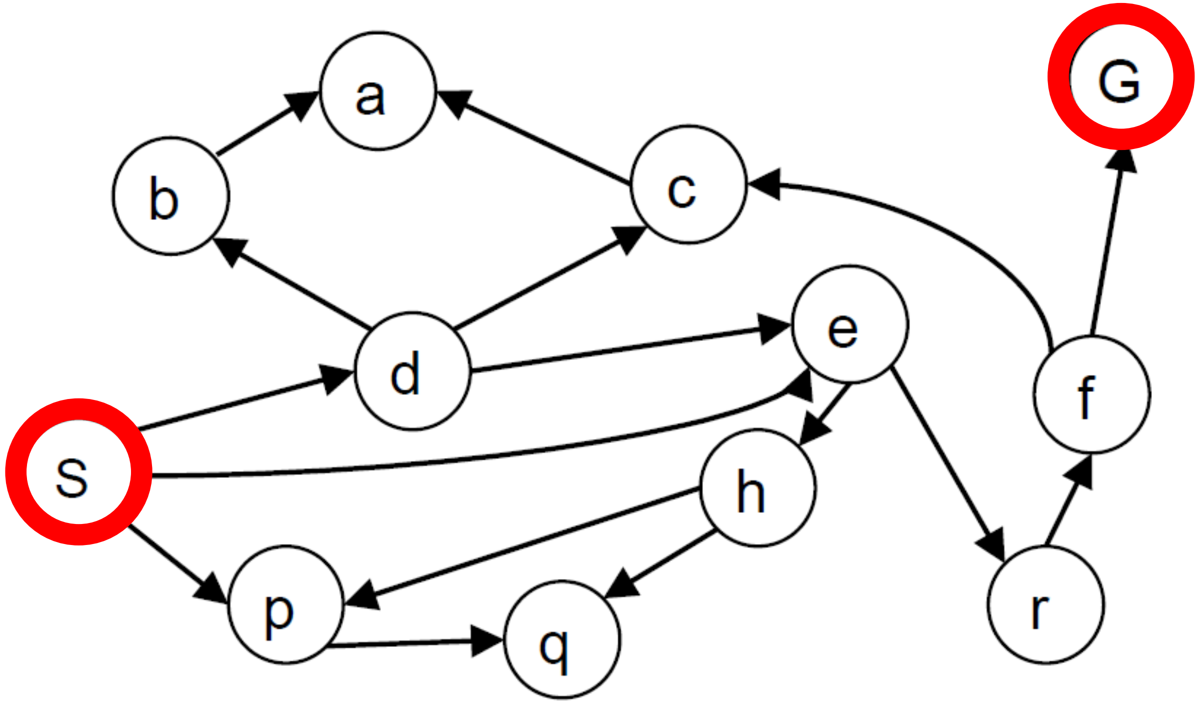
Breadth-first search

- Initialize the **frontier** using the **starting state**
- While the frontier is not empty
  - **Search strategy:** choose one of the nodes which is CLOSEST to the starting state
  - If the node contains the **goal state**, return solution
  - Else **expand** the node and add its children to the frontier



Breadth-first search

- Expand shallowest unexpanded node
- Implementation: *frontier* is FIFO (first-in, first out) (a queue)



Example from P. Abbeel and D. Klein

Breadth-first search

Expansion order:

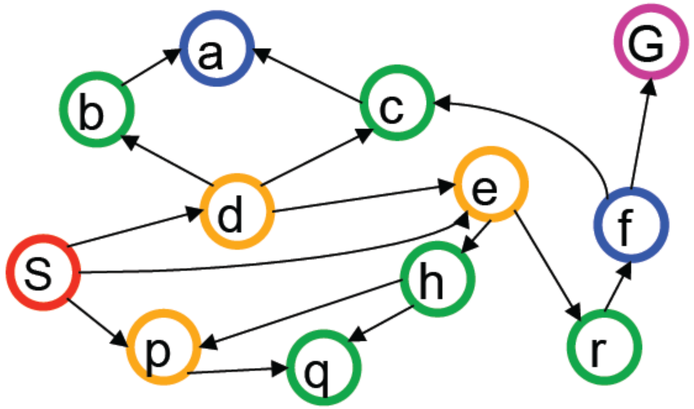
(s,

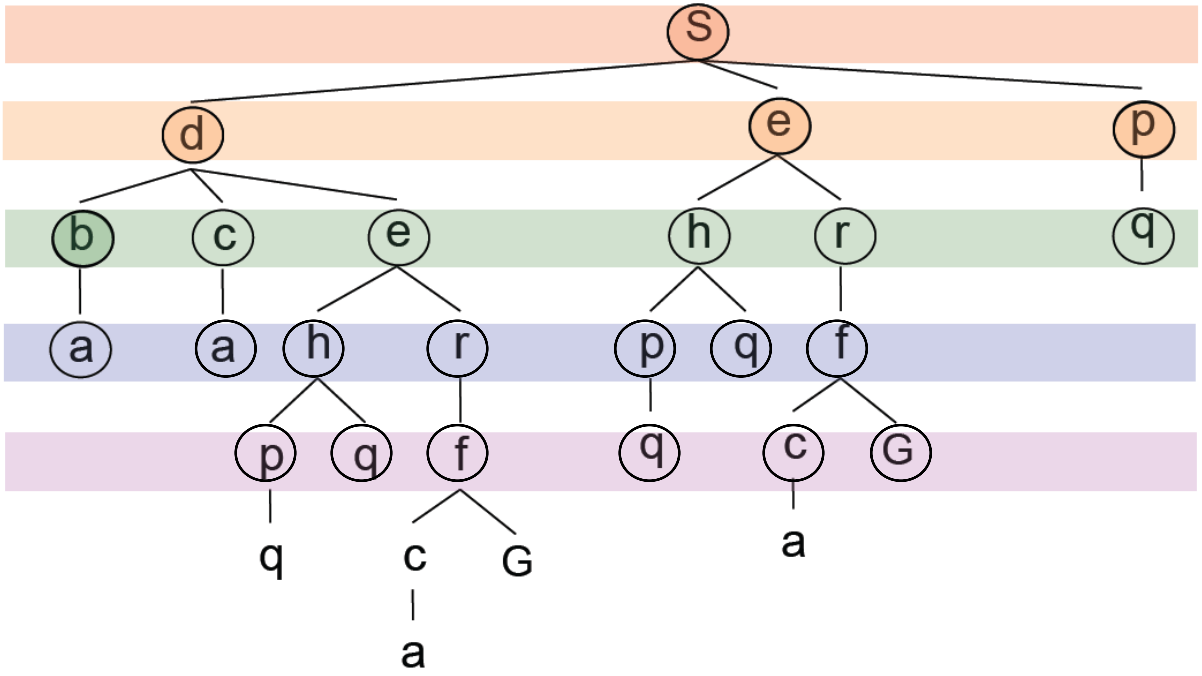
d,e,p,

b,c,e,h,r,q,

a,a,h,r,p,q,f,

p,q,f,q,c,G)





Properties of breadth-first search

- **Complete?**
  Yes (if branching factor *b* is finite).
  Even w/o repeated-state checking, it still works!!!
- **Optimal?**
  Yes – if cost = 1 per step (uniform cost search will fix this)
- **Time?**
  Number of nodes in a *b*-ary tree of depth *d*: $O(b^d)$
  (*d* is the depth of the optimal solution)
- **Space?**
  $O(b^d)$. --- much larger than DFS!

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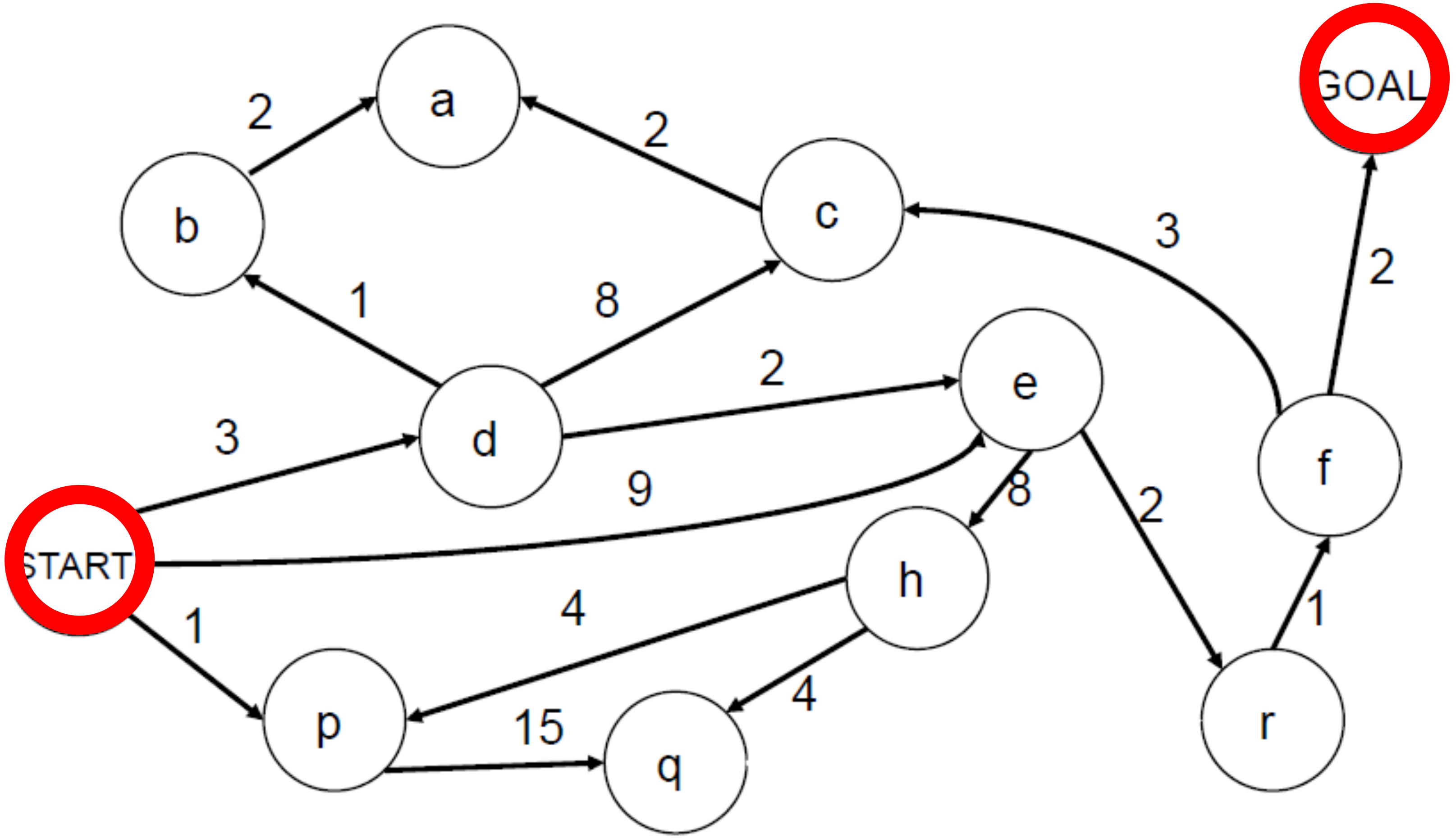
Uniform-cost search = Dijkstra’s algorithm

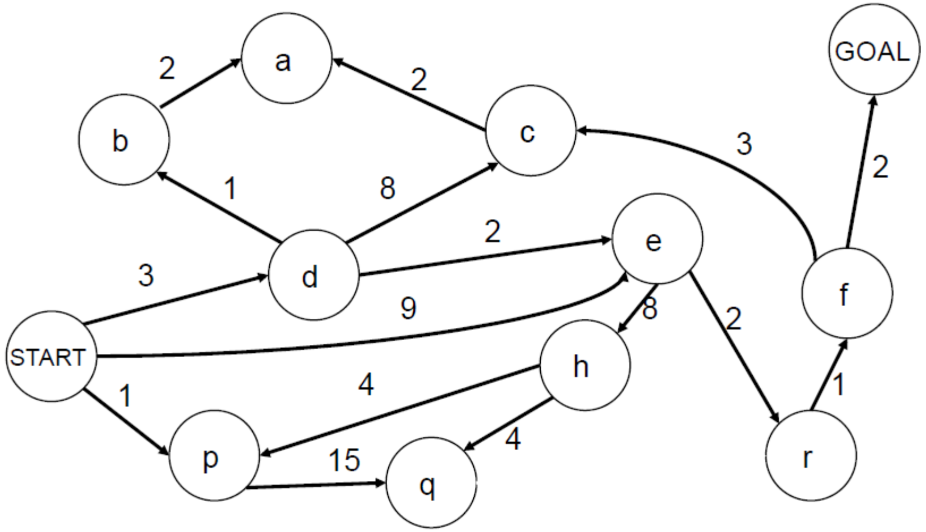
- For each frontier node, save the **total cost of the path from the initial state to that node**
- Expand the **frontier node with the lowest path cost**
- Implementation:
  frontier is a **priority queue ordered by path cost**
- Equivalent to breadth-first if step costs all equal
- Equivalent to Dijkstra’s algorithm, if Dijkstra’s algorithm is modified so that a node’s value is computed only when it becomes nonzero

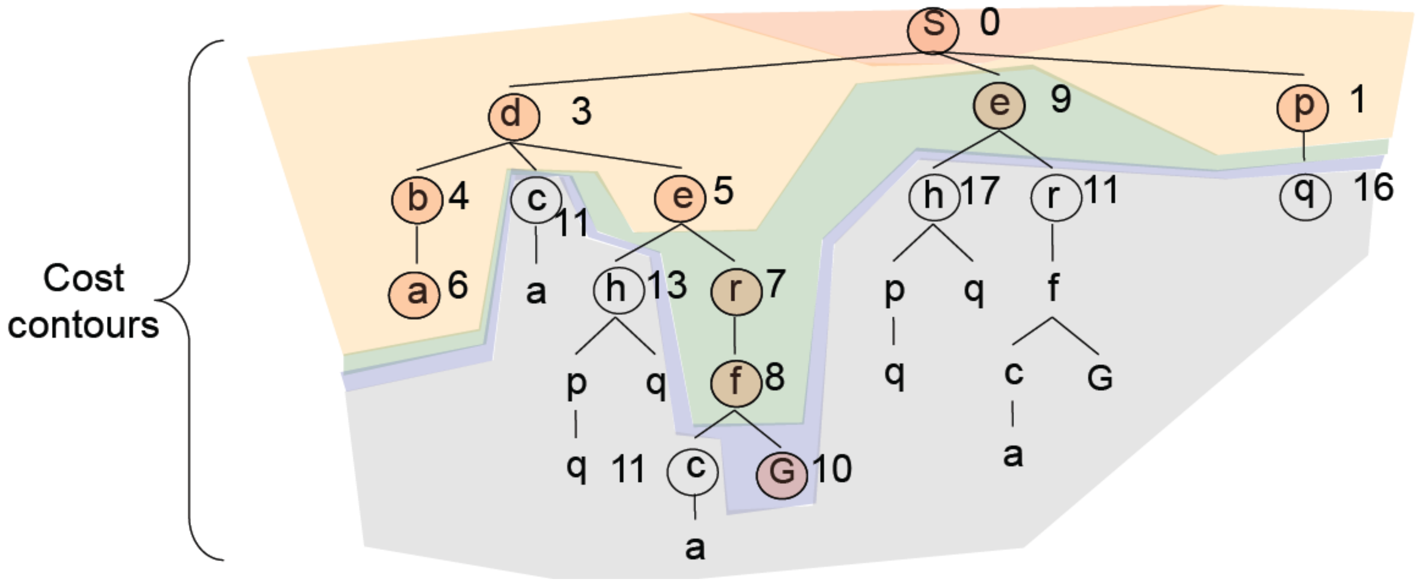
Uniform-cost search example



Uniform-cost search example

Expansion order:
(s,p(1),
d(3),b(4),
e(5),r(7),f(8)
e(9),
G(10))





Properties of uniform-cost search

- **Complete?**

  Yes (if branching factor b is finite).

  Even w/o repeated-state checking, it still works!!!

- **Optimal?**

  Yes

- **Time?**

  Number of nodes in a b-ary tree of depth d: $O\{b^d\}$

  Priority queue is $O\{\log_2 d\}$/node

- **Space?**

  $O\{b^d\}$ --- much larger than DFS! This might be a reason to use DFS.

Search strategies so far

Algorithm	Complete?	Optimal?	Time complexity	Space complexity	Implement the Frontier as a…
BFS	Yes	If all step costs are equal	$O\{b^d\}$	$O\{b^d\}$	Queue
DFS	No	No	$O\{b^m\}$	$O\{bm\}$	Stack
UCS	Yes	Yes	$O\{b^d \log_2 d\}$	$O\{b^d\}$	Priority Queue

Next time

- know how far it is, from the start point, to each node on the frontier.
- What if we also have an ESTIMATE of the distance from each node to the GOAL?