CS440/ECE448 Lecture 29: Review II

Final Exam Mon, May 6, 9:30–10:45

Covers all lectures after the first exam.

Same format as the first exam.

Location (if you're in Prof. Hockenmaier's sections) Materials Science and Engineering Building, Room 100 (http://ada.fs.illinois.edu/0034.html)

Conflict exam: Wed, May 8, 9:30–10:45

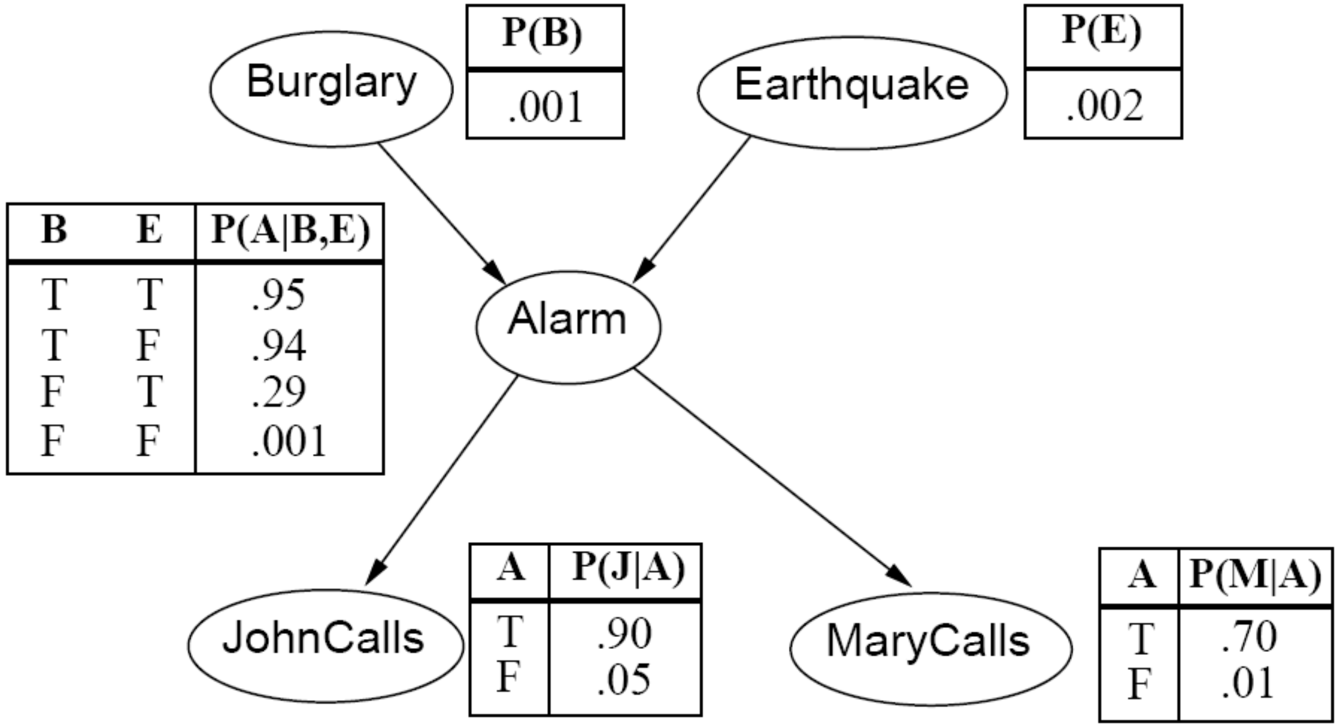
Location: Siebel 3403.

If you need to take your exam at DRES, make sure to notify DRES in advance

CS 440/ECE448 Lecture 19: Bayes Net Inference

Mark Hasegawa-Johnson, 3/2019 modified by Julia Hockenmaier 3/2019

Including slides by Svetlana Lazebnik, 11/2016



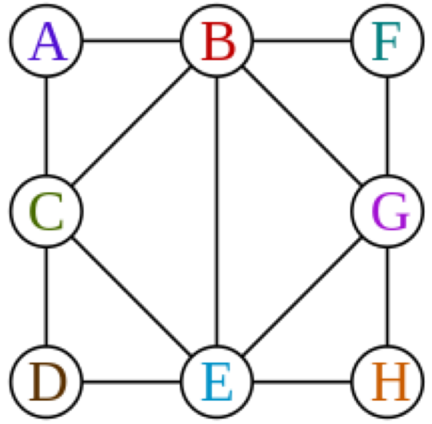
P(B)
.001

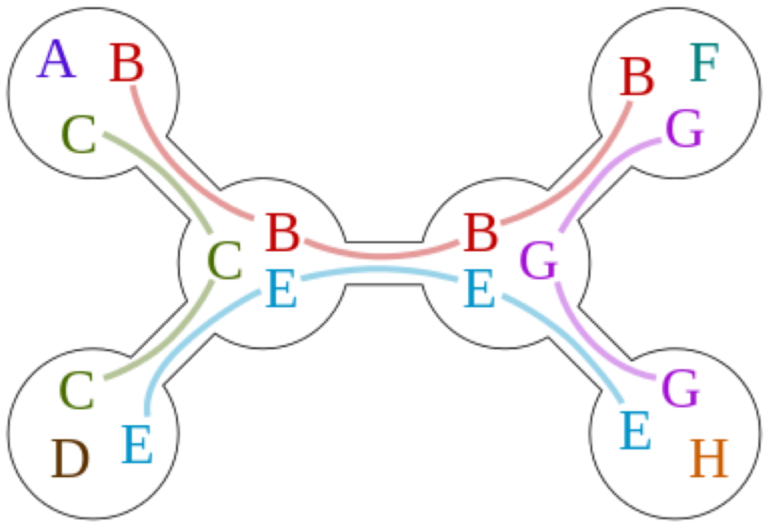
P(E)
.002

B	E	P(A|B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.05

A	P(M|A)
T	.70
F	.01



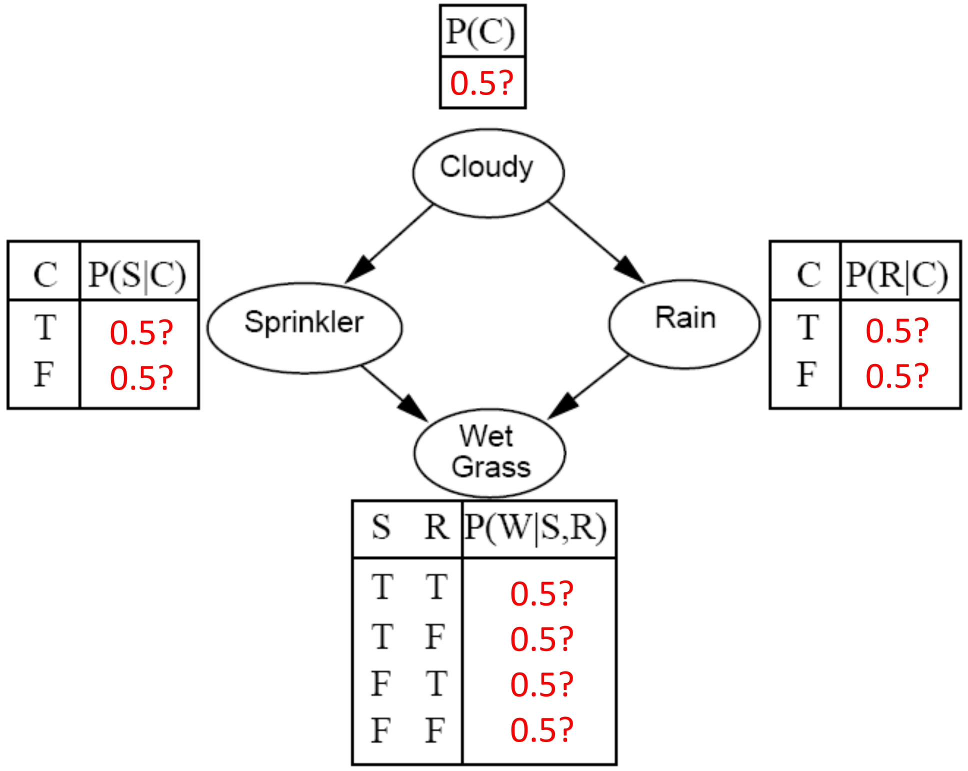


Parameter learning

- **Inference problem**: given values of evidence variables $\mathbf{E} = \mathbf{e}$, answer questions about query *variables* $\mathbf{X}$ using the posterior $P(\mathbf{X} \mid \mathbf{E} = \mathbf{e})$
- **Learning problem:** estimate the parameters of the probabilistic model $P(\mathbf{X} \mid \mathbf{E})$ given a *training sample* $\{(\mathbf{x}_1,\mathbf{e}_1), \ldots, (\mathbf{x}_n,\mathbf{e}_n)\}$
- **Learning from complete observations:** relative frequency estimates
- **Learning from data with missing observations:** EM algorithm

Missing data: the EM algorithm

- The EM algorithm starts ("Expectation Maximization") starts with an initial guess for each parameter value.
- We try to improve the initial guess, using the algorithm on the next two slides:
  - E-step
  - M-step



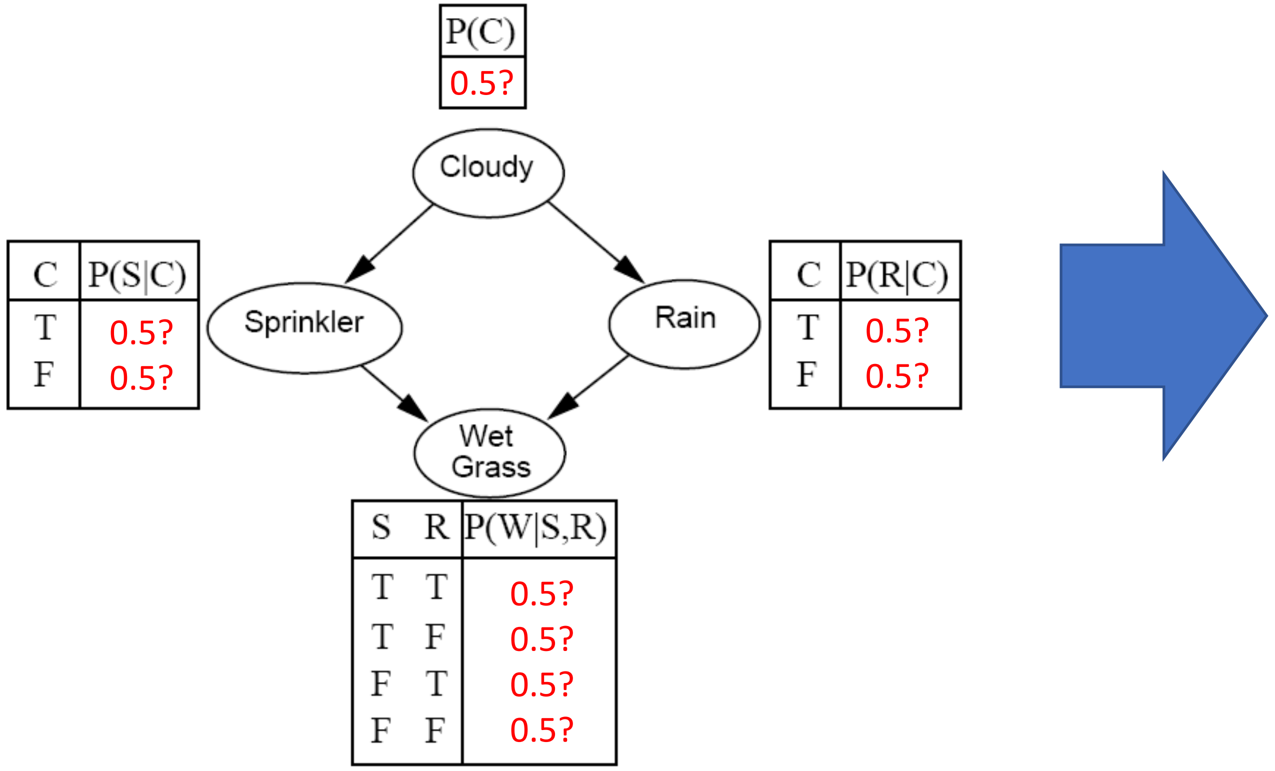
Training set

Sample	C	S	R	W
1	?	F	T	T
2	?	T	F	T
3	?	F	F	F
4	?	T	T	T
5	?	T	F	T
6	?	F	T	F
…	…	…	….	…

Missing data: the EM algorithm

- E-Step (Expectation): Given the model parameters, replace each of the missing numbers with a probability (a number between 0 and 1) using

$$P(C = 1|S, R, W) = \frac{P(C = 1, S, R, W)}{P(C = 1, S, R, W) + P(C = 0, S, R, W)}$$



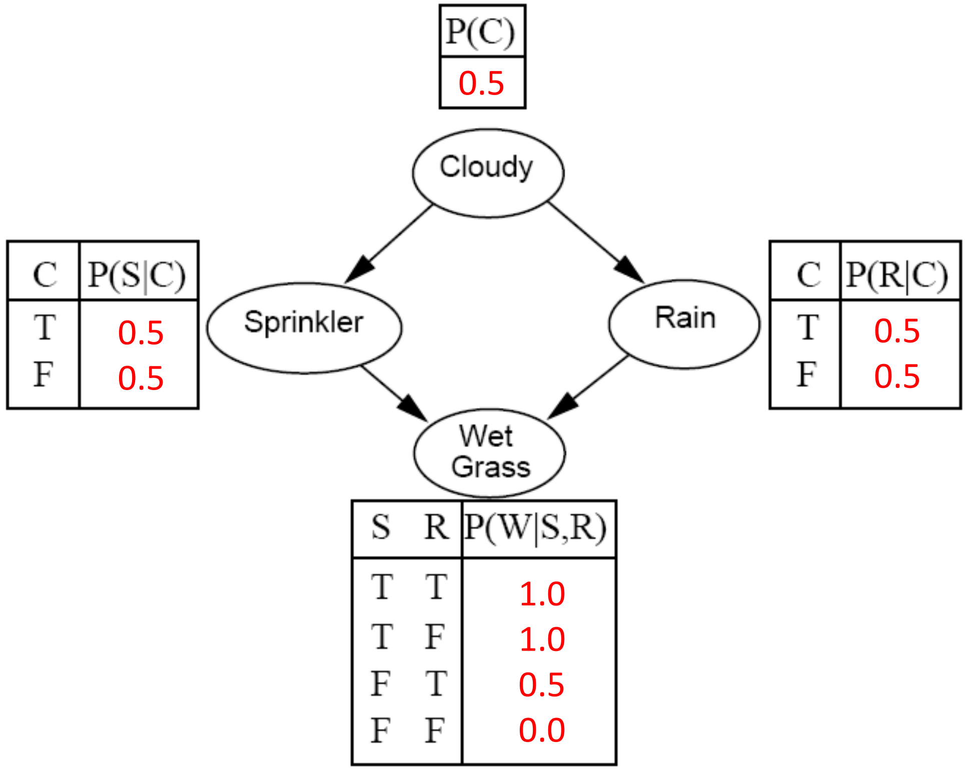
Training set

Sample	C	S	R	W
1	0.5?	F	T	T
2	0.5?	T	F	T
3	0.5?	F	F	F
4	0.5?	T	T	T
5	0.5?	T	F	T
6	0.5?	F	T	F
…	…	…	….	…

Missing data: the EM algorithm

- M-Step (Maximization): Given the missing data estimates, replace each of the missing model parameters using

$$P(\text{Variable} = \text{T}|\text{Parents} = \text{value}) = \frac{E[\#\text{ times Variable} = T, \text{Parents} = \text{value}]}{E[\#\text{times Parents} = \text{value}]}$$



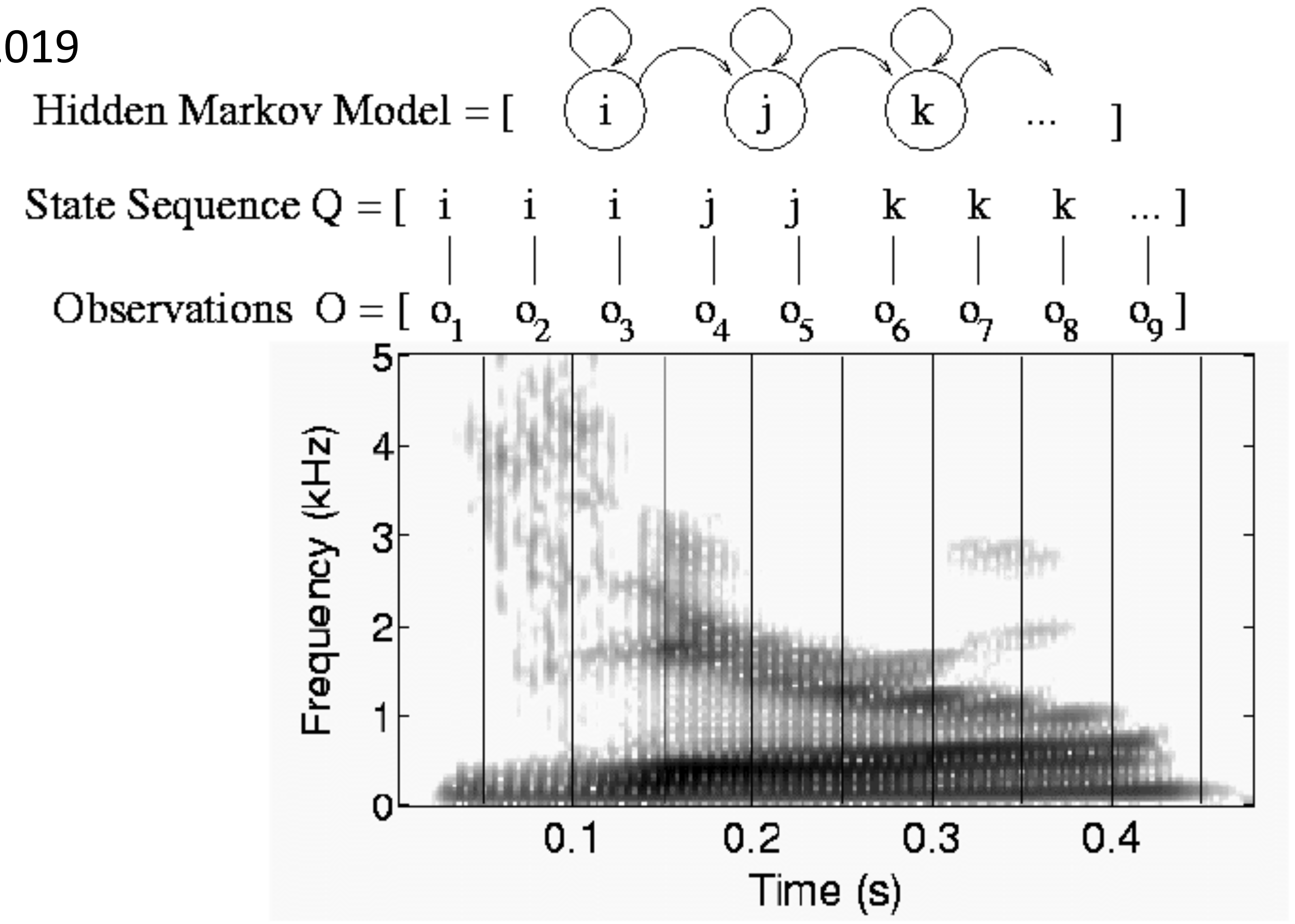
Training set

Sample	C	S	R	W
1	0.5?	F	T	T
2	0.5?	T	F	T
3	0.5?	F	F	F
4	0.5?	T	T	T
5	0.5?	T	F	T
6	0.5?	F	T	F
…	…	…	….	…

CS440/ECE448 Lecture 20: Hidden Markov Models

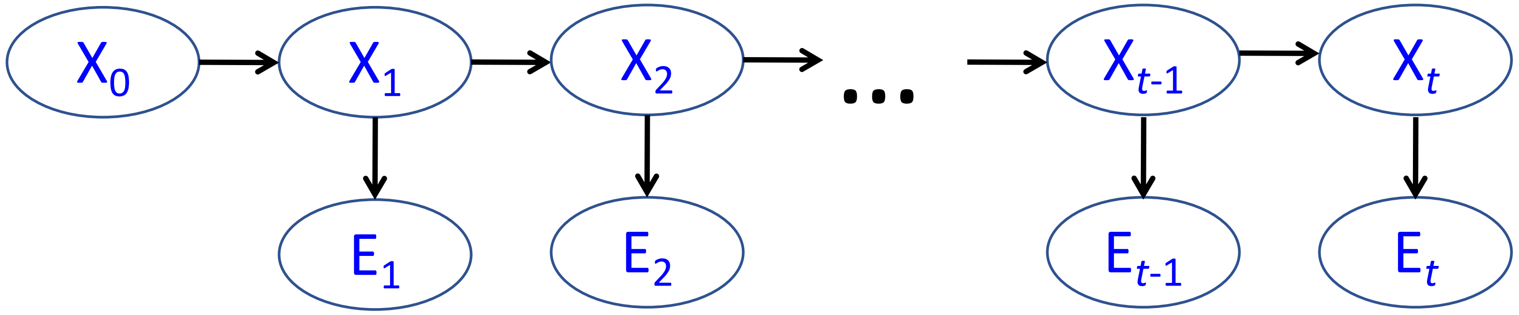
Slides by Svetlana Lazebnik, 11/2016

Modified by Mark Hasegawa-Johnson, 3/2019

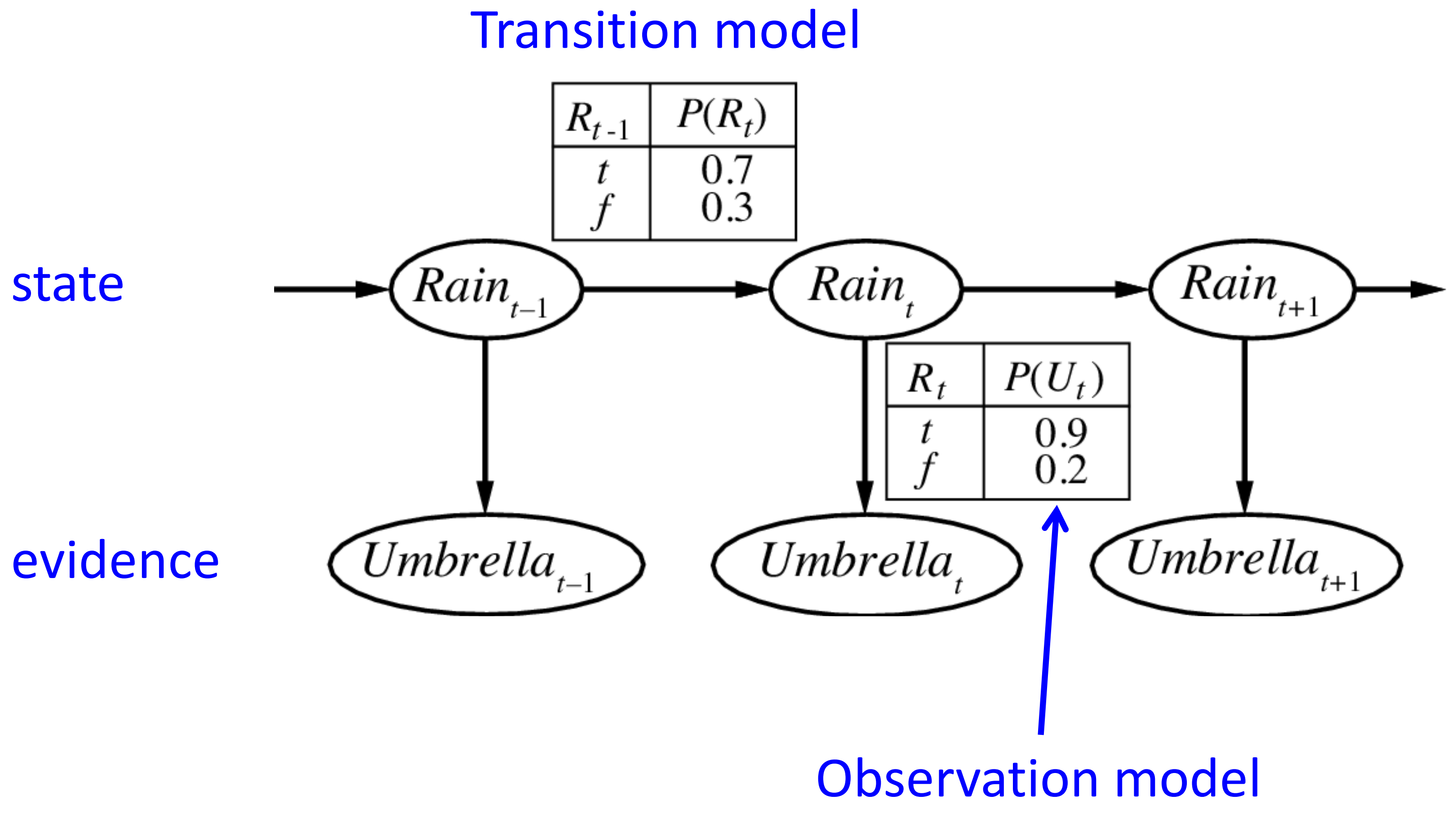

Hidden Markov Model = [i j k ...]
State Sequence Q = [i i i j j k k k ...]
Observations O = [o_1 o_2 o_3 o_4 o_5 o_6 o_7 o_8 o_9]
Frequency (kHz)
0
1
2
3
4
5
0.1
0.2
0.3
0.4
Time (s)

Hidden Markov Models

- At each time slice *t*, the state of the world is described by an **unobservable (hidden) variable** X_t and an **observable *evidence* variable** E_t
- **Transition model:** The current state is conditionally independent of all the other states given the state in the previous time step
  Markov assumption: $P(X_t \mid X_0, \dots, X_{t-1}) := P(X_t \mid X_{t-1})$
- **Observation model:** The evidence at time *t* depends only on the state at time *t*
  Markov assumption: $P(E_t \mid \mathbf{X}_{0:t}, \mathbf{E}_{1:t-1}) = P(E_t \mid X_t)$



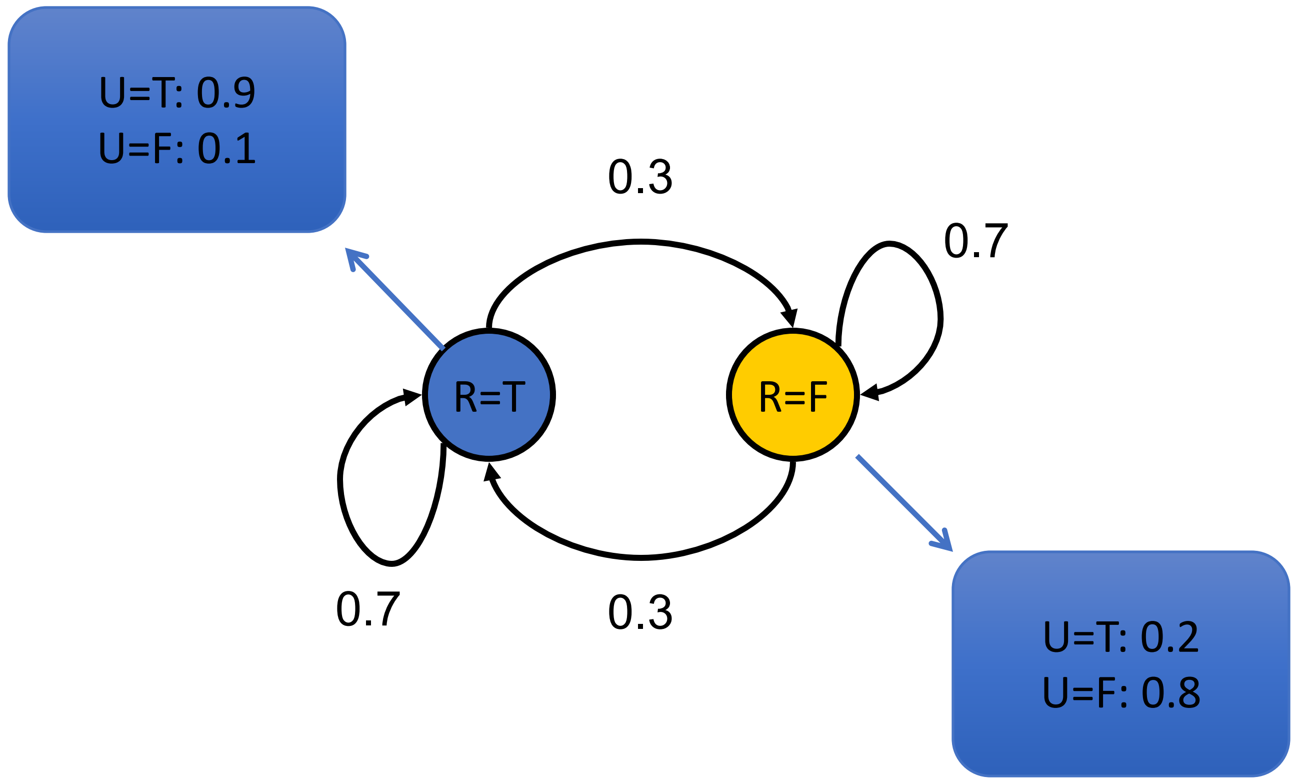
Example



R_{t-1}	$P(R_t)$
t	0.7
f	0.3

R_t	$P(U_t)$
t	0.9
f	0.2

An alternative visualization



Transition probabilities

	R_t = T	R_t = F
R_{t-1} = T	0.7	0.3
R_{t-1} = F	0.3	0.7

Observation (emission) probabilities

	U_t = T	U_t = F
R_t = T	0.9	0.1
R_t = F	0.2	0.8

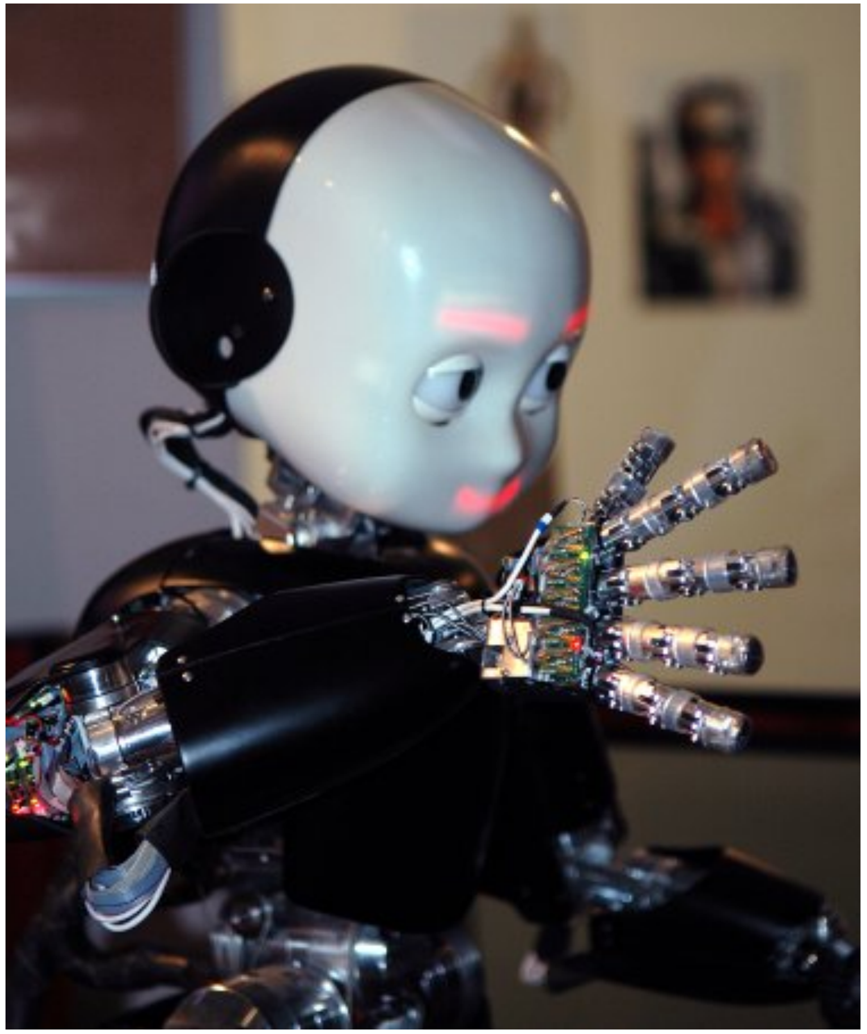
HMM Learning and Inference

- Inference tasks
  - **Filtering:** what is the distribution over the current state X_t given all the evidence so far, $\mathbf{e}_{1:t}$
  - **Smoothing:** what is the distribution of some state X_k given the entire observation sequence $\mathbf{e}_{1:t}$?
  - **Evaluation:** compute the probability of a given observation sequence $\mathbf{e}_{1:t}$
  - **Decoding:** what is the most likely state sequence $\mathbf{X}_{0:t}$ given the observation sequence $\mathbf{e}_{1:t}$?
- Learning
  - Given a training sample of sequences, learn the model parameters (transition and emission probabilities)
    - EM algorithm

CS440/ECE448 Lecture 21: Markov Decision Processes

Slides by Svetlana Lazebnik, 11/2016

Modified by Mark Hasegawa-Johnson, 3/2019



Markov Decision Processes (MDPs)

- Components that define the MDP. Depending on the problem statement, you either know these, or you learn them from data:
  - **States** s, beginning with initial state s_0
  - **Actions** a
    - Each state s has actions $A(s)$ available from it
  - **Transition model** $P(s' \mid s, a)$
    - *Markov assumption*: the probability of going to s' from s depends only on s and a and not on any other past actions or states
  - **Reward function** $R(s)$
- **Policy – the "solution" to the MDP:**
  - $\pi(s) \in A(s)$: the action that an agent takes in any given state

Maximizing expected utility

- The **optimal policy** π(s) should **maximize the *expected utility* over all possible state sequences** produced by following that policy:

$$\sum_{\substack{state\ sequences \\ starting\ from\ s_0}} P\big(sequence|s_0 = \pi(s_0)\big)U(sequence)$$

- How to define the **utility of a state sequence**?
  - **Sum of rewards** of individual states
  - Problem: **infinite state sequences**
  - **Solution:** *discount* individual state rewards by a factor γ between 0 and 1:

$$U([s_0, s_1, s_2, \ldots]) = R(s_0) + \gamma R(s_1) + \gamma^2 R(s_2) + \ldots$$

$$= \sum_{t=0}^{\infty} \gamma^t R(s_t) \le \frac{R_{\max}}{1-\gamma} \qquad (0 < \gamma < 1)$$

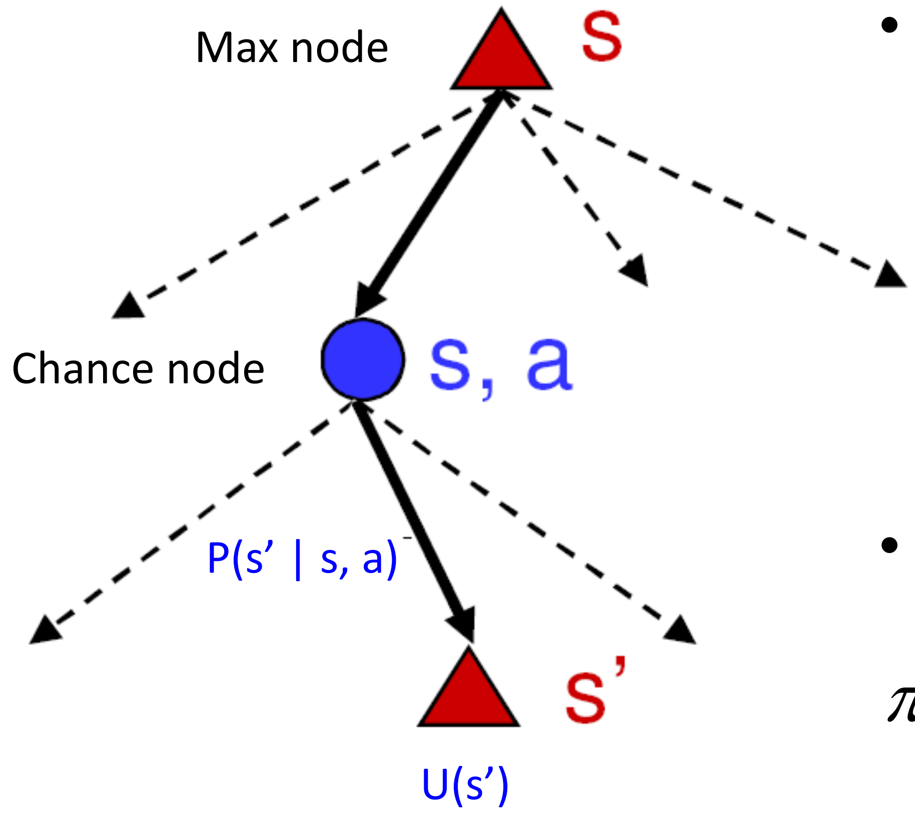
Utilities of **states**

- **Expected utility obtained by policy π starting in state s**:

$$U^{\pi}(s) = \sum_{\substack{state\ sequences \\ starting\ from\ s}} P\big(sequence|s, a = \pi(s)\big)U(sequence)$$

- The **"true" utility of a state**, denoted U(s), is the *best possible* expected sum of discounted rewards
  - if the agent executes the *best possible* policy starting in state s
- Reminiscent of minimax values of states…

Finding the utilities of **states**



- If state s' has utility U(s'), then what is the expected utility of taking action **a** in state **s**?

$$\sum_{s'} P(s' | s, a) U(s')$$

- How do we choose the optimal action?

$$\pi^*(s) = \arg\max_{a \in A(s)} \sum_{s'} P(s' | s, a) U(s')$$

- What is the recursive expression for **U(s)** in terms of the utilities of its successor states?

$$U(s) = R(s) + \gamma \max_a \sum_{s'} P(s' | s, a) U(s')$$

The Bellman equation

- Recursive relationship between the utilities of successive states:

$$\boxed{U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U(s')}$$

- For *N* states, we get *N* equations in *N* unknowns
  - Solving them solves the MDP
  - Nonlinear equations -> no closed-form solution, need to use an iterative solution method (is there a globally optimum solution?)
  - We could try to solve them through expectiminimax search, but that would run into trouble with infinite sequences
  - Instead, we solve them algebraically
  - Two methods: **value iteration** and **policy iteration**

Method 1: Value iteration

- Start out with every *U*(*s*) = 0
- Iterate until convergence
    - During the *i*th iteration, update the utility of each state according to this rule:

$$U_{i+1}(s) \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' | s, a) U_i(s')$$

- In the limit of infinitely many iterations, this is guaranteed to find the correct utility values
    - Error decreases exponentially, so in practice, don’t need an infinite number of iterations…

Method 2: Policy iteration

- Start with some **initial policy** π_0 and alternate between the following steps:
    - **Policy evaluation:** calculate $U^{\pi_i}(s)$ for every state s
    - **Policy improvement:** calculate a new policy π_{i+1} based on the updated utilities

- Notice it's kind of like **hill-climbing** in the N-queens problem.
    - **Policy evaluation**: Find ways in which the current policy is suboptimal
    - **Policy improvement**: Fix those problems

- Unlike Value Iteration, this is guaranteed to **converge in a *finite* number of steps**, as long as the **state space and action set are both finite**.

Method 2, Step 1: **Policy evaluation**

- Given a **fixed policy** π, calculate $U^\pi(s)$ for every state s

$$\boxed{U^\pi(s) = R(s) + \gamma \sum_{s'} P(s' \mid s, \pi(s)) U^\pi(s')}$$

- π(s) is fixed, therefore $P(s'|s,\pi(s))$ is an $s' \times s$ matrix, therefore we can solve a linear equation to get $U^\pi(s)$!
- Why is this "Policy Evaluation" formula so much easier to solve than the original Bellman equation?

$$\boxed{U(s) = R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' \mid s, a) U(s')}$$

CS 440/ECE448 Lecture 22: Reinforcement Learning

Slides by Svetlana Lazebnik, 11/2016

Modified by Mark Hasegawa-Johnson, 4/2019

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Reinforcement learning strategies

- **Model-based**
  - Learn the **model** of the MDP (**transition probabilities and rewards**) and try to **solve the MDP** concurrently
- **Model-free**
  - **Learn how to act** *without* explicitly learning the transition probabilities $P(s' \mid s, a)$
  - **Q-learning:** learn an **action-utility function** $Q(s,a)$ that tells us the value of doing action a in state s

Model-based reinforcement learning

- **Basic idea:**
  Try to **learn the model** of the MDP (transition probabilities and rewards) and **learn how to act** (solve the MDP) simultaneously
- **Learning the model:**
  - Keep track of how many times **state s' follows state s when you take action a**
  - **Update the transition probability** P(s' | s, a) according to these relative frequencies
  - **Keep track of the rewards** R(s)
- **Learning how to act:**
  - **Estimate the utilities** U(s) using Bellman's equations
  - Choose the **action that maximizes expected future utility**:

$$\pi^*(s) = \arg\max_{a \in A(s)} \sum_{s'} P(s' \mid s, a) U(s')$$

Exploration vs. exploitation

- **Exploration:** take a **new action** with **unknown consequences**
  - Pros:
    - Get a more accurate model of the environment
    - Discover higher-reward states than the ones found so far
  - Cons:
    - When you're exploring, you're not maximizing your utility
    - Something bad might happen
- **Exploitation:** go with the **best strategy found so far**
  - Pros:
    - Maximize reward as reflected in the current utility estimates
    - Avoid bad stuff
  - Cons:
    - Might also prevent you from discovering the true optimal strategy

Incorporating exploration

- **Idea:** explore more in the beginning, become more and more greedy over time
- **Standard ("greedy") selection of optimal action**:

$$a = \arg\max_{a' \in A(s)} \sum_{s'} P(s' \mid s, a') U(s')$$

- **Modified strategy** with exploration function *f*(*u*,*n*)

  f(*u*,*n*) trades off **greed** [preference for high utility *u*] against **curiosity** [preference for low observed frequencies *n*]

$$f(u,n) = \begin{cases} R^{+} & \text{if } n < N_e \\ u & \text{otherwise} \end{cases}$$

Set utility of a' to R^+ [= optimistic reward estimate] if a' in state s explored less than N_e [a constant] times

Set utility to actual observed utility

$$a = \arg\max_{a' \in A(s)} \underset{\substack{\uparrow \\ \text{exploration function}}}{f}\left(\sum_{s'} P(s' \mid s, a') U(s'), \underset{\substack{\uparrow \\ \text{Number of times we've taken action a' in state s}}}{N}(s, a') \right)$$

Model-free reinforcement learning

- **Idea:** learn how to act *without* explicitly learning the transition probabilities P(s' | s, a)
- **Q-learning:** learn an *action-utility function* Q(s,a) that tells us the value of doing action a in state s
- Relationship between Q-values and utilities:

$$U(s) = \max_a Q(s,a)$$

- Selecting an action: $\pi^*(s) = \arg\max_a Q(s,a)$
- Compare with: $\pi^*(s) = \arg\max_a \sum_{s'} P(s'|s,a)U(s')$
  - With Q-values, don't need to know the transition model to select the next action

Temporal difference (TD) learning

- **Equilibrium constraint** on Q values:

$$Q(s,a) = R(s) + \gamma \sum_{s'} P(s'|s,a) \max_{a'} Q(s',a')$$

- **Temporal difference (TD) update:**
  - Pretend that the currently observed transition (s,a,s’) is the *only* possible outcome.
    Call this “local quality” as $Q^{local}(s,a)$;
    it is computed using $Q(s,a)$.

$$Q^{local}(s,a) = R(s) + \gamma \max_{a'} Q(s',a')$$

  - Then interpolate between $Q(s,a)$ and $Q^{local}(s,a)$ to compute $Q^{new}(s,a)$.

$$Q^{new}(s,a) = (1-\alpha)Q(s,a) + \alpha Q^{local}(s,a)$$

Function approximation

- So far, we've assumed a lookup table representation for utility function U(s) or action-utility function Q(s,a)
- But what if the state space is really large or continuous?
- Alternative idea: approximate the utility function, e.g., as a weighted linear combination of *features*:

$$U(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots w_n f_n(s)$$

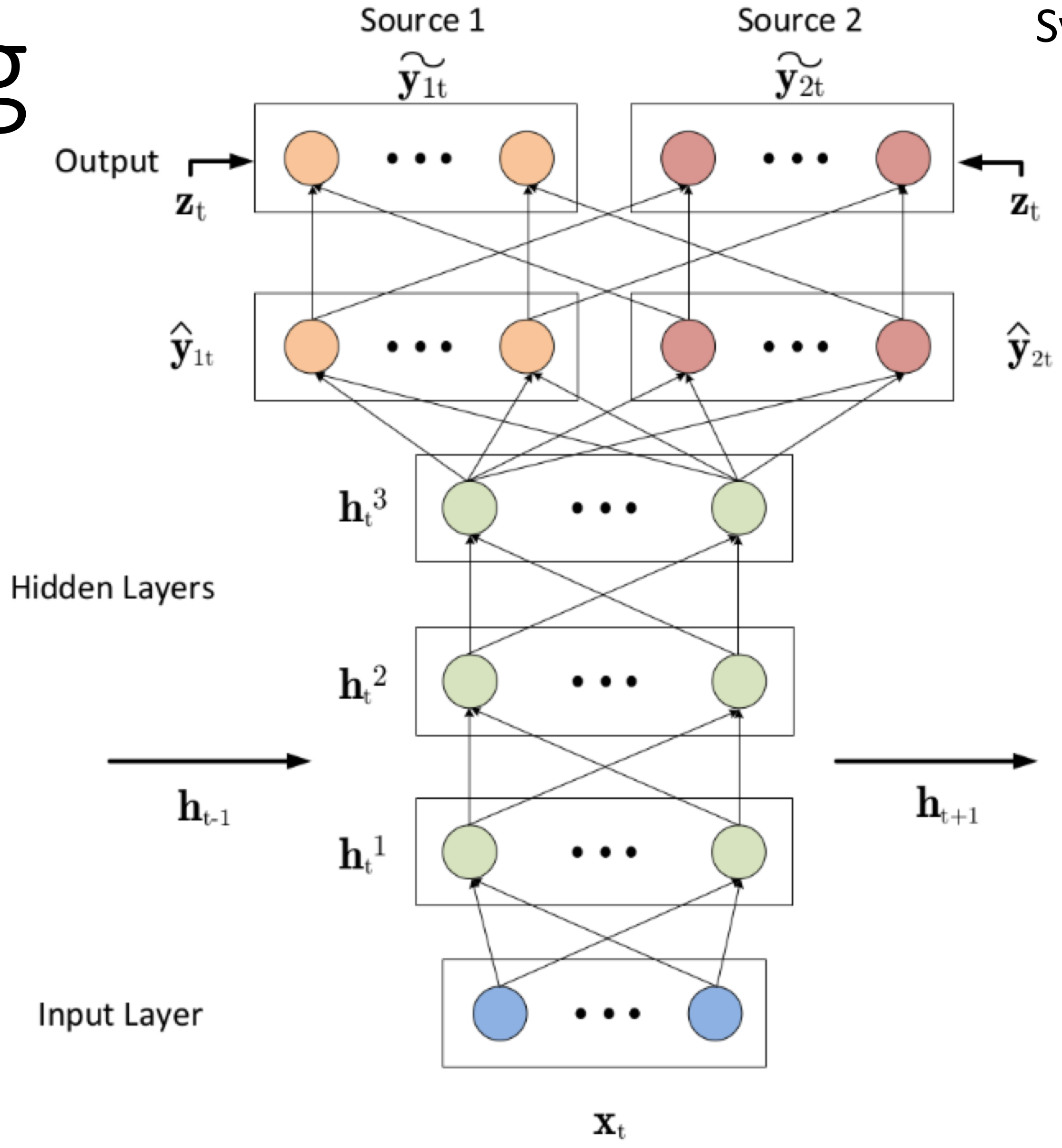
  - RL algorithms can be modified to estimate these weights
  - More generally, functions can be nonlinear (e.g., neural networks)
- Recall: features for designing evaluation functions in games
- Benefits:
  - Can handle very large state spaces (games), continuous state spaces (robot control)
  - Can *generalize* to previously unseen states

CS440/ECE448 Lecture 23: Deep Learning

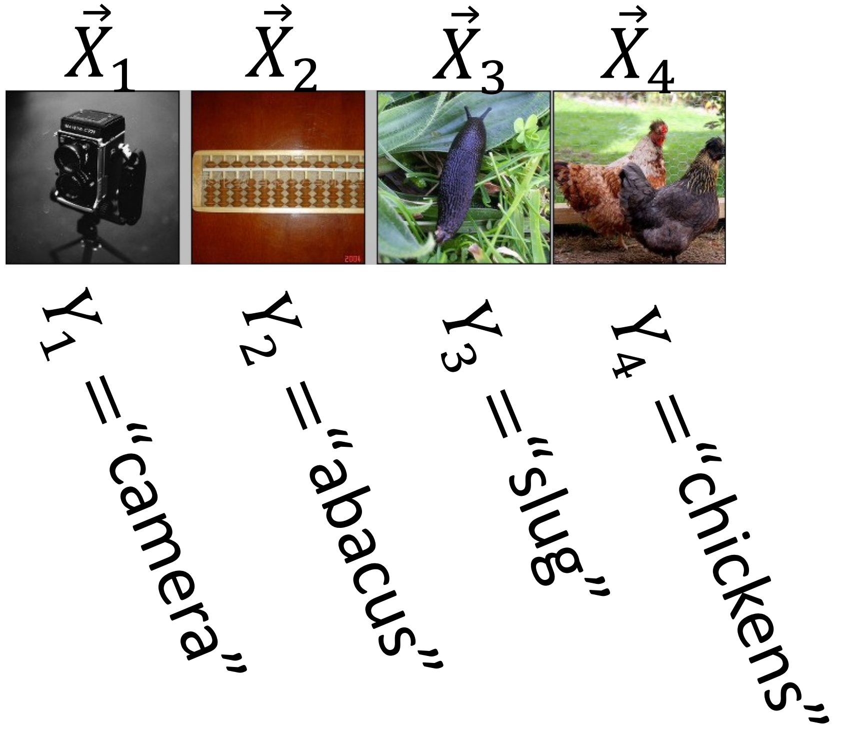
Mark Hasegawa-Johnson, 4/2019

Including Slides by

Svetlana Lazebnik, 10/2016



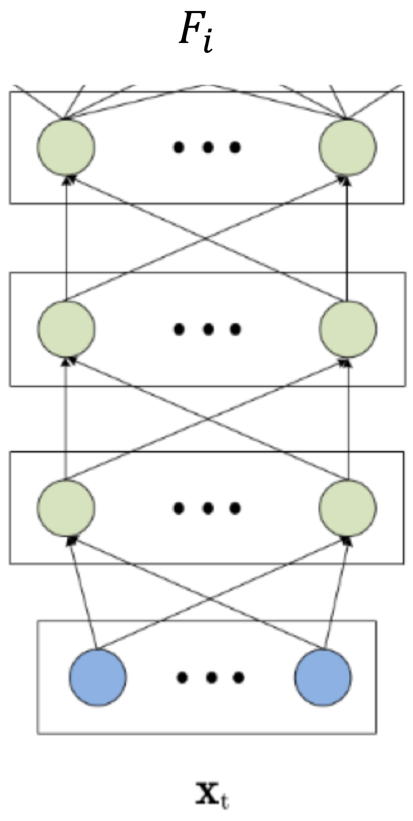
Notation



Usually we have two databases:

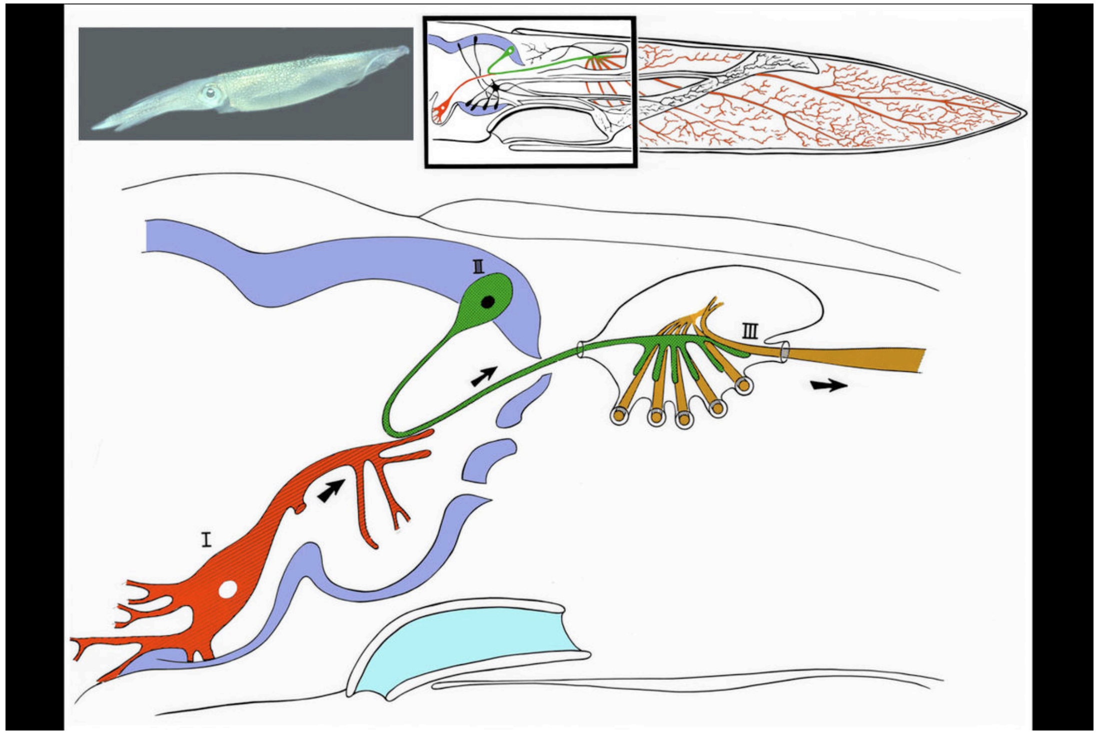
- A training database consists of N different training tokens (one token = one image, or sentence, or speech files, or whatever). We write them as vectors, $\vec{X}_i = [X_{i1}, \ldots, X_{iM}]$, for $1 \le i \le N$. Each one has an associated reference (ground truth) label Y_i.
- A testing database contains only the test tokens $\vec{X}_i$, for $\mathrm{N} + 1 \le i$

Notation



For both training and testing, we have to present the token $\vec{X}_i$ to the input of the neural net, and then the neural net computes some output $\vec{F}_i$.

Notation



A deep neural net has thousands of neurons (nodes).

Each neuron (node) has two key variables:

- The "affine", Z_{ij}, models the synapse of a biological neuron, collecting information from a lot of other neurons:

$$Z_{ij} = \sum_k A_{ik} W_{kj}$$

- The "activation," A_{ij}, models the axon of a biological neuron i.e., it's zero when the input is negative, and nonzero when the input is positive:

$$A_{ij} = g(Z_{ij})$$

Notation for a Neural Net without Layers

- A_{ij} is the j^{th} activation for the i^{th} token:
  - Some of the activations are provided by the input, i.e., $A_{ij} = X_{ij}$ for some of the j's.
  - Some of the activations are outputs, i.e., $F_{ij} = A_{ij}$ for some of the j's.
  - Some of the activations are neither inputs nor outputs. Those are called "hidden nodes."
  - Which ones are inputs, hidden, and outputs? Well, it depends on the particular neural network design, there's no way to know, in general.
- Z_{ij} is the j^{th} affine for the i^{th} token
- W_{kj} is the $(k, j)^{th}$ weight.

Notation for a Neural Net **with** Layers

- $A_{ij}^{(l)}$ is the j^{th} activation **in the $\boldsymbol{l^{th}}$ layer** for the i^{th} token:
  - The 0^{th} layer is the input, i.e., $A_{ij}^{(0)} = X_{ij}$.
  - The L^{th} layer is the output, i.e., $F_{ij} = A_{ij}^{(L)}$.
  - All other layers are “hidden layers.”
- $Z_{ij}^{(l)}$ is the j^{th} affine **in the $\boldsymbol{l^{th}}$ layer** for the i^{th} token
- $W_{kj}^{(l)}$ is the $(k, j)^{th}$ weight **in the $\boldsymbol{l^{th}}$ layer**.

$$Z_{ij}^{(l)} = \sum_k A_{ik}^{(l-1)} W_{kj}^{(l)}$$

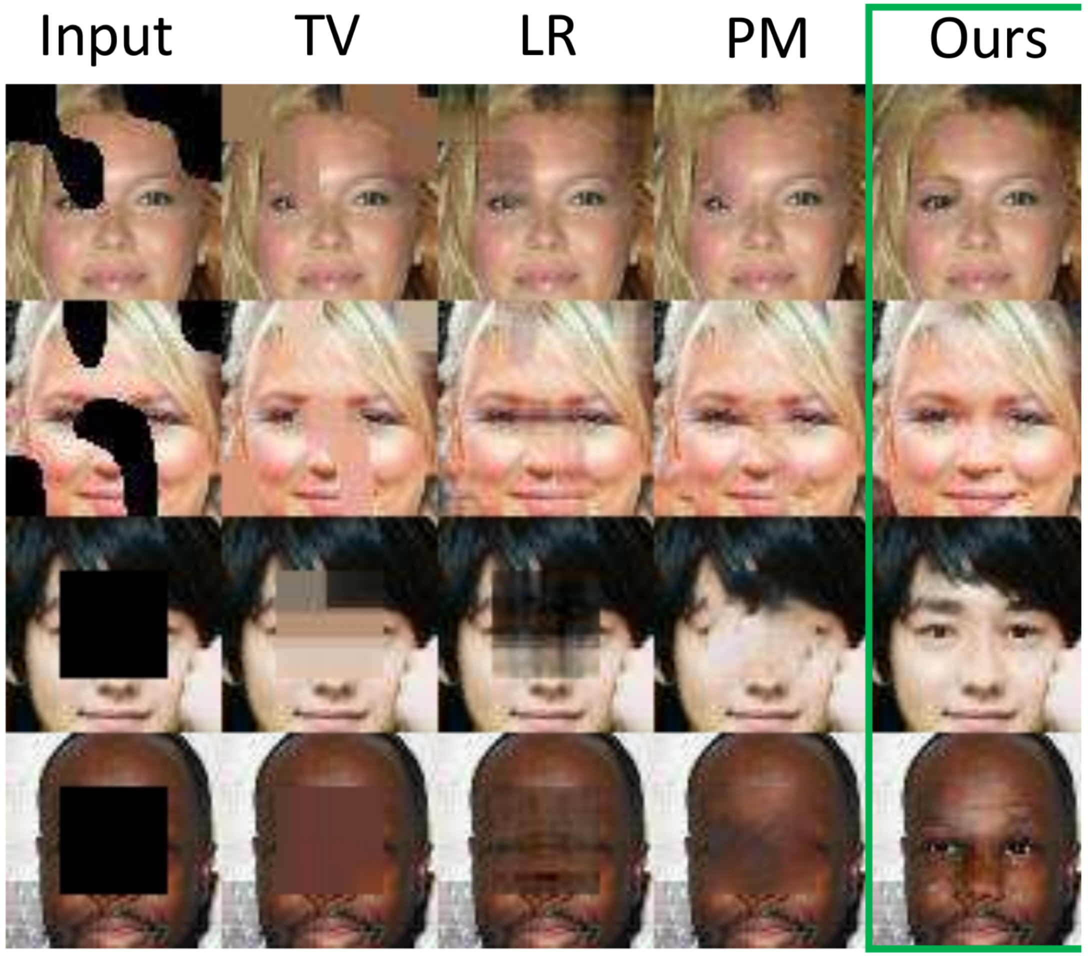
Forward Propagation (Using the Neural Net)

- We use a neural net by presenting a token $\vec{X}_i$, and computing the output $\vec{F}_i$.
- This is done by setting:
  - $A_{ij}^{(0)} = X_{ij}$
  - For $1 \leq l \leq L$:
    - $Z_{ij}^{(l)} = \sum_k A_{ik}^{(l-1)} W_{kj}^{(l)}$
    - $A_{ij}^{(l)} = g(Z_{ij}^{(l)})$
  - $F_{ij} = A_{ij}^{(L)}$
- This algorithm is called “forward propagation,” because information propagates forward through the network, from the 0^{th} layer to the L^{th} layer.

How well did it do?

- We test a neural net by computing $\vec{F}_i$ from $\vec{X}_i$, for each of the tokens $1 \le i \le N$, and then comparing the network output to the reference (ground truth) answer, Y_i.
  - During training: we measure error using training data, and try to train the network in order to reduce the error rate.
  - During ”development test:” we compare different networks on the development test data.
  - During “evaluation test:” our customer tests our network with data it’s never seen before.
- But… How do we compare $\vec{F}_i$ to Y_i? i.e., how we define “error” or “loss”?

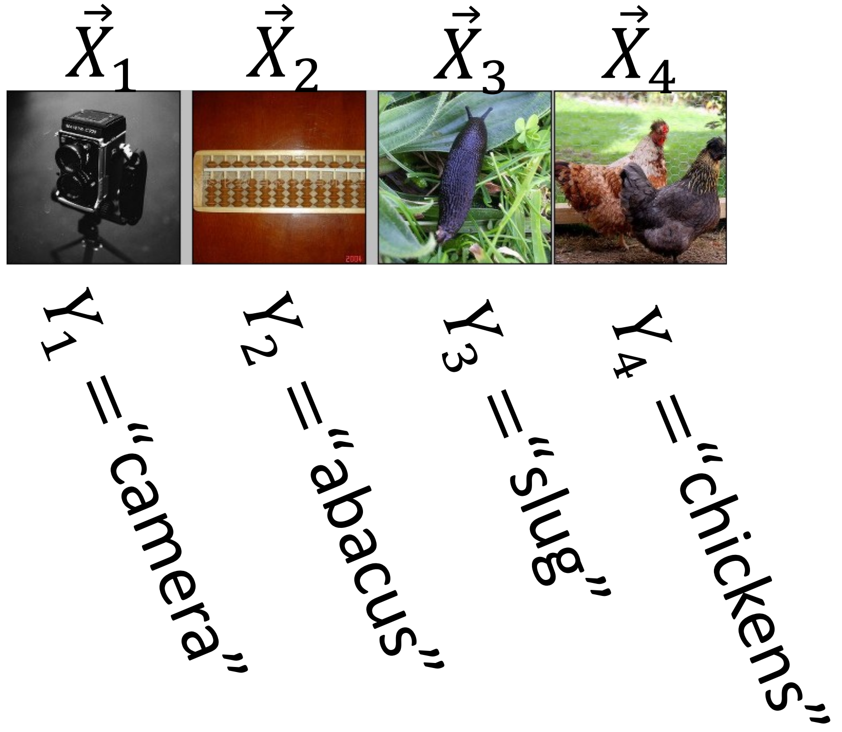
Regression problems: Sum-squared error



- For example, suppose that the network output is an image.
- An image is a vector, $\vec{F}_i = [F_{i1}, \ldots, F_{iM}]$
- The "right answer" is the image we were trying to reconstruct, $\vec{Y}_i = [Y_{i1}, \ldots, Y_{iM}]$.
- Then a reasonable loss function is sum-squared error (SSE):

$$L_{SSE} = \sum_{i=1}^{N} \sum_{j=1}^{M} \left(Y_{ij} - F_{ij}\right)^2$$

Classifier problems: Cross-entropy



- On the other hand, for this course, we usually want Y_i to be some category label, for example, $Y_i =$ "*chickens*".
- In that case, we can use a special kind of nonlinearity at the output of our neural network, called a softmax, that gives a probabilistic interpretation to the network outputs:

$$F_{ij} = P(Y_i = j^{th} \text{ type of category})$$

- Then a reasonable loss function is the log probability of the correct class:

$$L_{CE} = -\sum_{i=1}^{N} \ln F_{i,Y_i}$$

- This error criterion is called "cross entropy" for reasons that are fascinating but way beyond the scope of this course.

Classifier output: Softmax

- We want Y_i to be some category label, for example, $Y_i =$ "$boots$".
- In that case, we want F_{ij} to meet the criteria for a probability, i.e., we need $F_{ij} \geq 0$ and $\sum_j F_{ij} = 1$.
- In order to do that, we use a special kind of nonlinearity in the last layer of the neural net, called a softmax:

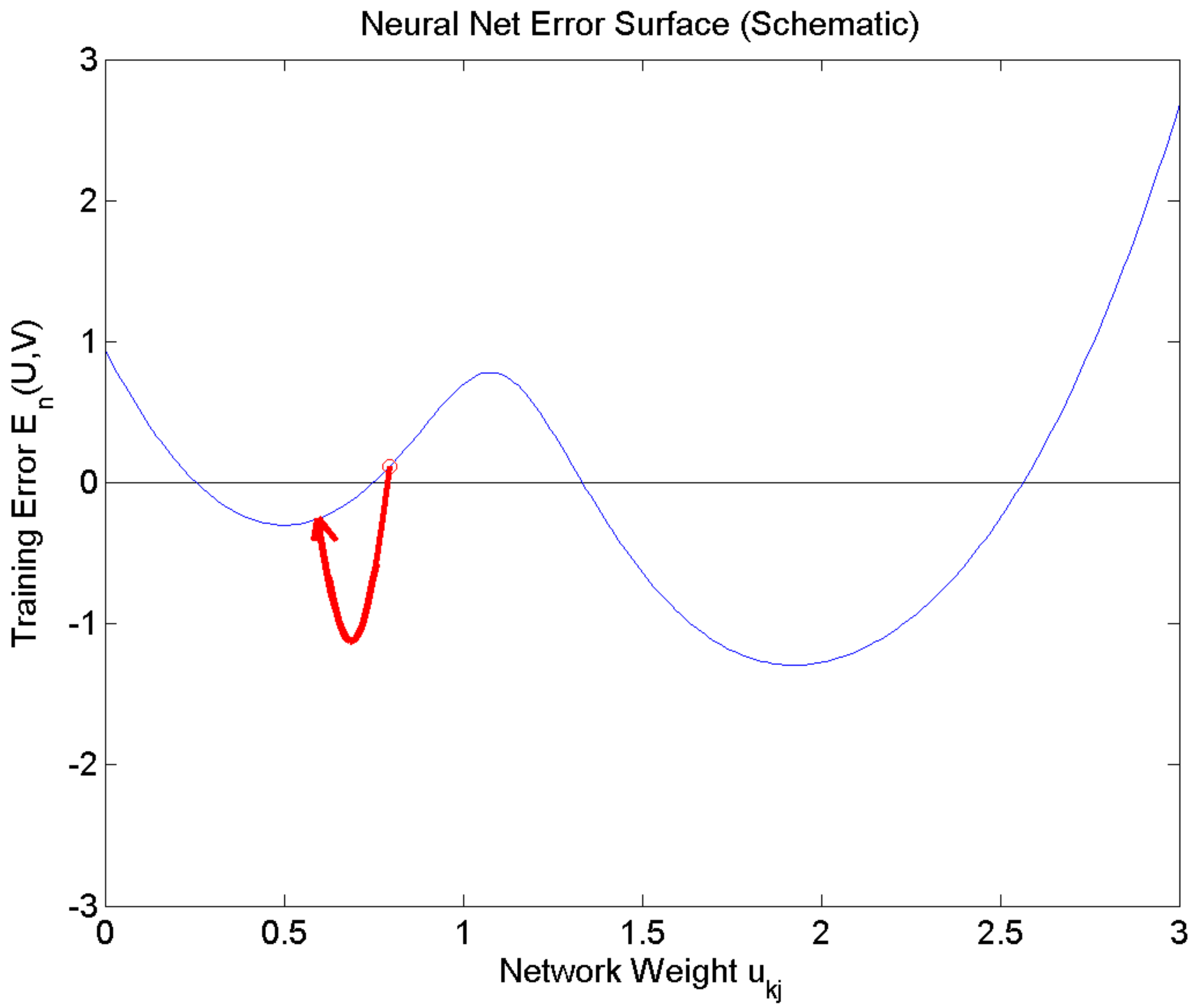
$$F_{ij} = \frac{e^{Z_{ij}^{(L)}}}{\sum_k e^{Z_{ik}^{(L)}}}$$

Training the Neural Net

A neural net is trained according to gradient descent:

$$W_{jk}^{(l)} = W_{jk}^{(l)} - \eta \frac{\partial L}{\partial W_{jk}^{(l)}}$$

So that the loss function, L, gradually approaches a local minimum.



Training the Neural Net: Notation

- Let’s use the following shorthand:

$$\delta(\text{Variable}) = \frac{\partial L}{\partial(\text{Variable})}$$

For example:

$$\delta W_{kj}^{(l)} = \frac{\partial L}{\partial W_{kj}^{(l)}}$$

Training the Neural Net: Last Layer

The cross entropy loss is:

$$L_{CE} = -\sum_{i=1}^{N} \ln F_{i,Y_i}$$

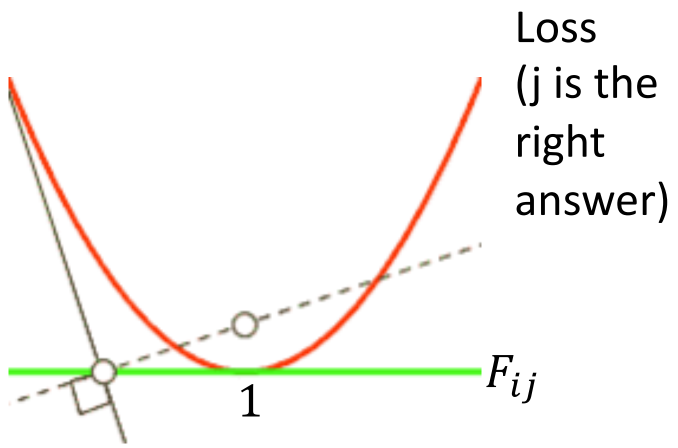
$$= -\sum_{i=1}^{N} \ln \frac{e^{Z_{i,Y_i}^{(L)}}}{\sum_k e^{Z_{ik}^{(L)}}}$$

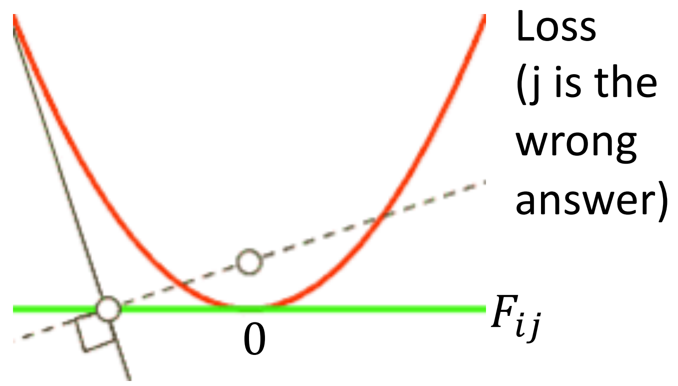
Its derivative is:

$$\delta Z_{ij}^{(L)} = \begin{cases} F_{ij} - 1 & j = Y_i \\ F_{ij} & j \neq Y_i \end{cases}$$

Here's how to remember that:

- If j is the right answer, then error is minimized ($\delta Z_{ij}^{(L)} = 0$) when $F_{ij} = 1$.
- If j is the wrong answer, then error is minimized ($\delta Z_{ij}^{(L)} = 0$) when $F_{ij} = 0$.



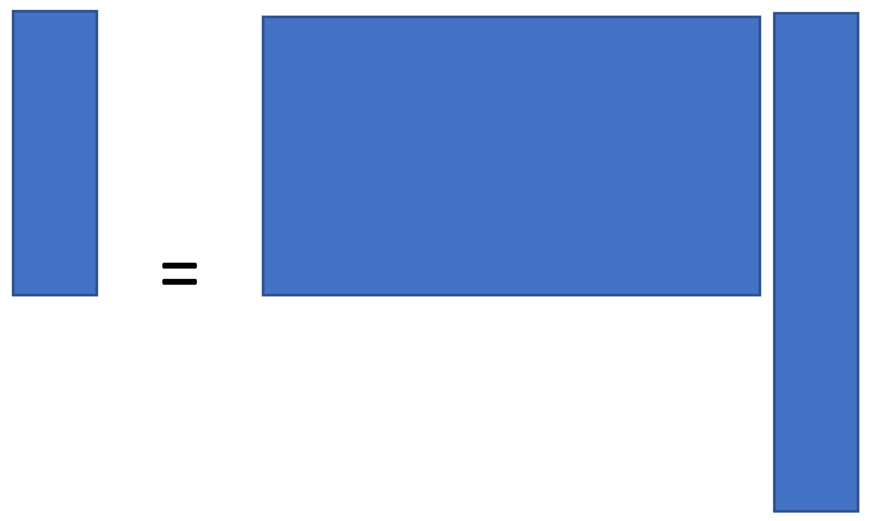


Credit: Tosha, distributed under CC-BY 1.0, https://commons.wikimedia.org/wiki/File:Parabola-antipodera.gif

Convolution versus Matrix Multiplication

A regular neural net uses a matrix multiplication in each layer:

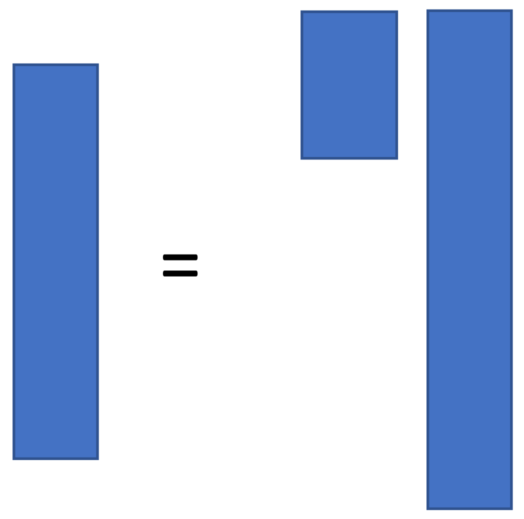
$$Z_{ij}^{(l)} = \sum_k A_{ik}^{(l-1)} W_{kj}^{(l)}$$



$$\vec{Z}_i^{(l)} = W^{(l)} \cdot \vec{A}_i^{(l-1)}$$

A convolutional neural net uses a convolution at each layer:

$$Z_{ij}^{(l)} = \sum_k A_{i,k}^{(l-1)} W_{j-k}^{(l)}$$



$$\vec{Z}_i^{(l)} = W^{(l)} * \vec{A}_i^{(l-1)}$$

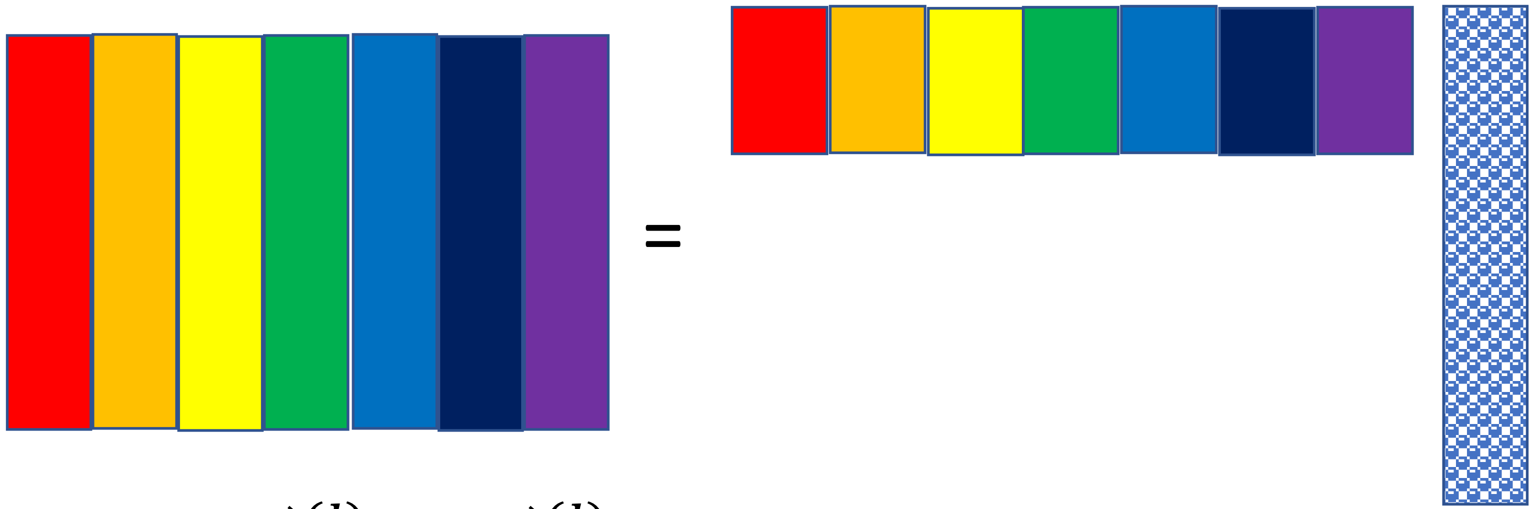
Convolution with Many Channels

Usually, we want the convolutional network to compute many different channels, c:

$$Z_{ij,c}^{(l)} = \sum_k A_{i,k}^{(l-1)} W_{j-k,c}^{(l)}$$

Each of the channels is computing a different type of feature (average, edge, etc.).

Each pixel, in each output channel, tells the degree to which that channel exists at that location in the image.

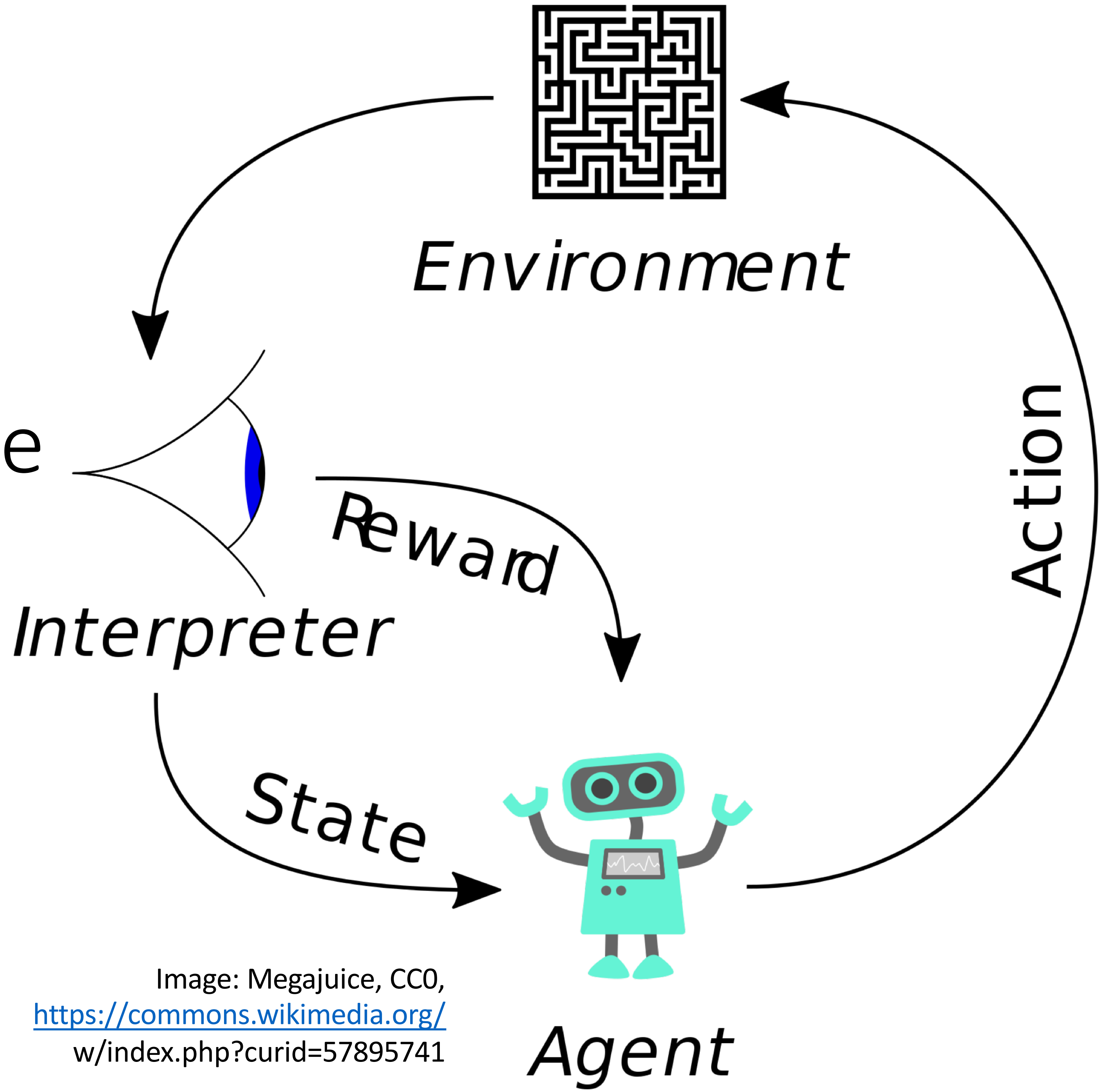


$$\vec{Z}_{i,1}^{(l)}, \dots, \vec{Z}_{i,7}^{(l)} = W_1^{(l)}, \dots, W_7^{(l)} * \vec{A}_i^{(l-1)}$$

Deep Reinforcement Learning

CS440/ECE448 Lecture 24

Slides by Svetlana Lazebnik, 11/2017
Modified by Mark Hasegawa-Johnson, 4/2019



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Deep Q learning

- Regular TD update: "nudge" Q(s,a) towards the target

$$Q(s,a) \leftarrow Q(s,a) + \alpha\left(\boxed{R(s) + \gamma \max_{a'} Q(s',a')} - Q(s,a)\right)$$

- Deep Q learning: encourage estimate to match the target by minimizing squared error:

$$L(w) = \left(\underbrace{\boxed{R(s) + \gamma \max_{a'} Q(s',a';w)}}_{\text{target}} - \underbrace{\boxed{Q(s,a;w)}}_{\text{estimate}}\right)^2$$

- Compare to supervised learning:

$$L(w) = \left(y - f(x;w)\right)^2$$

  - **Key difference**: the target in Q learning is not fixed – (s',a') is just one step ahead of (s,a)!

Online Q learning algorithm

- In state ***s***, perform action ***a***. Environment sends you to state ***s'***; choose the action ***a'*** that you'll perform there.
- Observe: $Q^{local}(s,a) = R(s) + \gamma \max_{a'} Q(s',a';W)$
- Update weights to reduce the error
$$L(W) = \left(Q^{local} - Q(s,a;W)\right)^2$$
- Gradient:
$$\nabla_W L = \left(Q(s,a;W) - Q^{local}\right)\nabla_W Q$$
- Weight update:
$$W \longleftarrow W - \eta\nabla_W L$$
- This is called *stochastic gradient descent* (SGD)
- "Stochastic" because the training sample (s,a,s',a') was chosen at random by our exploration function

Does Q-learning Converge?

- No!
- Because:

$$a = \operatorname{argmax} Q(s, a)$$

- If we always choose the action that is best, according to our current estimate of the Q-function, then we can never learn anything about any of the other actions!

Incorporating exploration (slide from last week)

- **Idea:** explore more in the beginning, become more and more greedy over time
- Standard ("greedy") selection of optimal action:

$$a = \arg\max_{a' \in A(s)} \sum_{s'} P(s' | s, a') U(s')$$

- Modified strategy:

$$a = \arg\max_{a' \in A(s)} \underset{\uparrow}{f} \left(\sum_{s'} P(s' | s, a') U(s'), \underset{\uparrow}{N}(s, a') \right)$$

exploration function

Number of times we've taken action a' in state s

$$f(u, n) = \begin{cases} R^+ & \text{if } n < N_e \\ u & \text{otherwise} \end{cases}$$

(optimistic reward estimate)

…but that doesn't work either:

$$f(u,n) = \begin{cases} R^{+} & \text{if } n < N_e \\ u & \text{otherwise} \end{cases}$$

- … which means that we get at least N_e samples of each action
- We can estimate Q(s,a) based on N_e samples
- But N_e is a constant, so it never $\rightarrow \infty$
- So Error never $\rightarrow 0$

Policy gradient methods

- Learning the policy directly can be much simpler than learning Q values
- We can train a neural network to output *stochastic policies*, or probabilities of taking each action in a given state
- *Softmax* policy:

$$\pi(s,a;u) = \frac{\exp\left(f(s,a;u)\right)}{\sum_{a'} \exp\left(f(s,a';u)\right)}$$

Policy gradient methods

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Policy gradient: the softmax function

- Notice that the softmax is normalized so that

$$\pi(s, a; u) \geq 0, \text{ and } \sum_a \pi(s, a; u) = 1$$

- So we can interpret $\pi(s, a; w)$ as some kind of probability. Something like "the probability that a is the best action to take from state s."
- In reality, there is no such probability. There is just one correct action. But the agent doesn't know what it is! So $\pi(s, a; u)$ is kind of like the agent's "degree of belief" that a is the best action (determined by parameters u).

Actor-critic algorithm

- Remember the relationship between the utility of a state, and the quality of an action:

$$U(s) = \max_a Q(s,a)$$

- If we don't know which action is best, then we could say that

$$U(s) \approx \sum_a \pi(s,a;u)Q(s,a;w)$$

- $\pi(s,a;u)$ is the "actor:" a neural net that tells the agent how to act.
- $Q(s,a;w)$ is the "critic:" a neural net that tells the agent how good or bad that action was.

Actor-critic algorithm

- Define objective function as total discounted reward:

$$J(u) = \mathbf{E}\left[R_1 + \gamma R_2 + \gamma^2 R_3 + \ldots\right]$$

- The gradient for a stochastic policy is given by

$$\nabla_u J = \mathbf{E}\left[\nabla_u \log \underbrace{\boxed{\pi(s,a;u)}}_{\text{Actor network}} \underbrace{\boxed{Q^\pi(s,a;w)}}_{\text{Critic network}}\right]$$

- Actor network update: $u \leftarrow u + \alpha \nabla_u J$
- Critic network update: use Q learning (following actor's policy)

CS440/ECE448 Artificial Intelligence

Lecture 25: Natural Language Processing with Neural Nets

Julia Hockenmaier

April 2019

Neural Language Models

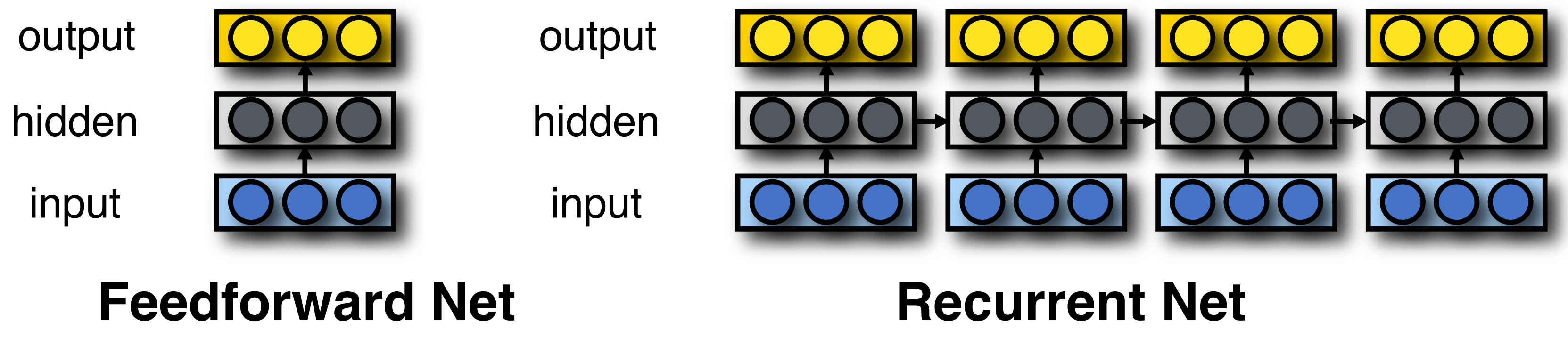
A neural LM defines a distribution over the V words in the vocabulary, conditioned on the preceding words.

- **Output layer:** V units (one per word in the vocabulary) with softmax to get a distribution
- **Input:** Represent each preceding word by its d-dimensional embedding.
  - **Fixed-length history** (n-gram): use preceding n–1 words
  - **Variable-length history**: use **a recurrent neural net**

Recurrent neural networks (RNNs)

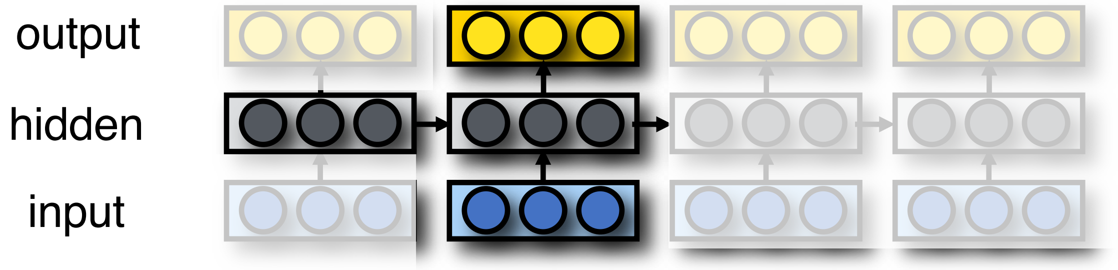
Basic RNN: Modify the standard feedforward architecture (which predicts a string $w_0 \ldots w_n$ one word at a time) such that the output of the current step (w_i) is given as additional input to the next time step (when predicting the output for w_{i+1}).

- "Output" — typically (the last) hidden layer.



Basic RNNs

Each time step corresponds to a feedforward net where the hidden layer gets its input not just from the layer below but also from the activations of the hidden layer at the previous time step



CS440/ECE448 Lecture 26: Speech

Mark Hasegawa-Johnson, 4/17/2019, CC-By 3.0

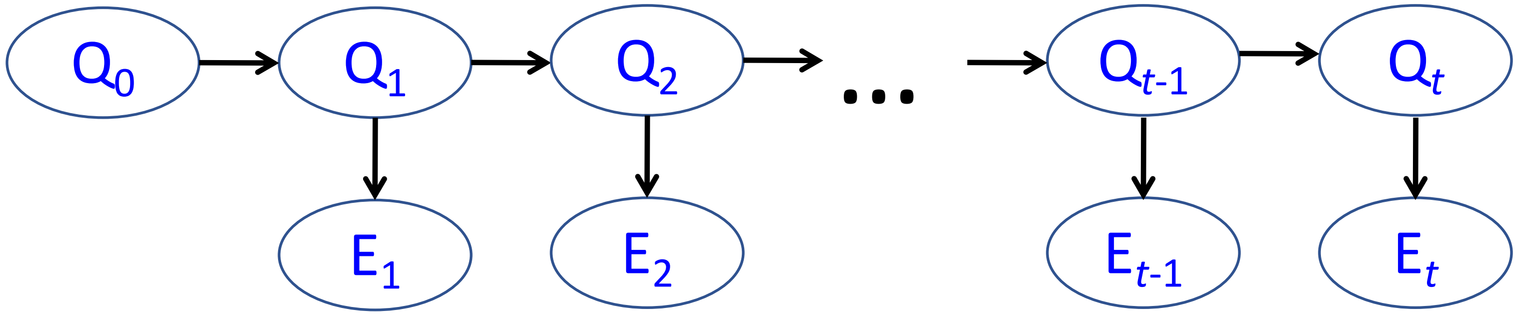
A Sequence Model you Know: HMM

You've seen this slide before, in lecture 20, on HMMs…

- <u>**Markov assumption for state transitions**</u>
  - The current state is conditionally independent of all the other states given the state in the previous time step

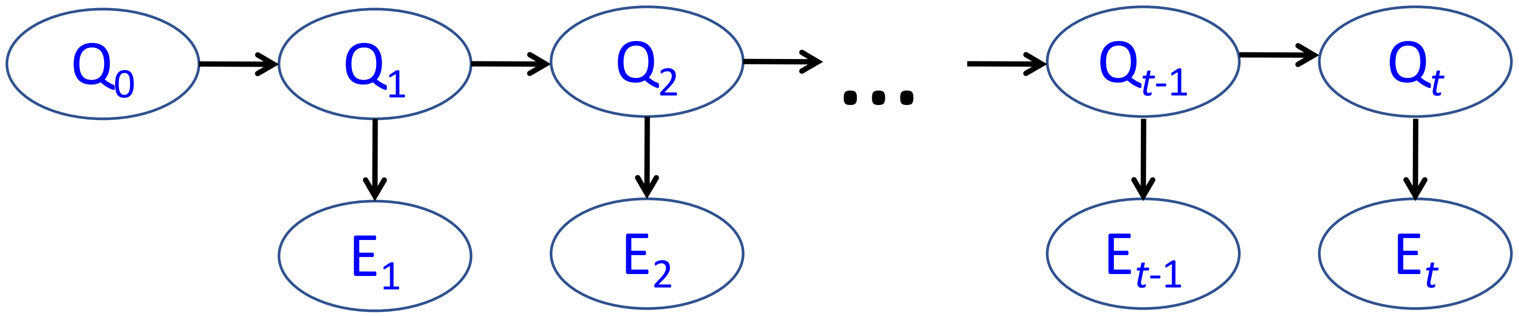
    $P(Q_t \mid \mathbf{Q}_{0:t-1}) = P(Q_t \mid Q_{t-1})$
- <u>**Markov assumption for observations**</u>
  - The evidence at time *t* depends only on the state at time *t*

  $P(E_t \mid \mathbf{Q}_{0:t}, \mathbf{E}_{1:t-1}) = P(E_t \mid Q_t)$



The Problem of Continuous Observations

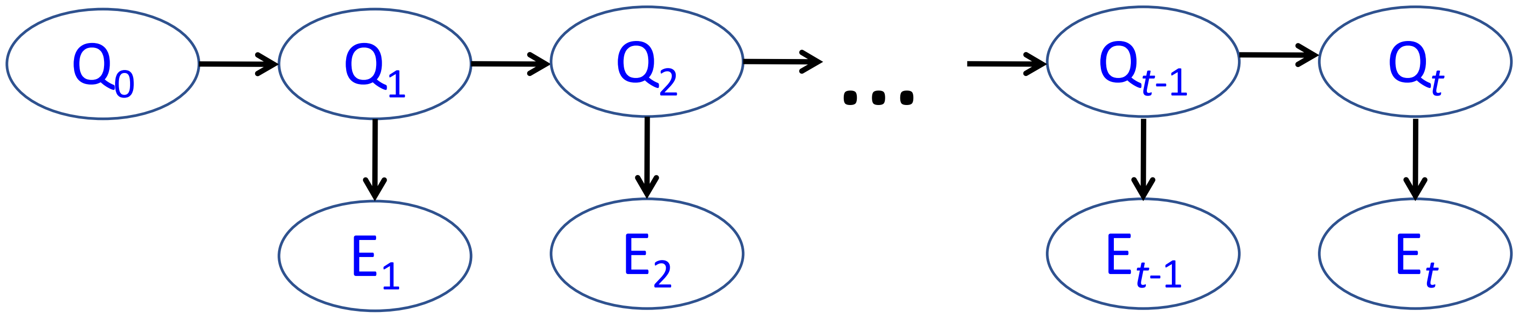
- But what about the likelihood? How can we model $P(E_t|Q_t)$?
- The big problem: E_t is continuous, not discrete, so we can't model $P(E_t|Q_t)$ using a lookup table!



Solutions to the Problem of Continuous Observations

Most systems model $P(E|Q)$ using one of these three standard methods:

1. Use a parameterized probability density, such as a Gaussian. In this case you learn senone-dependent parameters (μ_Q and σ_Q^2).
2. Quantize E (using vector quantization) to one of K different code vectors. Then you can learn the lookup table $P_W(E=k|Q)$ for $1 \le k \le K$.
3. <u>Use a neural net with a softmax output to compute $P(Q|E)$, then use Bayes' rule to get $P(E|Q)$ from $P(Q|E)$.</u>



Classifier output: Softmax

You’ve seen this slide before, in lecture 24, on Deep Learning….

- We want Q_t to be a senone, for example, $Q_t =$ “the jth type of phoneme ɑɪ”.
- In that case, we can force the neural net to learn want the neural net to compute a probability,

$$F_j = P(Q = j|E)$$

…if we just force F_j to meet the criteria for a probability, i.e., we need

$$F_j \geq 0, \qquad \sum_j F_j = 1$$

- In order to do that, we use a special kind of nonlinearity in the last layer of the neural net, called a softmax:

$$F_j = \frac{e^{Z_j}}{\sum_k e^{Z_k}}$$

Hybrid DNN-HMM: the problem

- The softmax computes $P(Q|E)$
- The HMM needs to know $P(E|Q)$
- How can we get $P(E|Q)$ from $P(Q|E)$?
- Answer: Bayes' rule!

Estimating p(E|Q) from p(Q|E)

Bayes rule:

$$P(E|Q) = \frac{P(Q|E)P(E)}{P(Q)}$$

… but notice, if our goal is to find the best possible state sequence $Q_1, \ldots, Q_T$, then we don't care about the $P(E)$ factor:

$$\underset{Q}{\operatorname{argmax}} P(E|Q) = \underset{Q}{\operatorname{argmax}} \frac{P(Q|E)}{P(Q)}$$

Hybrid DNN-HMM: the solution

$$P(E_1, E_2, Q_1, Q_2, \ldots | W) = P_W(Q_1 | Q_0) P(E_1 | Q_1) P_W(Q_2 | Q_1) P(E_2 | Q_2) \ldots$$

From the neural net

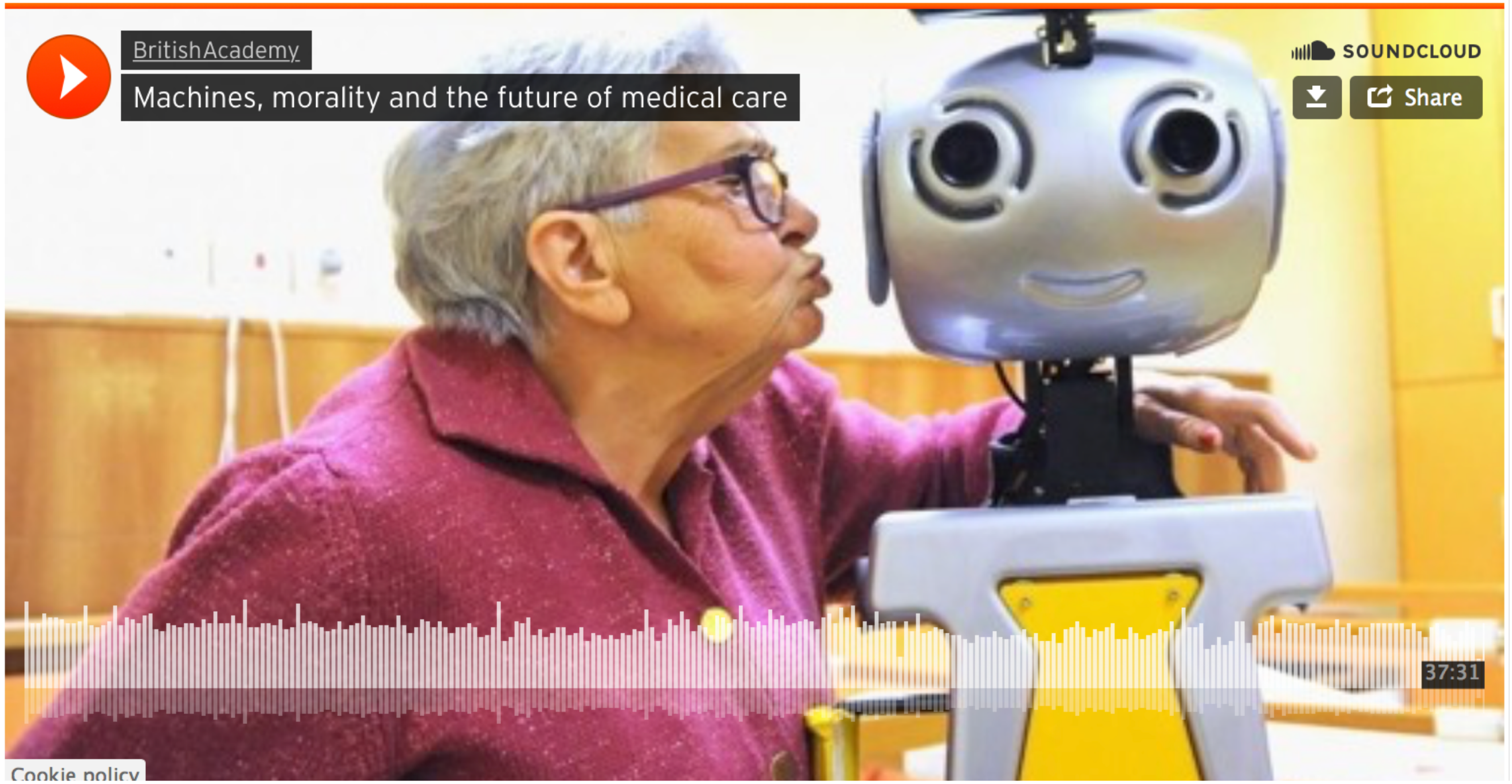
$$\propto \quad P_W(Q_1 | Q_0) \left(\frac{P(Q_1 | E_1)}{P(Q_1)} \right) P_W(Q_2 | Q_1) \left(\frac{P(Q_2 | E_2)}{P(Q_2)} \right) \ldots$$

HMM Parameters

Hybrid DNN-HMM: intuitive explanation

- Prior probability, p(Q), tells how frequently HMM state Q is, in normal conversations, ***if we don’t hear the speech***
- DNN computes a posterior probability, p(Q|E), saying how probable Q is ***given the available evidence***
- If p(Q|E) > p(Q), that means that ***the evidence favors Q more than usual***, so we should consider the possibility that this rare word has been spoken.
- If p(Q|E) is still a small number, that doesn’t really matter; what really matters is whether p(Q|E) > p(Q)

CS440/ECE448 Lecture 27: Societal Impacts of AI



Slides by Svetlana Lazebnik, 12/2017
Modified by Mark Hasegawa-Johnson, 4/2019

Image source: https://www.britac.ac.uk/audio/machines-morality-and-future-medical-care

AI and privacy

- Concerns
  - Personal data being inadvertently revealed or falling into the wrong hands
  - Personal data being misused by the parties who collected it
  - Personal data enabling individuals to be manipulated without their knowledge
- Potential solutions
  - Technological: encryption, differential confidentiality, anonymizing tools
  - Regulation: require the use of a technology; forbid disclosure

AI, bias, and fairness

- Concerns
  - AI will inadvertently absorb biases from data
  - Making important decisions based on biased data will exacerbate bias: especially for law enforcement, employment, loans, health insurance, etc.
  - Even well-intentioned applications can create negative side effects: filter bubbles, targeted advertising
  - Outcomes cannot be appealed because AI systems are opaque and proprietary
- Potential solutions
  - Regulation and transparency: e.g., right to explanation
  - More inclusivity among AI technologists: AI4ALL

AI ethics

- We should be aware of all these issues when developing AI technologies!
  - Privacy violations
  - Potential for deception, misuse and manipulation
  - Exacerbating bias and unfair outcomes
  - Lack of transparency and due process
  - Threats to human rights and dignity
  - Weaponization
  - Unintended consequences