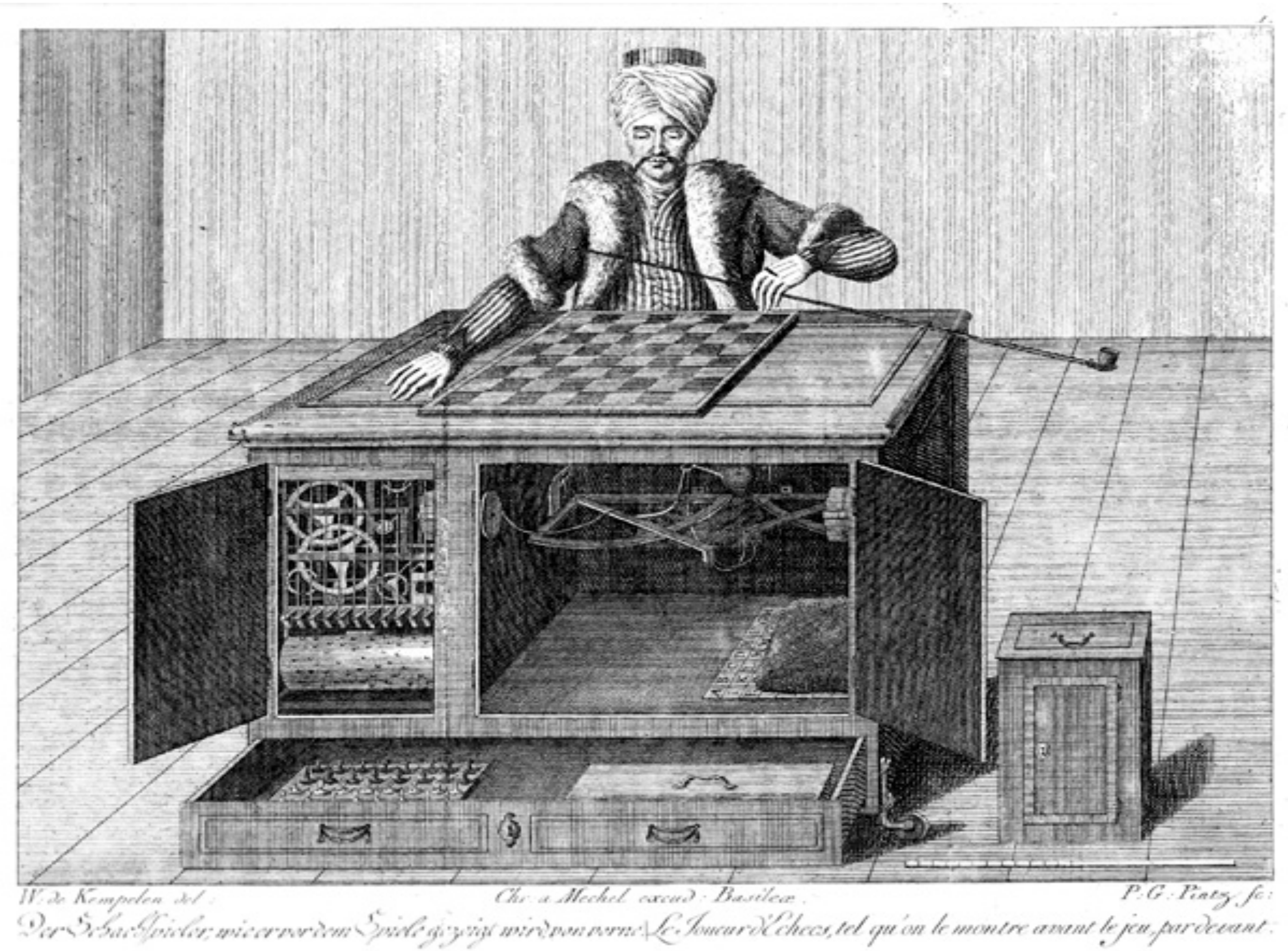
CS440/ECE448 Lecture 8: Two-Player Games

Slides by Svetlana Lazebnik 9/2016

Modified by Mark Hasegawa-Johnson 2/2019



Why study games?

- Games are a traditional hallmark of intelligence
- Games are easy to formalize
- Games can be a good model of real-world competitive or cooperative activities
    - Military confrontations, negotiation, auctions, etc.

Game AI: Origins

- Minimax algorithm: Ernst Zermelo, 1912
- Chess playing with evaluation function, quiescence search, selective search: Claude Shannon, 1949 (paper)
- Alpha-beta search: John McCarthy, 1956
- Checkers program that learns its own evaluation function by playing against itself: Arthur Samuel, 1956

Types of game environments

	Deterministic	**Stochastic**
Perfect information (fully observable)	Chess, checkers, go	Backgammon, monopoly
Imperfect information (partially observable)	Battleship	Scrabble, poker, bridge

Zero-sum Games

Alternating two-player zero-sum games

- Players take **turns**
- Each game **outcome** or **terminal state** has a **utility for each player** (e.g., 1 for win, 0 for loss)
- The **sum of both players' utilities is a constant**

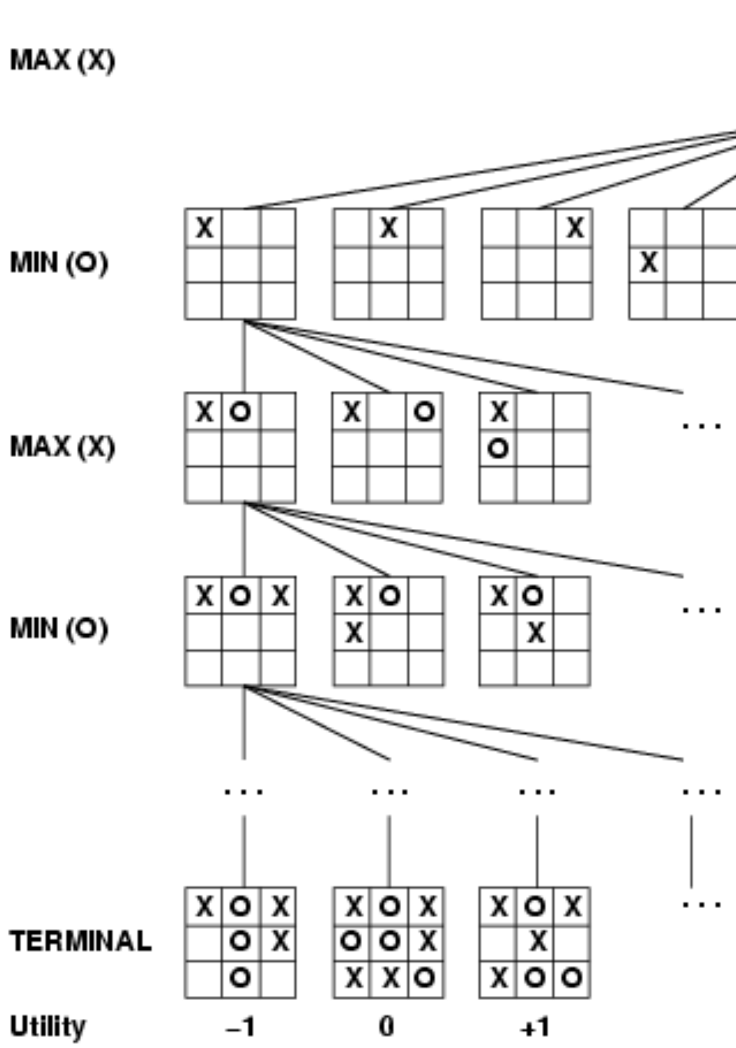
Games vs. single-agent search

We don’t know how the opponent will act

- The solution is **not a fixed sequence of actions** from start state to goal state, but a ***strategy*** **or** ***policy*** (a mapping from state to best move in that state)

Game tree

A game of tic-tac-toe between two players, “max” and “min”

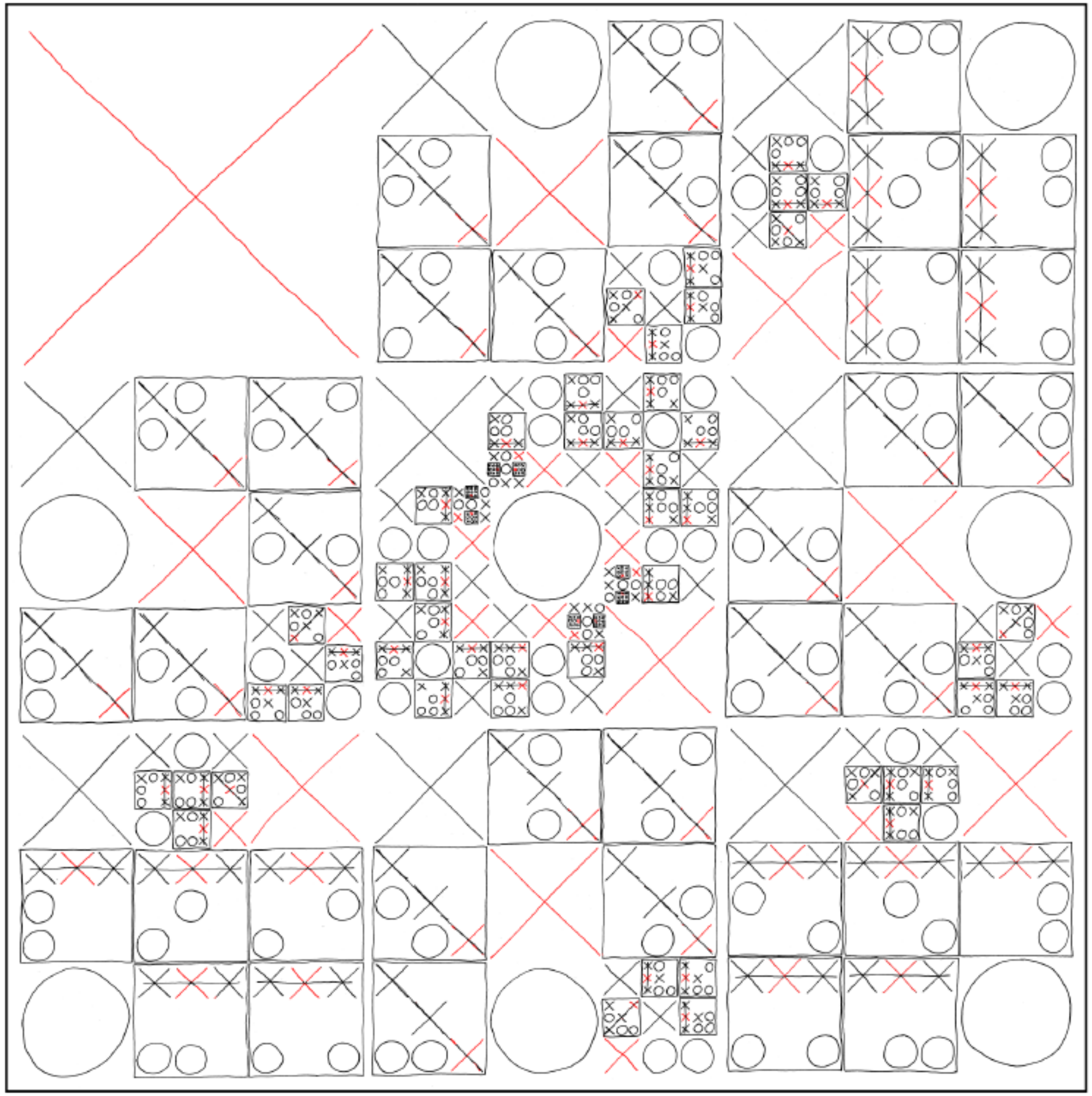


COMPLETE MAP OF OPTIMAL TIC-TAC-TOE MOVES

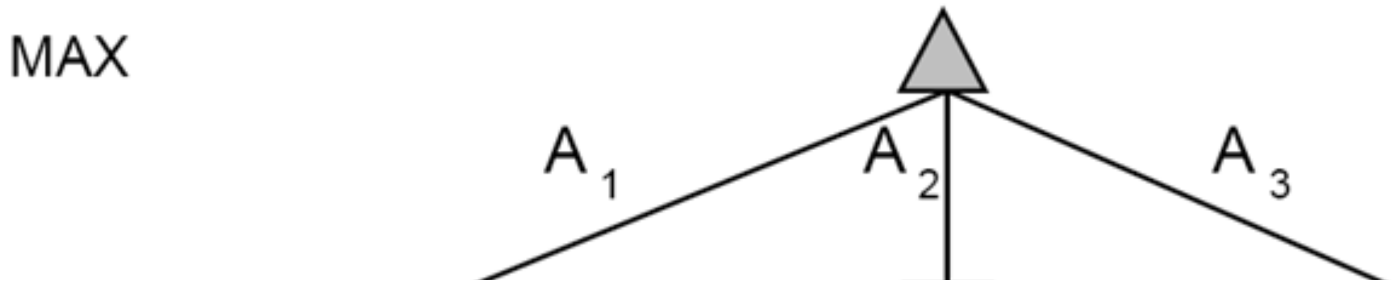
YOUR MOVE IS GIVEN BY THE POSITION OF THE LARGEST RED SYMBOL ON THE GRID. WHEN YOUR OPPONENT PICKS A MOVE, ZOOM IN ON THE REGION OF THE GRID WHERE THEY WENT. REPEAT.

http://xkcd.com/832/

MAP FOR X:



A more abstract game tree



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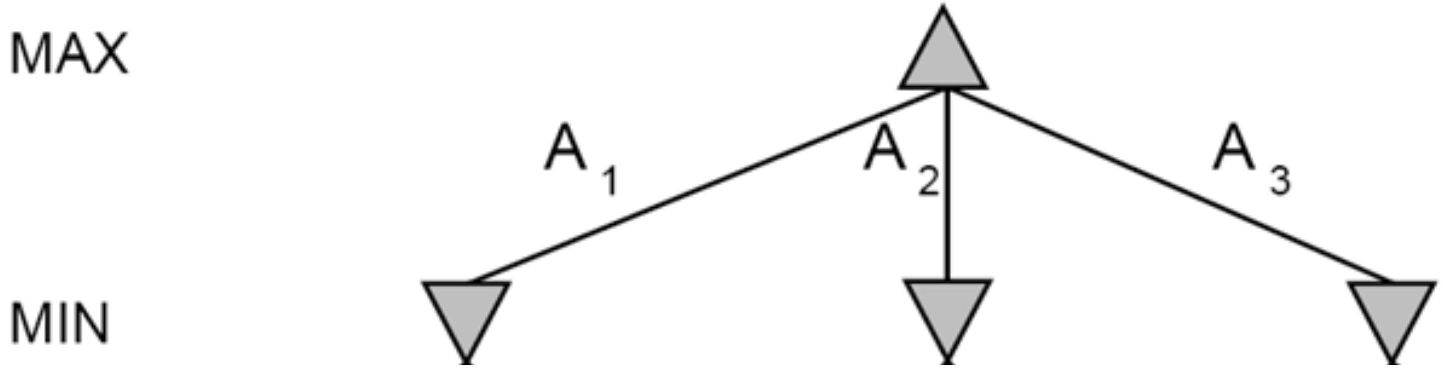

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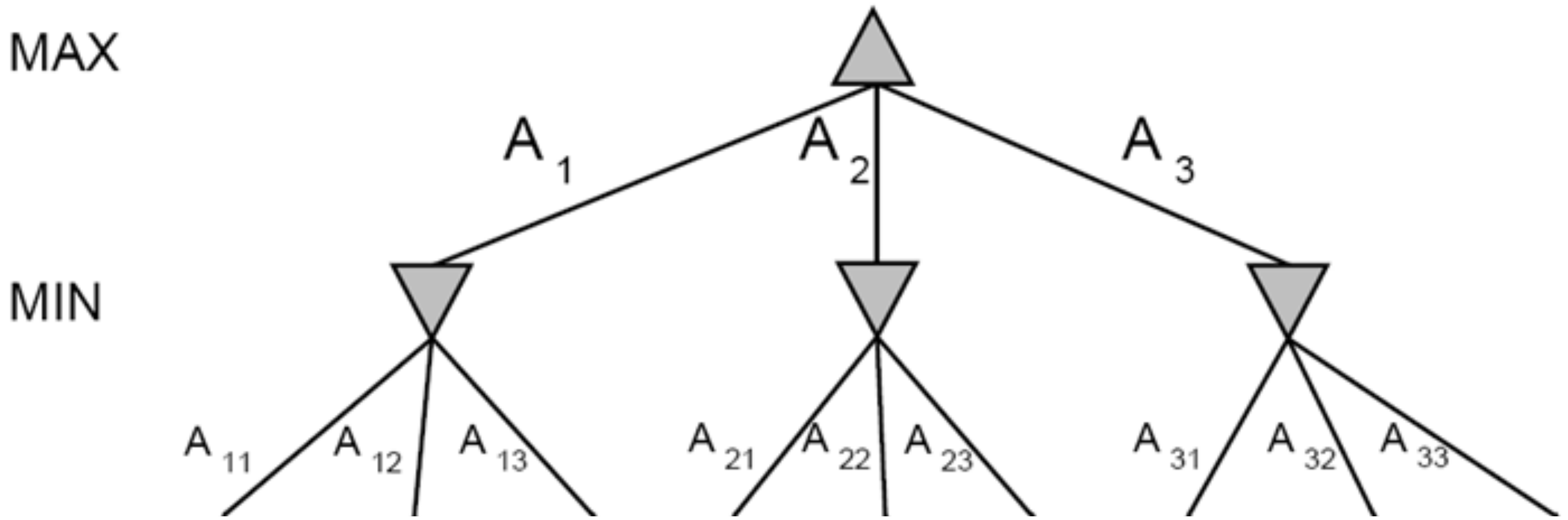

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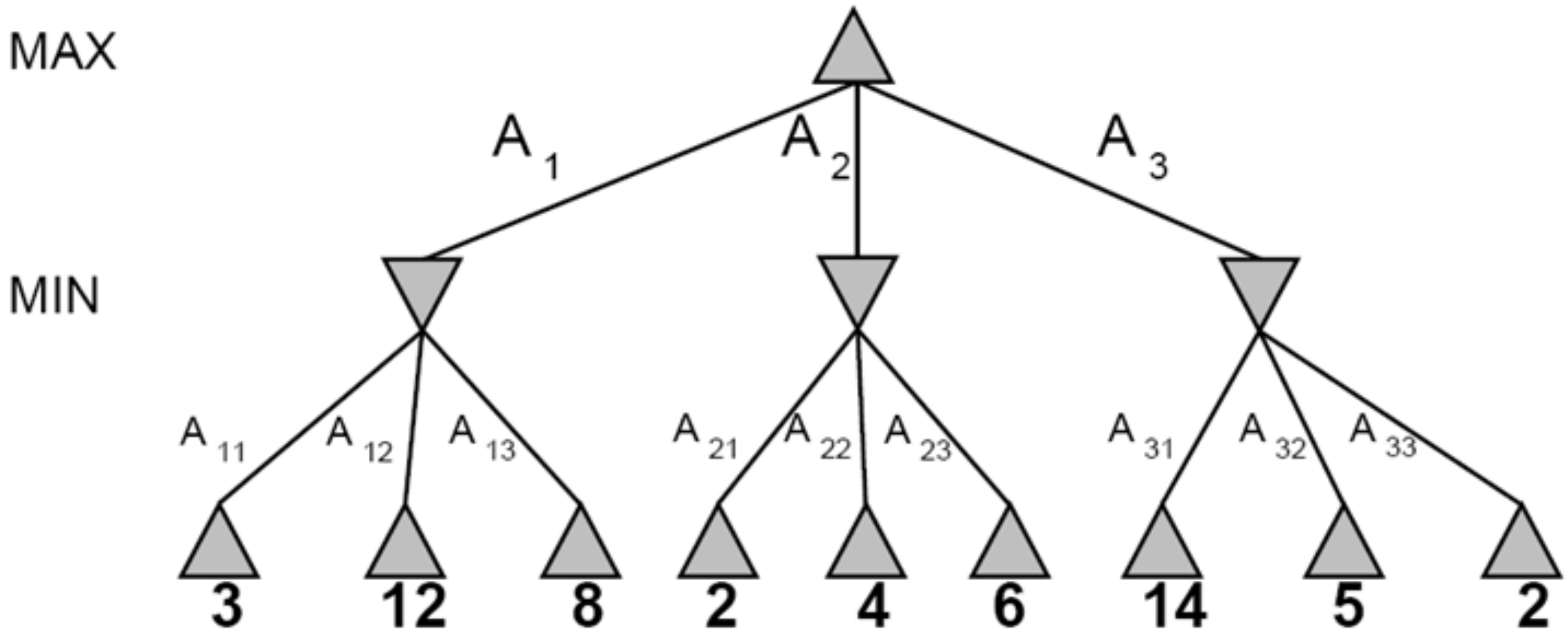


Terminal utilities (for MAX)

A *two-ply* game

ply = one move taken by one player

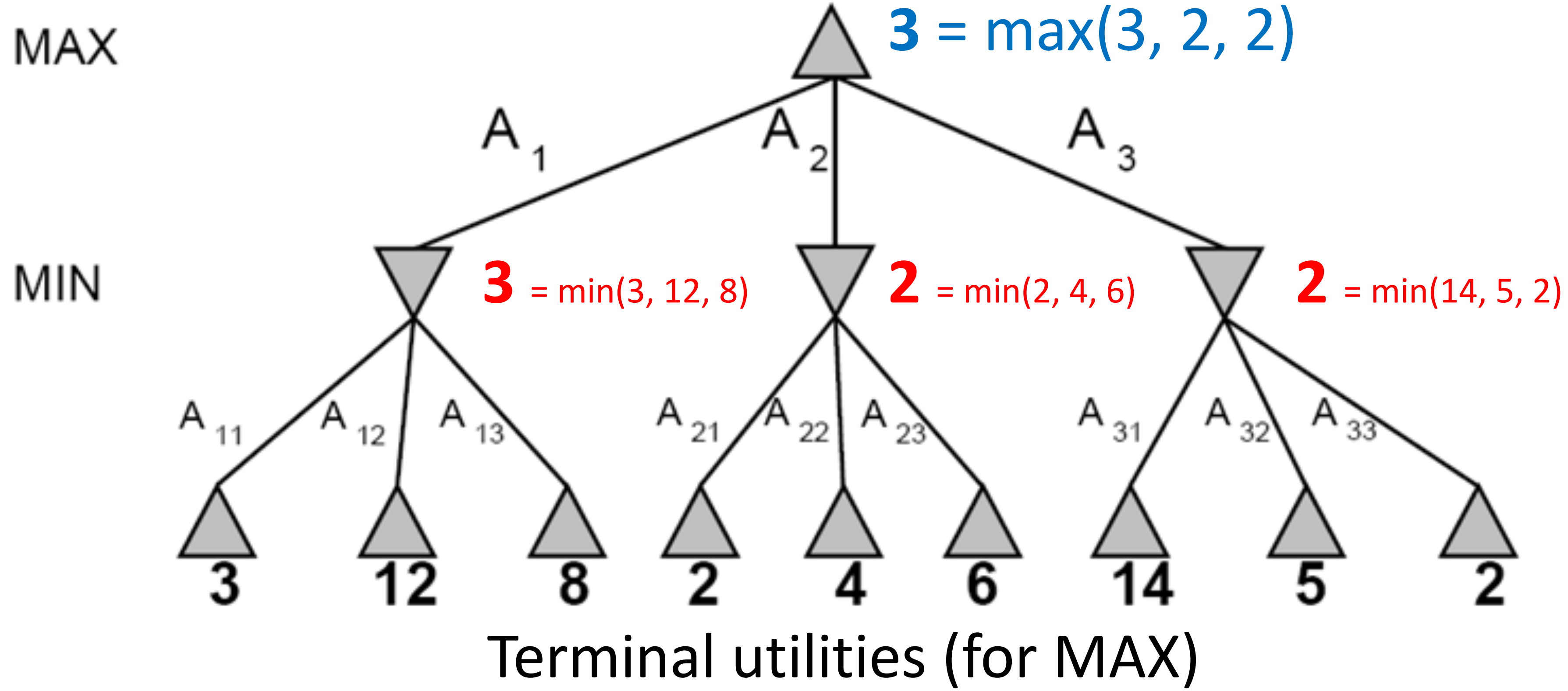
= one layer in the search tree

Minimax Search

Minimax assumptions

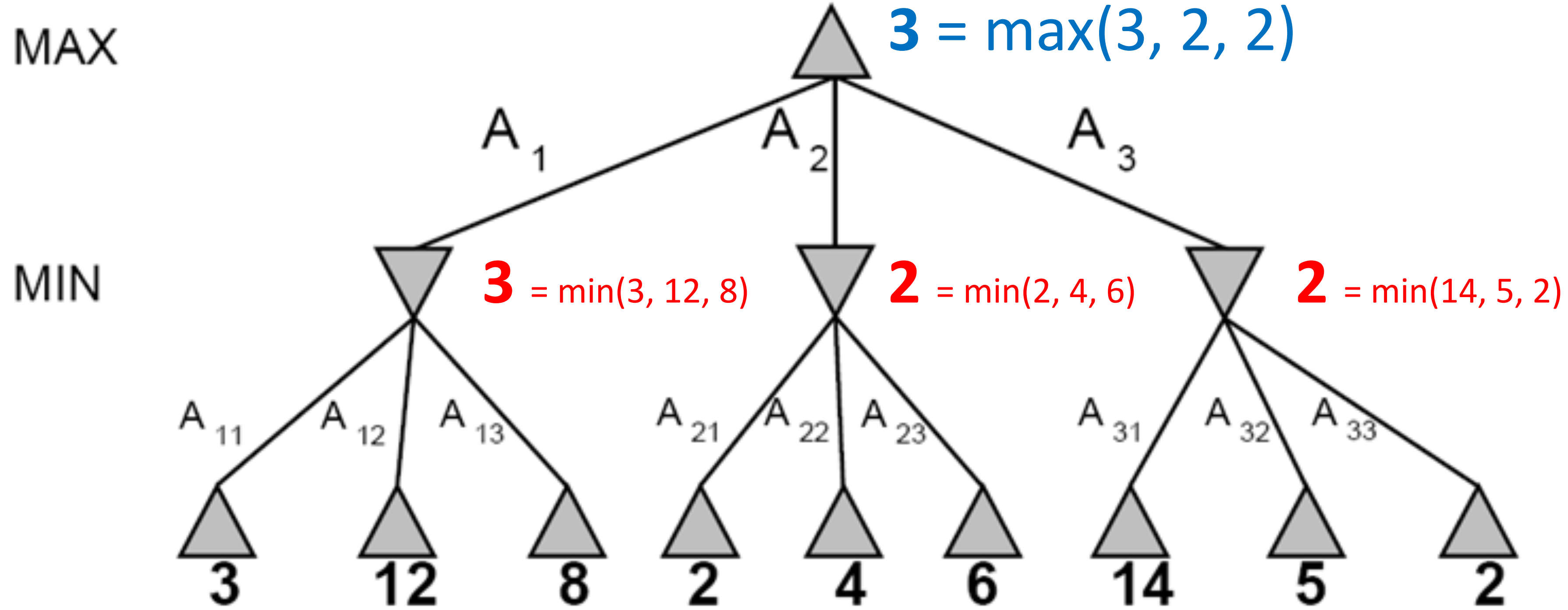
- I am MAX and my opponent is MIN
- Every possible outcome has a value (or "**utility**") for me (MAX).
- **Zero-sum game:**
  if the value to me is +V, then the value to my opponent (MIN) is –V.
- Phrased another way:
  - ***My* (MAX's) rational action**, on each move, is to choose a move that will **MAXIMIZE the value of the outcome**
  - ***My opponent (MIN)'s* rational action** is to choose a move that will **MINIMIZE the value of the outcome**
- **MAX and MIN will always choose the best (most rational) actions**

Game tree search



- **Minimax value of a node**: the utility (for MAX) of being in the corresponding state, *assuming perfect play on both sides*
- **Minimax strategy:**
  Choose the move that gives **the *best worst-case* payoff**

Computing the minimax value of a node



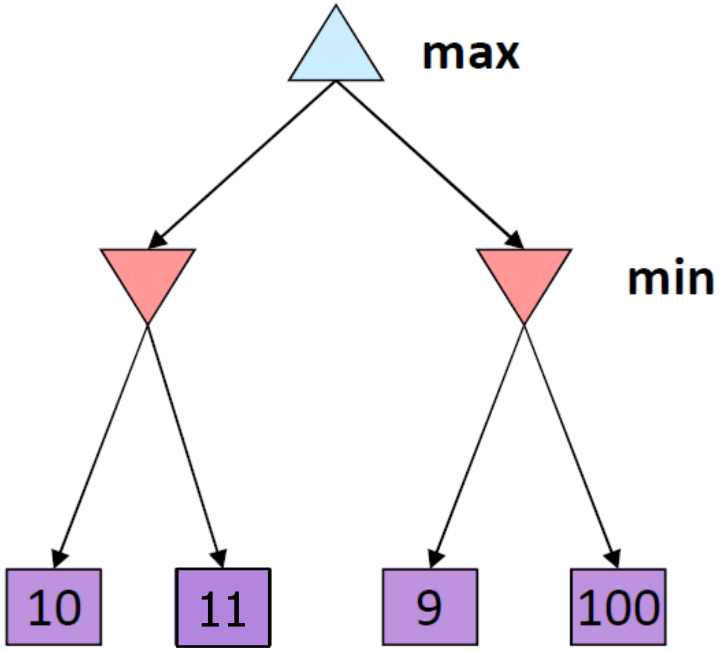
Terminal utilities (for MAX)

Minimax(*node*) =

- Utility(*node*) if *node* is terminal
- $\min_{action}$ **Minimax**(Succ(*node, action*)) if *player* = MIN
- $\max_{action}$ **Minimax**(Succ(*node, action*)) if *player* = MAX

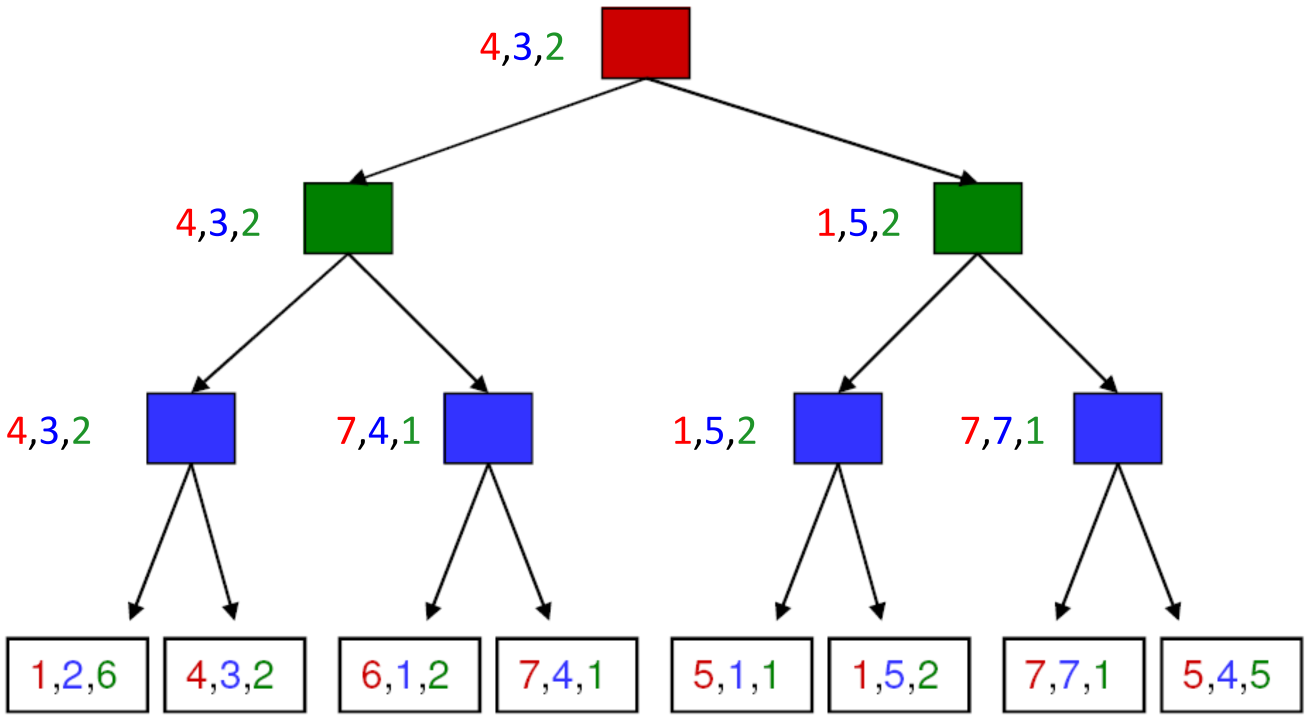
Optimality of minimax

- The minimax strategy is **optimal** against an **optimal opponent**
- What if your opponent is **suboptimal**?
  - Your utility will ALWAYS BE HIGHER than if you were playing an optimal opponent!
  - A different strategy may work better for a sub-optimal opponent, but it will necessarily be worse against an optimal opponent



Example from D. Klein and P. Abbeel

More general games

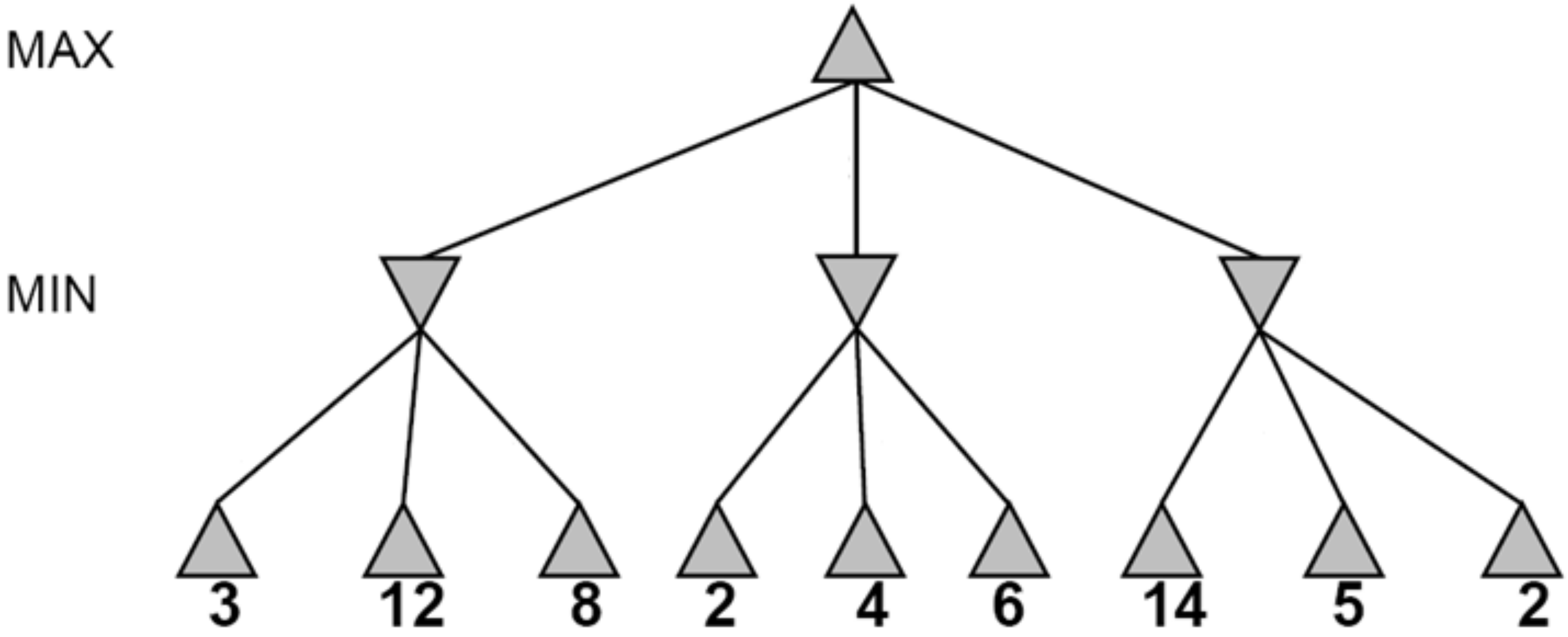


- More than two players (e.g. red, greed, blue), non-zero-sum
- Utilities are now tuples
- Each player maximizes their own utility at their node
- Utilities get propagated (*backed up*) from children to parents

Alpha-Beta Pruning

Alpha-beta pruning

It is possible to compute the *exact* minimax decision *without expanding every node in the game tree*



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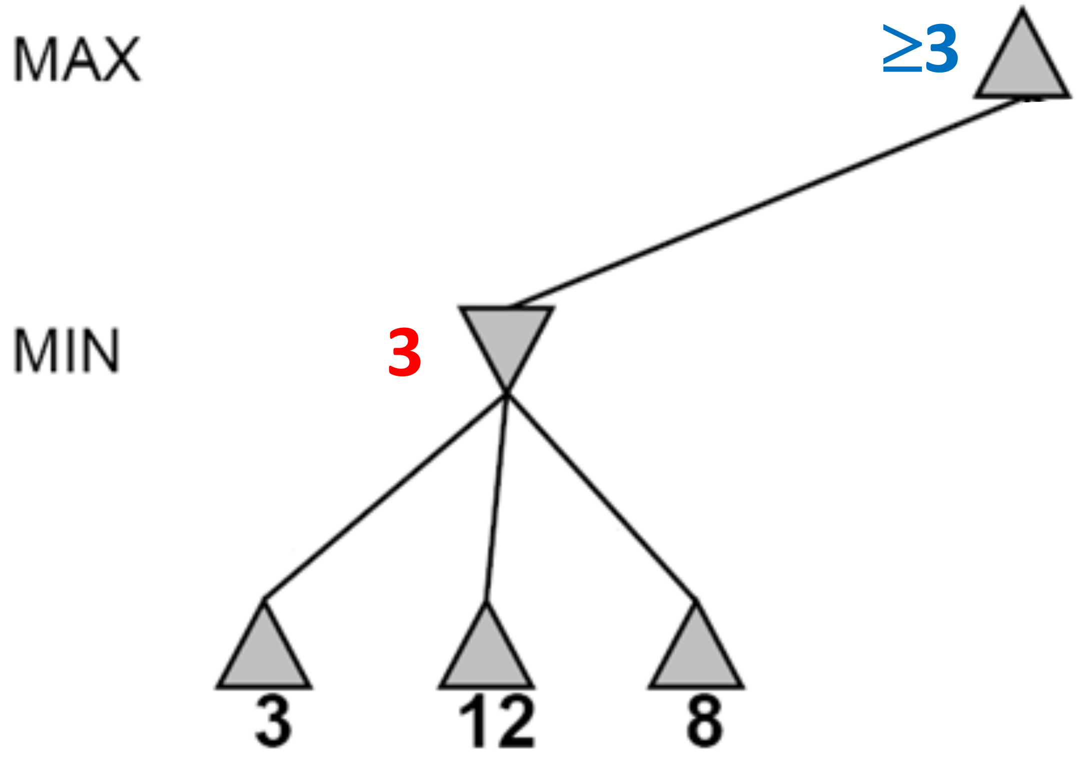

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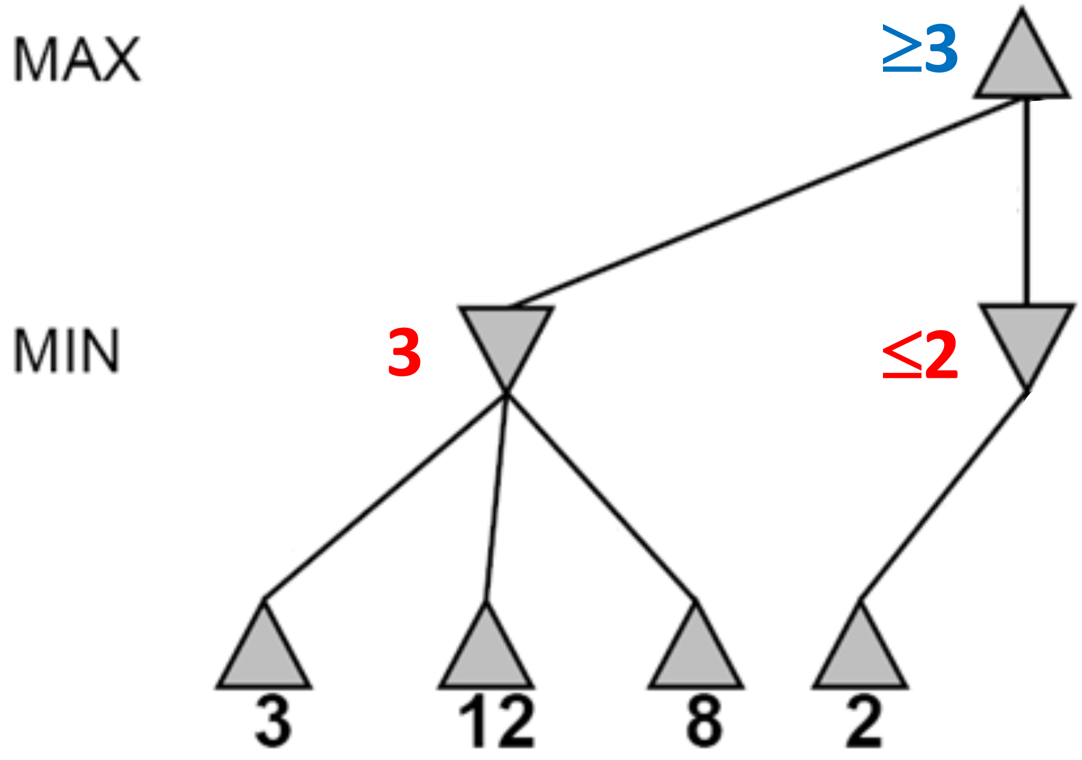

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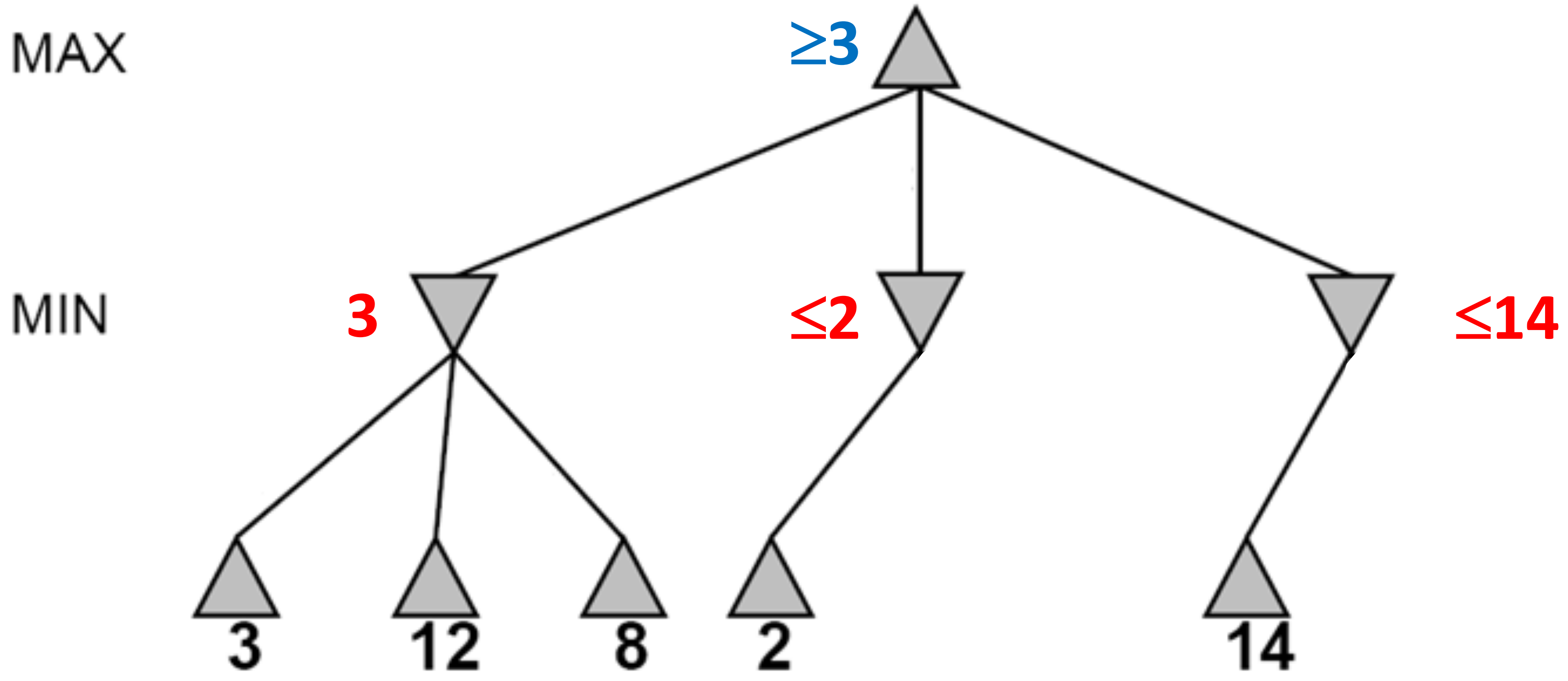

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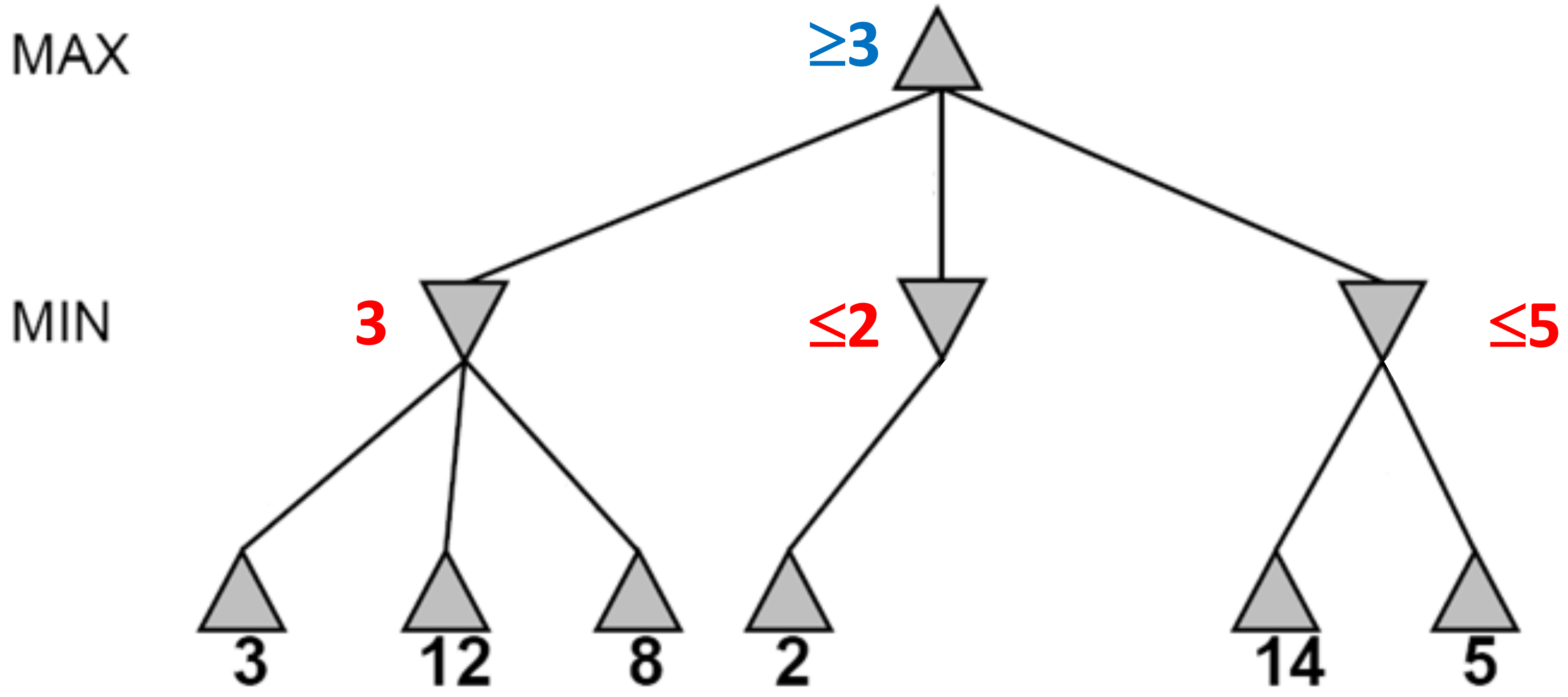

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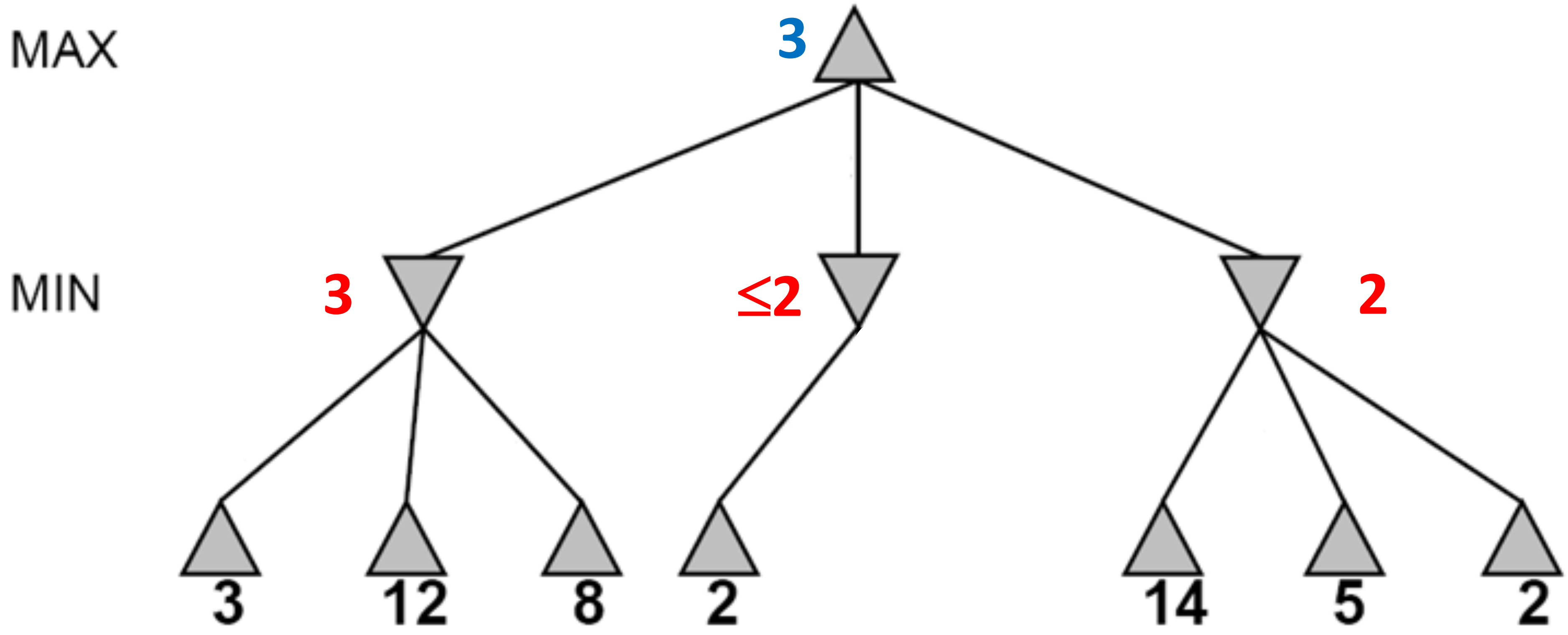


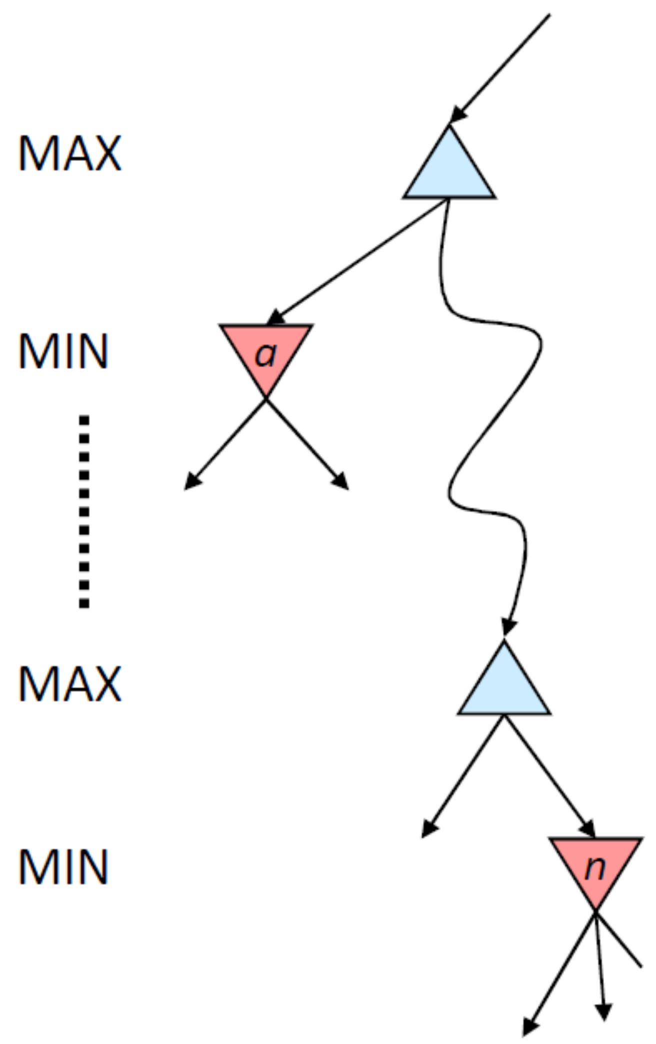
Alpha-Beta Pruning

Key point that I find most counter-intuitive:

- MIN needs to calculate which move MAX will make.
- MAX would never choose a suboptimal move.
- So if MIN discovers that, at a particular node in the tree, she can make a move that's REALLY REALLY GOOD for her…
- She can assume that MAX will never let her reach that node.
- … and she can prune it away from the search, and never consider it again.

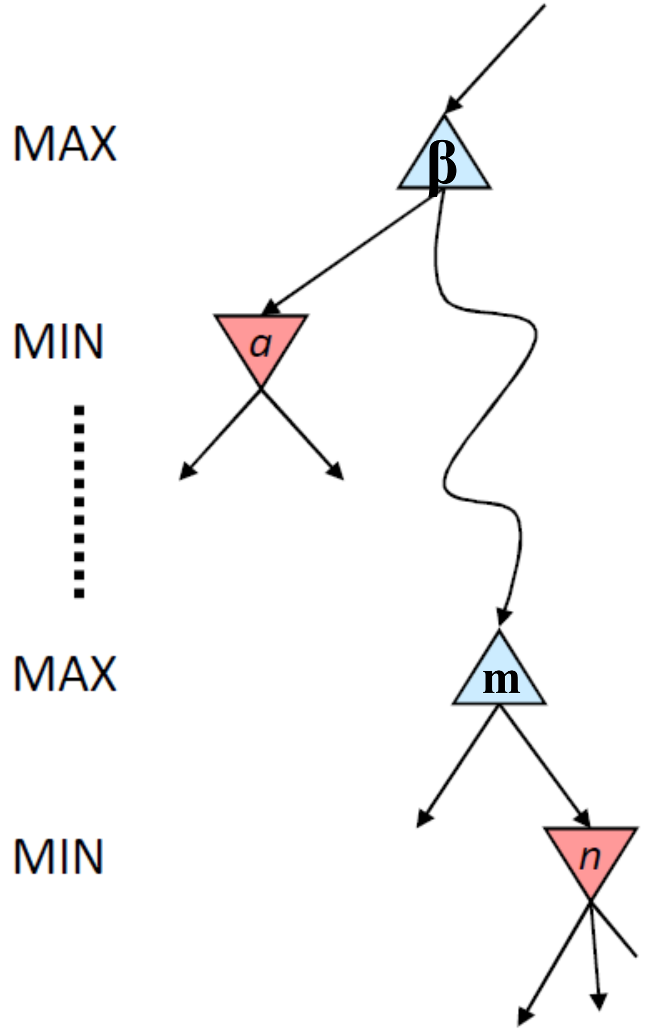
Alpha-beta pruning: MIN nodes

- We're at a **MIN node *n***
- α is the value of the **best choice for MAX** found so far at any choice point *above* node *n*
- More precisely: α **is the highest number that MAX knows how to force MIN to accept**
- We want to **compute the MIN-value at *n***
- As we loop over *n*'s children, the MIN-value decreases
- If it drops below α, MAX will never choose *n*, so we can ignore *n*'s remaining children



Alpha-beta pruning: MAX nodes

- We're at a **MAX node *m***
- β is the value of the **best choice for MIN** found so far at any choice point above node *n*
- More precisely: β is the **lowest number that MIN knows how to force MAX to accept**
- We want to **compute the MAX-value at *m***
- As we loop over *m*'s children, the MAX-value increases
- If it rises above β, MIN will never choose *m*, so we can ignore *m*'s remaining children



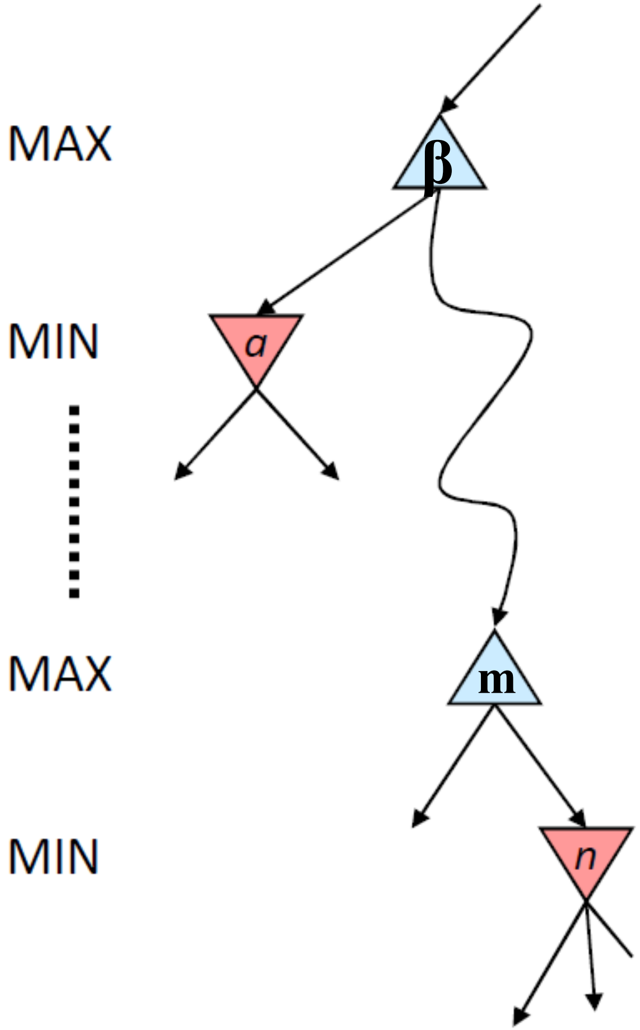
Alpha-beta pruning

An unexpected result:

- **α (current best choice for MAX)** is the **highest** number that MAX knows how to force MIN to accept
- **β (current best choice for MIN)** is the **lowest** number that MIN knows how to force MAX to accept

So

$$\alpha \leq \beta$$

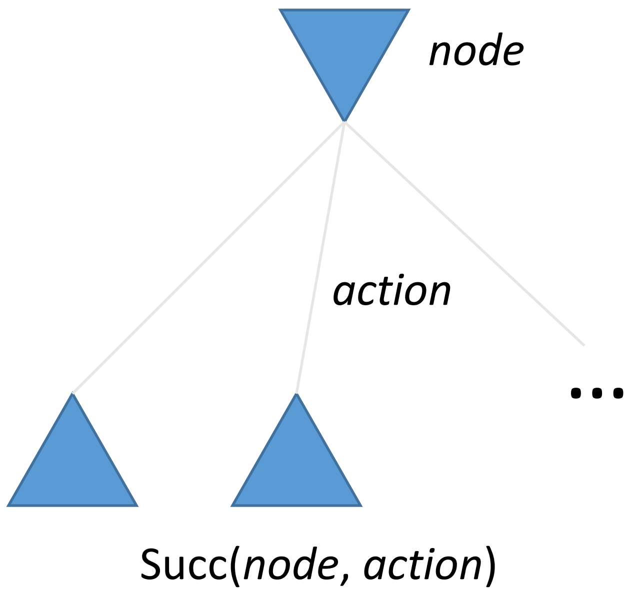


Alpha-beta pruning: MIN nodes

Function *action* = **Alpha-Beta-Search**(*node*)
    v = **Min-Value**(*node*, $-\infty$, ∞)
    return the *action* from *node* with value v

α: *best alternative available to the Max player*
β: *best alternative available to the Min player*

Function v = **Min-Value**(*node*, α, β)
    if Terminal(*node*) return Utility(*node*)
    $v = +\infty$
    for each *action* from *node*
        v = Min(v, **Max-Value**(Succ(*node*, *action*), α, β))
        if $v \leq \alpha$ return v
        β = Min(β, v)
    end for
    return v



Alpha-beta pruning: MAX nodes

Function *action* = **Alpha-Beta-Search**(*node*)

  v = **Max-Value**(*node*, $-\infty$, ∞)

  return the *action* from *node* with value v

α: best alternative available to the Max player

β: best alternative available to the Min player

Function v = **Max-Value**(*node*, α, β)

  if Terminal(*node*) return Utility(*node*)

  $v = -\infty$

  for each *action* from *node*

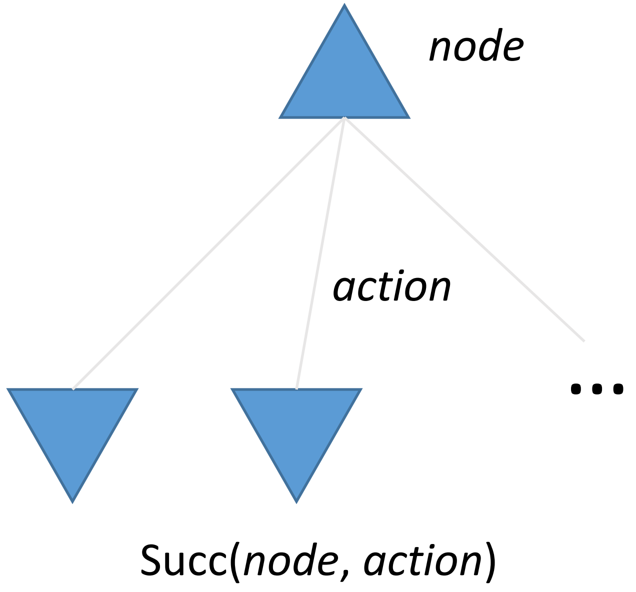
    v = Max(v, **Min-Value**(Succ(*node*, *action*), α, β))

    if $v \geq \beta$ return v

    α = Max(α, v)

  end for

  return v



Alpha-beta pruning

- Pruning does not affect final result
- Amount of pruning depends on move ordering
  - Should start with the "best" moves (highest-value for MAX or lowest-value for MIN)
  - For chess, can try captures first, then threats, then forward moves, then backward moves
  - Can also try to remember "killer moves" from other branches of the tree
- With perfect ordering, the time to find the best move is reduced to $O(b^{m/2})$ from $O(b^m)$
  - Depth of search is effectively doubled

Limited-Horizon Computation

Games vs. single-agent search

We don't know how the opponent will act

The solution is not a fixed sequence of actions from start state to goal state, but a ***strategy*** or ***policy*** (a mapping from state to best move in that state)

Efficiency is critical to playing well

- The time to make a move is limited
- The branching factor, search depth, and number of terminal configurations are huge

  Chess: branching factor ≈ 35 and depth ≈ 100 => search tree of 10^{154} nodes
  (Number of atoms in the observable universe ≈ 10^{80})

- This rules out searching all the way to the end of the game

Evaluation function

- Cut off search at a certain depth and compute the value of an **evaluation function** for a state instead of its minimax value

  The evaluation function may be thought of as the probability of winning from a given state or the *expected value* of that state

- A common evaluation function is a **weighted sum of *features***:

$$Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$$

  For chess, w_k may be the material value of a piece
  (pawn = 1, knight = 3, rook = 5, queen = 9)
  and $f_k(s)$ may be the advantage in terms of that piece

- Evaluation functions may be ***learned*** from game databases or by having the program play many games against itself

Cutting off search

Horizon effect:
You may incorrectly estimate the value of a state by overlooking an event that is just beyond the depth limit

For example, a damaging move by the opponent that can be delayed but not avoided

Possible remedies

- **Quiescence search:** do not cut off search at positions that are unstable – for example, are you about to lose an important piece?
- **Singular extension:** a strong move that should be tried when the normal depth limit is reached

Advanced techniques

- **Transposition table** to store previously expanded states
- **Forward pruning** to avoid considering all possible moves
- **Lookup tables** for opening moves and endgames

Chess playing systems

Baseline system: 200 million node evaluations per move (3 min), minimax with a decent evaluation function and quiescence search

5-ply ≈ human novice

Add alpha-beta pruning

10-ply ≈ typical PC, experienced player

Deep Blue: 30 billion evaluations per move, singular extensions, evaluation function with 8000 features, large databases of opening and endgame moves

14-ply ≈ Garry Kasparov

More recent state of the art (Hydra, ca. 2006): 36 billion evaluations per second, advanced pruning techniques

18-ply ≈ better than any human alive?

Summary

- A **zero-sum game** can be expressed as a **minimax tree**
- **Alpha-beta pruning** finds the correct solution.

  In the best case, it has half the exponent of minimax
  (can search twice as deeply with a given computational complexity).
- **Limited-horizon search** is *always necessary*
  (you can't search to the end of the game), and *always suboptimal*.
  - Estimate your utility, at the end of your horizon, using some type of learned utility function
  - Quiescence search: don't cut off the search in an unstable position (need some way to measure "stability")
  - Singular extension: have one or two "super-moves" that you can test at the end of your horizon