

# Last week: Q-learning for discrete s, a

- So far, we've assumed a *lookup table* representation for utility function U(s) or action-utility function Q(s,a)
- This does not work if the state space is really large or continuous

### This time: Function approximation

- Approximate Q(s, a) by a parameterized function, that is, by a function Q(s, a; W) that depends on some matrix of trainable parameters, W.
- Learn W by playing the game.

# Outline

- On-line Q-learning
- How to make Q-learning converge to the best answer
- How to make it converge more smoothly
- Policy learning and actor-critic networks
- Imitation learning

# Deep Q learning

• Train a deep neural network to output Q values:



Source: D. Silver

# Deep Q learning

• SARSA update: "nudge" Q(s,a) toward value we observe it to have in the most recent action:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \left( R(s) + \gamma \max_{a'} Q(s',a') - Q(s,a) \right)$$

 Deep Q learning: train neural network weights, w, in order to minimize a loss function that penalizes differences between Q(local) and Q(predicted):

$$L(w) = (R(s) + \gamma \max_{a'} Q(s', a'; w) - Q(s, a; w))^{2}$$

Q(local):

s'=state you actually reach by performing a in s,

*a'*=action you will actually perform there.

Q(predicted): What the network predicts for action *a* in state *s* 

# Deep Q learning

- Regular TD update: "nudge" Q(s,a) towards the target  $Q(s,a) \leftarrow Q(s,a) + \alpha (R(s) + \gamma \max_{a'} Q(s',a') - Q(s,a))$
- Deep Q learning: encourage estimate to match the target by minimizing squared error:

$$L(w) = (R(s) + \gamma \max_{a'} Q(s', a'; w) - Q(s, a; w))^{2}$$

target

estimate

• Compare to supervised learning:

$$L(w) = (y - f(x; w))^2$$

<u>Key difference</u>: the target in Q learning is not fixed – (s',a') is just one step ahead of (s,a)!

### Online Q learning algorithm

- In state s, perform action a. Environment sends you to state s'; choose the action a' that you'll perform there.
- Observe:  $Q^{local}(s, a) = R(s) + \gamma \max_{a'} Q(s', a'; W)$
- Update weights to reduce the error

$$L(W) = \left(Q^{local} - Q(s, a; W)\right)^2$$

• Gradient:

$$\nabla_W L = \left(Q(s, a; W) - Q^{local}\right) \nabla_W Q$$

• Weight update:

$$W \leftarrow W - \eta \nabla_W L$$

- This is called stochastic gradient descent (SGD)
- "Stochastic" because the training sample (s,a,s',a') was chosen at random by our exploration function

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#### Convergence of neural networks



- A general neural net (e.g., a classifier) is trained to minimize the training corpus error.
- Test corpus error might be very different!
- Barron showed: generalization error is *G* <(#hidden nodes/#training tokens)
- As #training tokens  $\rightarrow \infty$ ,  $G \rightarrow 0$

#### Does Q-learning Converge?

- No!
- Because:

$$a = \operatorname{argmax} Q(s, a)$$

 If we always choose the action that is best, according to our current estimate of the Q-function, then we can never learn anything about any of the other actions!

### Incorporating exploration (slide from last week)

- Idea: explore more in the beginning, become more and more greedy over time
- Standard ("greedy") selection of optimal action:

$$a = \underset{a' \in A(s)}{\operatorname{arg\,max}} \sum_{s'} P(s'|s,a') U(s')$$

• Modified strategy:

$$a = \underset{a' \in A(s)}{\operatorname{arg\,max}} f\left(\sum_{s'} P(s'|s,a')U(s'), N(s,a')\right)$$

exploration<br/>functionNumber of times<br/>we've taken action a'<br/>in state s<br/>(optimistic reward<br/>estimate) $f(u,n) = \begin{cases} R^+ & \text{if } n < N_e \\ u & \text{otherwise} \end{cases}$ (optimistic reward<br/>estimate)

#### ...but that doesn't work either:

$$f(u,n) = \begin{cases} R^+ & \text{if } n < N_e \\ u & \text{otherwise} \end{cases}$$

- ... which means that we get at least  $N_e$  samples of each action
- We can estimate Q(s,a) based on  $N_e$  samples
- But  $N_e$  is a constant, so it never  $\rightarrow \infty$
- So Error never  $\rightarrow 0$

# **Epsilon-greedy exploration**

- At each time step:
  - With probability  $\epsilon$ , choose an action at random
  - With probability  $1 \epsilon$ , choose a=argmax Q(s,a)

- As 
$$n \to \infty, \epsilon \to 0$$
, for example,  $\epsilon = 1/n$ 

- Result:
  - As you play the game infinite times, each action is sampled an infinite number of samples, so Q converges, but also,
  - As you play the game infinite times, you start to exploit your knowledge more and more frequently, so that you converge to the best possible policy.
  - ... actually, it doesn't always work in practice. To guarantee success, you need a few more tweaks, e.g., Re-Trace algorithm, Munos et al., 2016.

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# Dealing with training instability

#### • Challenges

- Target values are not fixed
- Successive experiences are correlated and dependent on the policy
- Policy may change rapidly with slight changes to parameters, leading to drastic change in data distribution
- Solutions
  - Freeze target Q network
  - Use experience replay

Mnih et al. <u>Human-level control through deep reinforcement learning</u>, *Nature* 2015

#### Experience replay

• At each time step:

*s*<sub>t</sub>, *a*<sub>t</sub>, *r*<sub>t+1</sub>, *s*<sub>t+1</sub>

- Take action a<sub>t</sub> according to epsilon-greedy policy
- Store experience  $(s_t, a_t, r_{t+1}, s_{t+1})$  in replay memory buffer
- Randomly sample *mini-batch* of experiences from the buffer

$$\begin{array}{c} s_{1}, a_{1}, r_{2}, s_{2} \\ s_{2}, a_{2}, r_{3}, s_{3} \\ s_{3}, a_{3}, r_{4}, s_{4} \\ \dots \\ \rightarrow s_{t}, a_{t}, r_{t+1}, s_{t+1} \end{array}$$

Mnih et al. Human-level control through deep reinforcement learning, Nature 2015

### Experience replay

At each time step:

- Take action a<sub>t</sub> according to epsilon-greedy policy
- Store experience  $(s_t, a_t, r_{t+1}, s_{t+1})$  in replay memory buffer
- Randomly sample *mini-batch* of experiences from the buffer
- Perform update to reduce objective function

$$\mathbf{E}_{s,a,s'}\left[\left(R(s)+\gamma\max_{a'}Q(s',a';w^{-})-Q(s,a;w)\right)^2\right]$$

Keep parameters of *target network* fixed during the entire mini-batch; only update between mini-batches

Mnih et al. Human-level control through deep reinforcement learning, Nature 2015

#### Deep Q learning in Atari



Mnih et al. Human-level control through deep reinforcement learning, Nature 2015

# Deep Q learning in Atari

- End-to-end learning of Q(s,a) from pixels s
- Output is Q(s,a) for 18 joystick/button configurations
- Reward is change in score for that step



Mnih et al. Human-level control through deep reinforcement learning, Nature 2015

#### Deep Q learning in Atari

- Input state s is stack of raw pixels from last 4 frames
- Network architecture and hyperparameters fixed for all games



Mnih et al. Human-level control through deep reinforcement learning, Nature 2015

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# Policy gradient methods

- Learning the policy directly can be much simpler than learning Q values
- We can train a neural network to output *stochastic policies*, or probabilities of taking each action in a given state
- *Softmax* policy:

$$\pi(s,a;u) = \frac{\exp(f(s,a;u))}{\sum_{a'} \exp(f(s,a';u))}$$

# Policy gradient: the softmax function

• Notice that the softmax is normalized so that

 $\pi(s, a; u) \ge 0$ , and  $\sum_{a} \pi(s, a; u) = 1$ 

- So we can interpret π(s, a; w) as some kind of probability.
   Something like "the probability that a is the best action to take from state s."
- In reality, there is no such probability. There is just one correct action. But the agent doesn't know what it is! So π(s, a; u) is kind of like the agent's "degree of belief" that a is the best action (determined by parameters u).

#### Actor-critic algorithm

• Remember the relationship between the utility of a state, and the quality of an action:

$$U(s) = \max_{a} Q(s, a)$$

• If we don't know which action is best, then we could say that

$$U(s) \approx \sum_{a} \pi(s, a; u) Q(s, a; w)$$

- $\pi(s, a; u)$  is the "actor:" a neural net that tells the agent how to act.
- Q(s, a; w) is the "critic:" a neural net that tells the agent how good or bad that action was.

#### Actor-critic algorithm

- Define objective function as total discounted reward:  $J(u) = \mathbf{E} \left[ R_1 + \gamma R_2 + \gamma^2 R_3 + \dots \right]$
- The gradient for a stochastic policy is given by

$$\nabla_{u} J = \mathbf{E} \begin{bmatrix} \nabla_{u} \log \pi(s, a; u) & Q^{\pi}(s, a; w) \end{bmatrix}$$
Actor Critic
network network

- Actor network update:  $u \leftarrow u + \alpha \nabla_u J$
- Critic network update: use Q learning (following actor's policy)

### Advantage actor-critic

- The raw Q value is less meaningful than whether the reward is better or worse than what you expect to get
- Introduce an *advantage function* that subtracts a baseline number from all Q values

$$A^{\pi}(s,a) = Q^{\pi}(s,a) - V^{\pi}(s)$$

– Estimate V using a value network

• Advantage actor-critic:

$$\nabla_{u} J = \mathbf{E} \Big[ \nabla_{u} \log \pi(s, a; u) A^{\pi}(s, a; w) \Big]$$

# Asynchronous advantage actor-critic (A3C)



Asynchronously update global parameters

Mnih et al. <u>Asynchronous Methods for Deep Reinforcement</u> <u>Learning</u>. ICML 2016

# Asynchronous advantage actor-critic (A3C)



TORCS car racing simulation video

Mnih et al. <u>Asynchronous Methods for Deep Reinforcement</u> <u>Learning</u>. ICML 2016

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# **Imitation learning**





- In some applications, you cannot bootstrap yourself from random policies
  - High-dimensional state and action spaces where most random trajectories fail miserably
  - Expensive to evaluate policies in the physical world, especially in cases of failure
- **Solution:** learn to imitate sample trajectories or demonstrations
  - This is also helpful when there is no natural reward formulation

# Learning visuomotor policies



- Underlying state x: true object position, robot configuration
- Observations o: image pixels
- Two-part approach:
  - Learn guiding policy π(a|x) using trajectory-centric RL and control techniques
  - Learn visuomotor policy
     π(a|o) by imitating π(a|x)

S. Levine et al. End-to-end training of deep visuomotor policies. JMLR 2016

#### Learning visuomotor policies



Overview video, training video

S. Levine et al. End-to-end training of deep visuomotor policies. JMLR 2016

# Conclusions

- 1. What is deep Q-learning?
- 2. How to make Q-learning converge to the best answer?
- 3. How to make it converge more smoothly?
- 4. What are policy learning and actor-critic networks?
- 5. What is imitation learning?

- 1. Estimate Q(s,a) using a neural net.
- 2. Epsilon-greedy usually works.
- 3. Experience replay.
- 4. Actor network: Pr(a). Critic network: Q(s, a), to train the actor.
- 5. Learn to imitate an expert player.