1 Resolution Refutation

Consider the following English sentences. They are already translated into FOPC sentences.

(1) DaeHoon is a CS student.
   \[\text{CS\_student}(\text{DaeHoon})\]

(2) Every CS student has been to the CRCE (Campus Recreation Center).
   \[\forall x (\text{CS\_student}(x) \Rightarrow \text{Visited}(x, \text{CRCE}))\]

(3) There is a racquetball court in CRCE.
   \[\exists x (\text{Is\_racquetball\_court}(x) \land \text{Located\_in}(x, \text{CRCE}))\]

(4) No graduate student plays squash.
   \[\forall x (\text{Is\_grad\_student}(x) \Rightarrow \neg \text{Plays\_squash}(x))\]

(5) DaeHoon is a graduate student.
   \[\text{Is\_grad\_student}(\text{DaeHoon})\]

(6) Everyone who plays racquetball also plays squash.
   \[\forall x (\text{Plays\_racquetball}(x) \Rightarrow \text{Plays\_squash}(x))\]

(7) If a person who has a racquetball racquet has been somewhere with a racquetball court then the person plays racquetball.
   \[\forall x \forall y ((\text{Has\_racquetball\_racquet}(x) \land \text{Visited}(x, y)) \land (\exists z (\text{Is\_racquetball\_court}(z) \land \text{Located\_in}(z, y))) \Rightarrow \text{Plays\_racquetball}(x))\]
1. (This problem is already solved for your convenience.) Convert FOPC sentences (1) to (7) into clause form. Number each clause using the original labels (1) through (7). If sentence (i) is converted into several clauses, number them in the style (i.1), (i.2), etc. Remember to standardize the variables apart and Skolemize properly.

   (1) \{CS\_{student}(DaeHoon)\}
   (2) \{\lnot CS\_{student}(x_1), Visited(x_1, CRCE)\}
   (3.1) \{Is\_{racquetball\_court}(Sk_1)\}
   (3.2) \{Located\_{in}(Sk_1, CRCE)\}
   (4) \{\lnot Is\_{grad\_student}(x_2), \lnot Plays\_{squash}(x_2)\}
   (5) \{Is\_{grad\_student}(DaeHoon)\}
   (6) \{\lnot Plays\_{racquetball}(x_3), Plays\_{squash}(x_3)\}
   (7) \{\lnot Has\_{racquetball\_racquet}(x_4), \lnot Visited(x_4, y_1), \lnot Is\_{racquetball\_court}(z_1), \lnot Located\_{in}(z_1, y_1), Plays\_{racquetball}(x_4)\}

2. (2 points) We want to use resolution refutation to prove that DaeHoon does not have a racquetball racquet. Give the FOPC representation of the negated goal, and convert it into clause form. Number this clause as (8).

   Goal: \lnot Has\_{racquetball\_racquet}(DaeHoon)
   Negated Goal: Has\_{racquetball\_racquet}(DaeHoon)
   Clause Form: (8) \{ Has\_{racquetball\_racquet}(DaeHoon) \}

3. (10 points) Use resolution refutation to prove that DaeHoon does not have a racquetball racquet. That is, give a derivation of the empty clause {} from clauses (1) through (8). In each step please clearly indicate the clauses and unifiers used, and write down and label the derived clauses. Remember to standardize the variables apart.

   (9) \{Visited(DaeHoon, CRCE) \}
   \{x_1 = DaeHoon\}
   (1) and (2)
   (10) \{ \lnot Plays\_{squash}(DaeHoon) \}
   \{x_2 = DaeHoon\}
   (4) and (5)
   (11) \{ \lnot Plays\_{racquetball}(DaeHoon) \}
   \{x_3 = DaeHoon\}
   (6) and (10)
   (12) \{\lnot Has\_{racquetball\_racquet}(x_5), \lnot Visited(x_5, y_1), \lnot Located\_{in}(Sk_1, y_2), Plays\_{racquetball}(x_5)\}
   \{ z_1 = Sk_1 \}
   (3.1) and (7)
(13) \{-\text{Has} \_ \text{racquetball} \_ \text{racquet}(x_6), \neg \text{Visited}(x_6,CRCE), \text{Plays} \_ \text{racquetball}(x_6)\} \\
\{ y_2 = CRCE \} \\
(3.2) \text{and} \ (12)

(14) \{-\text{Has} \_ \text{racquetball} \_ \text{racquet}(DaeHoon), \text{Plays} \_ \text{racquetball}(DaeHoon)\} \\
\{ x_6 = DaeHoon \} \\
(9) \text{and} \ (13)

(15) \{-\text{Has} \_ \text{racquetball} \_ \text{racquet}(DaeHoon)\} \\
\{} \\
(11) \text{and} \ (14)

(16) \{}
\{}
(8) \text{and} \ (15)

4. (10 points) Give the proof tree of your derivation. The lowest node is the empty clause. You are allowed to draw this part by hand and attach the scanned drawing with your submission or hand in the hard copy of the tree. Make sure your submission is clearly legible. Solution in a separate file.