Lecture 18: Mutual Exclusion
Why Mutual Exclusion?

- **Bank’s Servers in the Cloud**: Two of your customers make simultaneous deposits of $10,000 into your bank account, each from a separate ATM.
  - Both ATMs read initial amount of $1000 concurrently from the bank’s cloud server
  - Both ATMs add $10,000 to this amount (locally at the ATM)
  - Both write the final amount to the server
  - **What’s wrong?**
Why Mutual Exclusion?

- **Bank’s Servers in the Cloud**: Two of your customers make simultaneous deposits of $10,000 into your bank account, each from a separate ATM.
  - Both ATMs read initial amount of $1000 concurrently from the bank’s cloud server
  - Both ATMs add $10,000 to this amount (locally at the ATM)
  - Both write the final amount to the server
  - You lost $10,000!
- The ATMs need *mutually exclusive* access to your account entry at the server
  - or, mutually exclusive access to executing the code that modifies the account entry
More Uses of Mutual Exclusion

- **Distributed File systems**
  - Locking of files and directories
- **Accessing objects** in a safe and consistent way
  - Ensure at most one server has access to object at any point of time
- **Server coordination**
  - Work partitioned across servers
  - Servers coordinate using locks
- **In industry**
  - Chubby is Google’s locking service
  - Many cloud stacks use Apache Zookeeper for coordination among servers
Problem Statement for Mutual Exclusion

- **Critical Section** Problem: Piece of code (at all processes) for which we need to ensure there is at most one process executing it at any point of time.
- Each process can call three functions
  - `enter()` to enter the critical section (CS)
  - `AccessResource()` to run the critical section code
  - `exit()` to exit the critical section
Our Bank Example

ATM1:
enter(S);
// AccessResource()
obtain bank amount;
add in deposit;
update bank amount;
// AccessResource() end
exit(S); // exit

ATM2:
enter(S);
// AccessResource()
obtain bank amount;
add in deposit;
update bank amount;
// AccessResource() end
exit(S); // exit
Approaches to Solve Mutual Exclusion

• **Single OS:**
  • If all processes are running in one OS on a machine (or VM), then
  • Semaphores, mutexes, condition variables, monitors, etc.
Approaches to Solve Mutual Exclusion (2)

• Distributed system:
  • Processes communicating by passing messages

Need to guarantee 3 properties:
• **Safety** (essential) – At most one process executes in CS (Critical Section) at any time
• **Liveness** (essential) – Every request for a CS is granted eventually
• **Ordering** (desirable) – Requests are granted in the order they were made
Processes Sharing an OS: Semaphores

- Semaphore == an integer that can only be accessed via two special functions
- Semaphore S=1; // Max number of allowed accessors

1. wait(S) (or P(S) or down(S)):

```c
while(1) { // each execution of the while loop is atomic
    if (S > 0) {
        S--; // atomic
        break;
    }
}
```

Each while loop execution and S++ are each atomic operations – supported via hardware instructions such as compare-and-swap, test-and-set, etc.

2. signal(S) (or V(S) or up(s)):

```c
S++; // atomic
```
Our Bank Example Using Semaphores

Semaphore S=1; // shared
ATM1:
    wait(S);
    // AccessResource()
    obtain bank amount;
    add in deposit;
    update bank amount;
    // AccessResource() end
    signal(S); // exit

Semaphore S=1; // shared
ATM2:
    wait(S);
    // AccessResource()
    obtain bank amount;
    add in deposit;
    update bank amount;
    // AccessResource() end
    signal(S); // exit
• In a distributed system, cannot share variables like semaphores
• So how do we support mutual exclusion in a distributed system?
System Model

- Before solving any problem, specify its System Model:
  - Each pair of processes is connected by reliable channels (such as TCP).
  - Messages are eventually delivered to recipient, and in FIFO (First In First Out) order.
  - Processes do not fail.
    - Fault-tolerant variants exist in literature.
Central Solution

- Elect a central master (or leader)
  - Use one of our election algorithms!
- Master keeps
  - A **queue** of waiting requests from processes who wish to access the CS
  - A special **token** which allows its holder to access CS
- Actions of any process in group:
  - **enter()**
    - Send a request to master
    - Wait for token from master
  - **exit()**
    - Send back token to master
Central Solution

- Master Actions:
  - On receiving a request from process $P_i$
    - if (master has token)
      - Send token to $P_i$
    - else
      - Add $P_i$ to queue
  - On receiving a token from process $P_i$
    - if (queue is not empty)
      - Dequeue head of queue (say $P_j$), send that process the token
    - else
      - Retain token
Analysis of Central Algorithm

• Safety – at most one process in CS
  • Exactly one token
• Liveness – every request for CS granted eventually
  • With $N$ processes in system, queue has at most $N$ processes
  • If each process exits CS eventually and no failures, liveness guaranteed
• FIFO Ordering is guaranteed, in order of requests received at master
Analyzing Performance

Efficient mutual exclusion algorithms use fewer messages, and make processes wait for shorter durations to access resources. Three metrics:

- **Bandwidth**: the total number of messages sent in each enter and exit operation.
- **Client delay**: the delay incurred by a process at each enter and exit operation (when no other process is in, or waiting) (We will prefer mostly the enter operation.)
- **Synchronization delay**: the time interval between one process exiting the critical section and the next process entering it (when there is only one process waiting)
Analysis of Central Algorithm

- **Bandwidth**: the total number of messages sent in each *enter* and *exit* operation.
  - 2 messages for enter
  - 1 message for exit

- **Client delay**: the delay incurred by a process at each enter and exit operation (when no other process is in, or waiting)
  - 2 message latencies (request + grant)

- **Synchronization delay**: the time interval between one process exiting the critical section and the next process entering it (when there is *only one* process waiting)
  - 2 message latencies (release + grant)
But...

- The master is the performance bottleneck and SPoF (single point of failure)
Ring-based Mutual Exclusion

Currently holds token, can access CS
Ring-based Mutual Exclusion

Cannot access CS anymore

Here’s the token!

Token: ●
Ring-based Mutual Exclusion

Currently holds token, can access CS

Token: ●
Ring-based Mutual Exclusion

- $N$ Processes organized in a virtual ring
- Each process can send message to its successor in ring
- Exactly 1 token
- `enter()`
  - Wait until you get token
- `exit() // already have token`
  - Pass on token to ring successor
- If receive token, and not currently in `enter()`, just pass on token to ring successor
Analysis of Ring-based Mutual Exclusion

- Safety
  - Exactly one token
- Liveness
  - Token eventually loops around ring and reaches requesting process (no failures)
- Bandwidth
  - Per enter(), 1 message by requesting process but up to $N$ messages throughout system
  - 1 message sent per exit()
Analysis of Ring-Based Mutual Exclusion (2)

- Client delay: 0 to $N$ message transmissions after entering enter()
  - Best case: already have token
  - Worst case: just sent token to neighbor
- Synchronization delay between one process’ exit() from the CS and the next process’ enter() :
  - Between 1 and $(N-1)$ message transmissions.
  - Best case: process in enter() is successor of process in exit()
  - Worst case: process in enter() is predecessor of process in exit()
• Client/Synchronization delay to access CS still $O(N)$ in Ring-Based approach.
• Can we make this faster?
Before solving any problem, specify its System Model:

- Each pair of processes is connected by reliable channels (such as TCP).
- Messages are eventually delivered to recipient, and in FIFO (First In First Out) order.
- Processes do not fail.
Ricart-Agrawala’s Algorithm

- Classical algorithm from 1981
- Invented by Glenn Ricart (NIH) and Ashok Agrawala (U. Maryland)

- No token
- Uses the notion of causality and multicast
- Has lower waiting time to enter CS than Ring-Based approach
Key Idea: Ricart-Agrawala Algorithm

- enter() at process Pi
  - multicast a request to all processes
    - Request: <T, Pi>, where T = current Lamport timestamp at Pi
    - Wait until all other processes have responded positively to request
  - Requests are granted in order of causality
  - <T, Pi> is used lexicographically: Pi in request <T, Pi> is used to break ties (since Lamport timestamps are not unique for concurrent events)
Messages in RA Algorithm

- **enter() at process $P_i$**
  - set state to **Wanted**
  - multicast “Request” $<T_i, P_i>$ to all processes, where $T_i =$ current Lamport timestamp at $P_i$
  - wait until *all* processes send back “Reply”
  - change state to **Held** and enter the CS

- **On receipt of a Request $<T_j, P_j>$ at $P_i$ ($i \neq j$):**
  - if (state = **Held**) or (state = **Wanted** & ($T_i, i < (T_j, j)$))
    // lexicographic ordering in $(T_j, P_j)$
    add request to local queue (of waiting requests)
  - else send “Reply” to $P_j$

- **exit() at process $P_i$**
  - change state to **Released** and “Reply” to *all* queued requests.
Example: Ricart-Agrawala Algorithm

Request message
\(<T, Pi> = \langle102, 32\rangle\)
Example: Ricart-Agrawala Algorithm

N32 state: Held.
Can now access CS
Example: Ricart-Agrawala Algorithm

N12 state: Wanted

N12

Request message <115, 12>

N6

N3

N32

N32 state: Held.
Can now access CS

N80

Request message <110, 80>

N5

N80 state: Wanted
Example: Ricart-Agrawala Algorithm

- **N12 state:** Wanted
- **Request message** `<115, 12>`
- **Reply messages**
- **N3 state:**
- **N6 state:**
- **N80 state:** Wanted
- **N5 state:**
- **N32 state:** Held.
  - Can now access CS
- **Request message** `<110, 80>`
Example: Ricart-Agrawala Algorithm

N12 state: Wanted

N3 state:

Request message

N6

N32 state: Held.
Can now access CS
Queue requests:

Request message

N80

N5

N80 state:
Wanted

Reply messages

Request message

<115, 12>

<110, 80>

<115, 12>, <110, 80>
Example: Ricart-Agrawala Algorithm

N12 state: Wanted
Request message <115, 12>

N3 state: Wanted
Reply messages

N6

N80 state: Wanted
Request message <110, 80>
Queue requests: <115, 12>, <110, 80>

N32 state: Held.
Can now access CS
Queue requests: <115, 12>, <110, 80>

N5

Queue requests: <115, 12> (since > (110, 80))
Example: Ricart-Agrawala Algorithm

N12 state: Wanted

N6

N3

N32 state: Held.
Can now access CS
Queue requests: <115, 12>, <110, 80>

N12

N3

Request message <115, 12>

Reply messages

N80

N5

N80 state: Wanted
Queue requests: <115, 12>

Request message <110, 80>
Example: Ricart-Agrawala Algorithm

- **N12 state:** Wanted (waiting for N80’s reply)
- **N80 state:** Held. Can now access CS. Queue requests: <115, 12>
- **N32 state:** Released. Multicast Reply to <115, 12>, <110, 80>
- **N5**
- **N6**
- **N3**

Request messages:
- N12 request: <115, 12>
- N80 request: <110, 80>

Reply messages:
- N32 multicasts reply to N12 and N3.
Analysis: Ricart-Agrawala’s Algorithm

- Safety
  - Two processes $P_i$ and $P_j$ cannot both have access to CS
    - If they did, then both would have sent Reply to each other
    - Thus, $(T_i, i) < (T_j, j)$ and $(T_j, j) < (T_i, i)$, which are together not possible
    - What if $(T_i, i) < (T_j, j)$ and $P_i$ replied to $P_j$’s request before it created its own request?
      - Then it seems like both $P_i$ and $P_j$ would approve each others’ requests
      - But then, causality and Lamport timestamps at $P_i$ implies that $T_i > T_j$, which is a contradiction
      - So this situation cannot arise
Analysis: Ricart-Agrawala’s Algorithm (2)

• Liveness
  • Worst-case: wait for all other \((N-1)\) processes to send Reply

• Ordering
  • Requests with lower Lamport timestamps are granted earlier
Performance: Ricart-Agrawala’s Algorithm

- Bandwidth: $2(N-1)$ messages per enter() operation
  - $N-1$ unicasts for the multicast request + $N-1$ replies
  - $N$ messages if the underlying network supports multicast (1 multicast + $N-1$ unicast replies)
  - $N-1$ unicast messages per exit operation
    - 1 multicast if the underlying network supports multicast
- Client delay: one round-trip time
- Synchronization delay: one message transmission time
Ok, but ...

- Compared to Ring-Based approach, in Ricart-Agrawala approach
  - Client/synchronization delay has now gone down to $O(1)$
  - But bandwidth has gone up to $O(N)$
- Can we get *both* down?
Maekawa’s Algorithm: Key Idea

- Ricart-Agrawala requires replies from *all* processes in group.
- Instead, get replies from only *some* processes in group.
- But ensure that only process one is given access to CS (Critical Section) at a time.
Maekawa’s Voting Sets

- Each process $P_i$ is associated with a voting set $V_i$ (of processes)
- Each process belongs to its own voting set
- The intersection of any two voting sets must be non-empty
  - Same concept as Quorums!
- Each voting set is of size $K$
- Each process belongs to $M$ other voting sets
- Maekawa showed that $K=M=\sqrt{N}$ works best
- One way of doing this is to put $N$ processes in a $\sqrt{N}$ by $\sqrt{N}$ matrix and for each $P_i$, its voting set $V_i =$ row containing $P_i$ + column containing $P_i$. Size of voting set $= 2*\sqrt{N-1}$
Example: Voting Sets with N=4

P1’s voting set = V1

V1

V2

V3

V4

p1  p2

p3  p4
Maekawa: Key Differences From Ricart-Agrawala

- Each process requests permission from only its voting set members
  - Not from all
- Each process (in a voting set) gives permission to at most one process at a time
  - Not to all
Actions

- state = *Released*, voted = false
- enter() at process $P_i$:
  - state = *Wanted*
  - Multicast *Request* message to all processes in $V_i$
  - Wait for *Reply (vote)* messages from all processes in $V_i$ (including vote from self)
  - state = *Held*
- exit() at process $P_i$:
  - state = *Released*
  - Multicast *Release* to all processes in $V_i$
Actions (2)

- When $P_i$ receives a Request from $P_j$:
  
  ```
  if (state == Held OR voted = true)
      queue Request
  else
      send Reply to $P_j$ and set voted = true
  ```

- When $P_i$ receives a Release from $P_j$:
  
  ```
  if (queue empty)
      voted = false
  else
      dequeue head of queue, say $P_k$
      Send Reply only to $P_k$
      voted = true
  ```
Safety

• When a process $P_i$ receives replies from all its voting set $V_i$ members, no other process $P_j$ could have received replies from all its voting set members $V_j$
  • $V_i$ and $V_j$ intersect in at least one process say $P_k$
  • But $P_k$ sends only one Reply (vote) at a time, so it could not have voted for both $P_i$ and $P_j$
Liveness

- A process needs to wait for at most \((N-1)\) other processes to finish CS
- But does not guarantee liveness
- Since can have a deadlock
- Example: all 4 processes need access
  - P1 is waiting for P3
  - P3 is waiting for P4
  - P4 is waiting for P2
  - P2 is waiting for P1
  - No progress in the system!
- There are deadlock-free versions
Performance

- Bandwidth
  - $2\sqrt{N}$ messages per enter()
  - $\sqrt{N}$ messages per exit()
  - Better than Ricart and Agrawala’s ($2*(N-I)$ and $N-I$ messages)
  - $\sqrt{N}$ quite small. $N \sim 1$ million $\Rightarrow \sqrt{N} = 1K$
- Client delay: One round trip time
- Synchronization delay: 2 message transmission times
Why $\sqrt{N}$?

- Each voting set is of size $K$
- Each process belongs to $M$ other voting sets
- Total number of voting set members (processes may be repeated) $= K*N$
- But since each process is in $M$ voting sets
  - $K*N/M = N \Rightarrow K = M \ (1)$
- Consider a process $P_i$
  - Total number of voting sets = members present in $P_i$’s voting set and all their voting sets $= (M-1)*K + 1$
  - All processes in group must be in above
  - To minimize the overhead at each process ($K$), need each of the above members to be unique, i.e.,
    - $N = (M-1)*K + 1$
    - $N = (K-1)*K + 1$ (due to $1$)
    - $K \sim \sqrt{N}$
Failures?

• There are fault-tolerant versions of the algorithms we’ve discussed
  • E.g., Maekawa

• One other way to handle failures: Use Paxos-like approaches!
Chubby

- Google’s system for locking
- Used underneath Google’s systems like BigTable, Megastore, etc.
- Not open-sourced but published
- Chubby provides *Advisory* locks only
  - Doesn’t guarantee mutual exclusion unless every client checks lock before accessing resource

*Reference: http://research.google.com/archive/chubby.html*
Chubby (2)

- Can use not only for locking but also writing small configuration files
- Relies on Paxos-like (consensus) protocol
- Group of servers with one elected as Master
  - All servers replicate same information
- Clients send read requests to Master, which serves it locally
- Clients send write requests to Master, which sends it to all servers, gets majority (quorum) among servers, and then responds to client
- On master failure, run election protocol
- On replica failure, just replace it and have it catch up
Summary

- Mutual exclusion important problem in cloud computing systems
- Classical algorithms
  - Central
  - Ring-based
  - Ricart-Agrawala
  - Maekawa
- Industry systems
  - Chubby: a coordination service
  - Similarly, Apache Zookeeper for coordination
Announcements

- Reminder: HW3 due 10/28
- MP3 due 11/6 (demos 11/7)
- You should have started on both by now.