Homework 3

CS425/ECE428 Fall 2021

Due: Friday, October 29 at 11:59 p.m.

This assignment has 6 pages and 5 questions, worth a total of 60 points. Solutions must be submitted via Gradescope. Solutions must be typed, not hand-written, but you may include hand-drawn diagrams.

You must acknowledge any sources you used to arrive at your solutions, other than the course materials and textbook. If you work in a group on homework assignments, please list the names of your collaborators, but make sure to write your own solution.

1. Consider the following modification of the Bully algorithm: The initiating node (which we assume does not fail) sends an Election message only to the process with the highest id. If it does not get a response after a timeout, it then sends an Election message to the process with the second highest id. If after another timeout it gets no response, it tries the third highest id, and so on. If no higher numbered processes respond, it sends a Coordinator message to all lower-numbered processes.

```
class ModifiedBully:
    def run_election(self, failed_leader = None):
        """
        'failed_leader': previous leader that has failed, to avoid
           sending a message to it
        """
        self.leader = None
        # process pid's in reverse order. self.group
        # is a list of all processes in the group
        for pid in sorted(self.group, reverse=True):
           if pid == failed_leader:
             continue # skip previous leader
           elif pid == self.pid:
             # I am the new leader
             self.leader = self.pid
             for pid2 in self.group:
                if pid2 < self.pid:
                  unicast(pid2, "Coordinator")
             break
           unicast(pid, "Election")
           sleep(TIMEOUT)
           if self.leader is not None:
             break

    def receive_message(self, message):
       if message.contents == "Coordinator":
         self.leader = message.sender
       elif message.contents == "Election":
       # fill in the rest
```

   (a) (2 points) Complete the `receive_message` function (pseudocode OK).

   For the following parts, consider a distributed system of 8 processes that uses the modified Bully algorithm for leader election (including your solution to part (a)). The processes are called $\{P_1, \ldots, P_8\}$ with P_i having PID i. Initially all 8 processes are alive and P_8 is the leader. Then P_8 fails, P_4 detects this, and initiates the election. Assume one-way message transmission time is T, and timeout is set using the knowledge of T.

   (b) (2 points) If no other node fails during the election run, how many *total* messages will be sent by *all* processes in this election run?

   (c) (2 points) If no other node fails during the election run, how long will it take for the election to finish?

(d) (2 points) Now assume that immediately after P_4 detects P_8's failure and initiates the election, P_7 fails. How many *total* messages will be sent by *all* processes in this election run?

(e) (2 points) For the above scenario (where P_7 fails right after P_4 initiates election upon detecting P_8's failure), how long will it take for the election to finish?

(f) (4 points) What are the best- and worst-case turnaround times for this election algorithm?

2. Consider a system of N processes, $\{P_1, \ldots, P_N\}$ arranged in a ring. Each process can only communicate to its ring successor; i.e., P_i can only send a message to P_{i+1}, and P_N can only send a message to P_1. Furthermore, each message is a pair of two signed 16-bit integers, i.e., they can each take values from $-32\,768$ to $32\,767$. Assume that $N \leq 1000$. Assume that that there are no failures, and the communication channel delivers all messages correctly and exactly once.

(a) (6 points) In this problem each process has a 16-bit signed integer input x_k ($-32\,768 \leq x_k \leq 32\,767$), and an output variable y_k, initialized to None (meaning *undecided*). A consensus algorithm is designed to calculate the maximum of all the input variables. In other words, the safety condition is that, at any point y_k is either None or $y_k = \max_{i\in\{1,\ldots,N\}} x_i$.

We will adapt the (first version of the) ring-based election algorithm for this problem.

- A process P_i initiates the algorithm by sending $(0, x_i)$ to its ring successor. (The 0 indicates this is a *proposal*) message.
- When a process P_j receives $(0, x)$ from its ring predecessor:
  - if $x > x_j$, it forwards $(0, x)$ to its successor.
  - if $x < x_j$, it sends replaces x with x_j and sends $(0, x_j)$ to its successor.
  - if $x = x_j$, it concludes that $x = x_j$ is the minimum value, sets $y_j = x$ and sends $(1, x)$ to its successor, (1 indicating it is a *decided* message.)
- When a process P_j receives $(1, x)$ and its y_j is None, it sets $y_j = x$ and forwards $(1, x)$ to its successor. If y_j is not None it ignores any received messages.

Note that multiple processes may initiate the algorithm simultaneously. Here is a Python implementation:

```
PROPOSAL = 0
DECIDED = 1
X_K = # my input
Y_K = None

def start_consensus():
    # send forwards two integers to the successor in the ring
    send(PROPOSAL, X_K)

# receive two integers from the neighbor
def receive_message(a,b):
    if Y_K is not None:
        continue # ignore messages after decided state
    if a == PROPOSAL:
        if b > X_K: # forward
            send(PROPOSAL, b)
        elif b < X_K: # replace b with X_K
            send(PROPOSAL, X_K)
        else: # b == X_K
            Y_K = b
            send(DECIDED, Y_K)
    elif a == DECIDED:
        # set variable, forward decision
        Y_K = b
        send(DECIDED, b)
```

Does the algorithm described above guarantee safety condition for the problem? If yes, prove how. If not, (i) describe a scenario where safety is violated, and (ii) suggest modifications to the algorithm that would guarantee the safety condition. You do not need to submit code but you should have enough detail in your modified algorithm for the grader to be able to implement the modification.

(b) (4 points) In this problem, the inputs x_k are all either 0 or 1. In this case, the output variable should be set to represent the majority value. In other words, if y_k is not None, it must be 1 if a majority of processes have input 1, 0 if a majority of processes have input 0, and 2 if there is a tie. Describe an algorithm for solving this problem (using either Python or pseudocode). Again assume that multiple processes may initiate your algorithm simultaneously.

3. Consider the following algorithm:

- A process, say P_i initiates consensus by multicasting a Query message to the group
- Each process P_j *unicasts* a reply back to P_i with a message Value(x_j) that includes its input
- After receiving replies from everyone or a timeout, it computes the minimum of all received values (including its own) $y_i = \min x_j$ and multicasts Decision(y_i) to the group.

Assume that we operate in a synchronous system with a maximum one-way delay of T for a *unicast* message transmission, including any processing time. Assume that unicast channels are reliable.

(a) (3 points) For the multicast of the Query message, should R-multicast or B-multicast be used, and why?

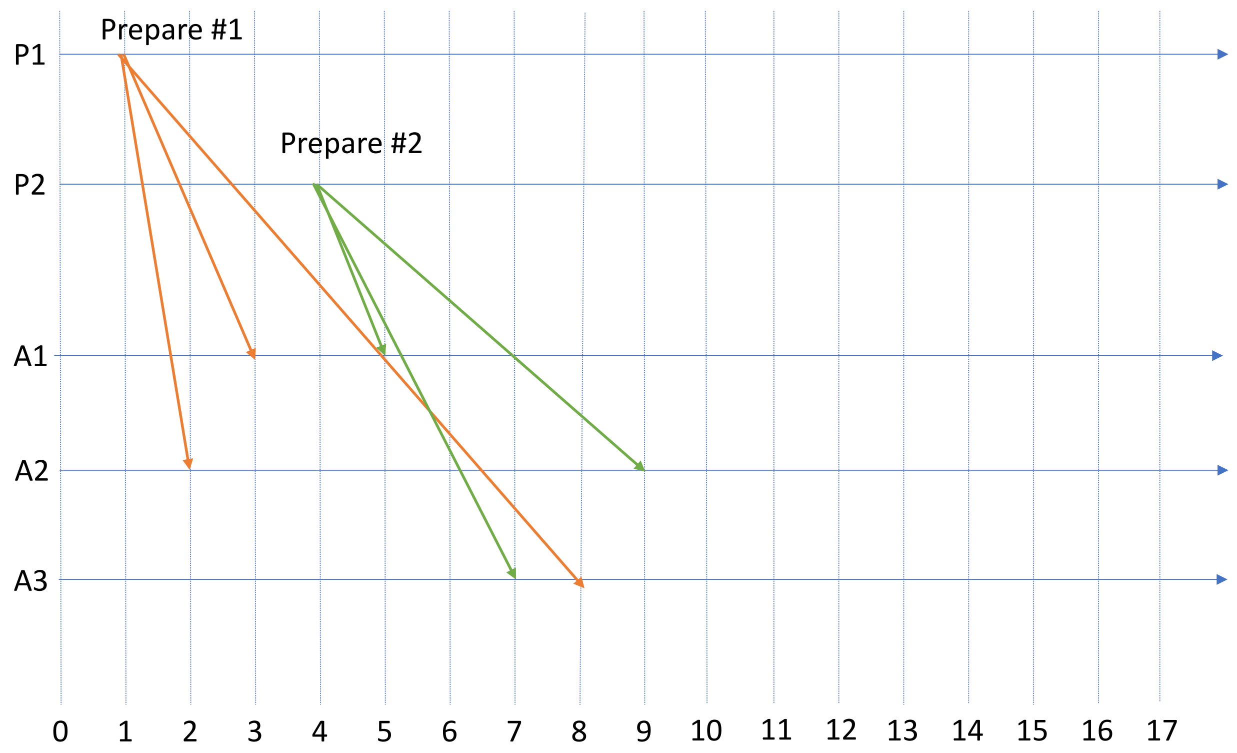
(b) (2 points) What should the timeout value be?

(c) (3 points) For the multicast of the Decision message, should R-multicast or B-multicast be used, and why?

(d) (2 points) Assuming no failures, how long will the algorithm take to complete?

(e) (3 points) This algorithm can lead to a safety failure while trying to achieve consensus. Explain how it could happen. (For this part, you may need to consider the possibility of process failures.)

4. Consider a system implementing Paxos with two proposers, $P1$ and $P2$, and three acceptors, $A1, A2, A3$. Assume that the input value for P1 is 1 and the input value for P2 is 2. Answer the following sub-questions, each of which is unrelated to the other two sub-questions.



(a)

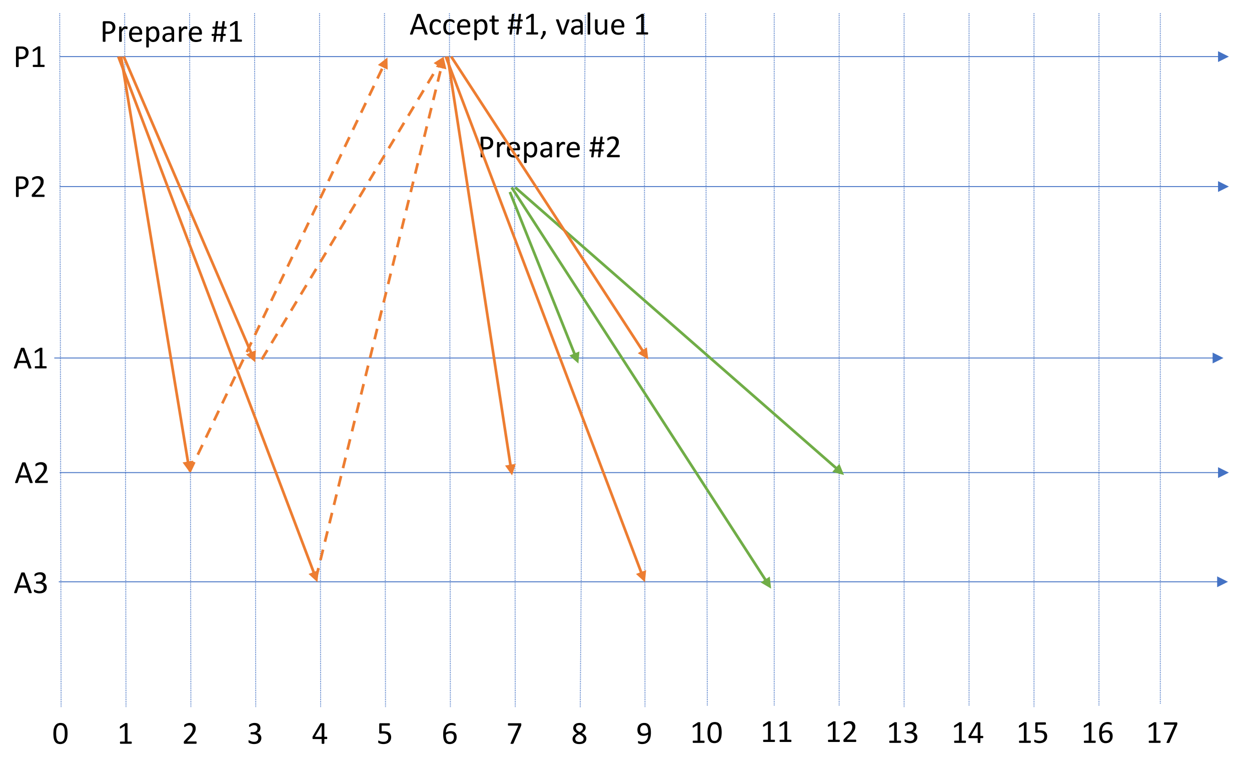
Refer to the figure above. It shows the Prepare messages sent by P1 and P2. The responses (if any) are not shown. Assume no other proposals are initiated.

i. (1 point) Which processes will reply back to P1's Prepare messages?

ii. (1 point) Which processes will reply back to P2's Prepare messages?

iii. (2 points) Assuming each Promise message take *exactly* 2 time units, at what time will P1 send Accept messages? At what time will P2 send Accept messages?

iv. (2 points) Assuming each Accept message takes *at least* 1 time units and, as above, the Promise messages take *exactly* 2 time units, is it possible for for proposal #1 to be accepted by a majority of acceptors? If no, explain why not. If yes, explain how.



(b)

Refer to the figure above. P1 send Prepare messages at time 1, receives a Promise from all acceptors, and sends Accept messages at time 6. Meanwhile, P2 sends a prepare for proposal #2 at time 7.

i. (1 point) Which processes will reply to P2's Prepare message?

ii. (1 point) Which processes will accept P1's proposal?

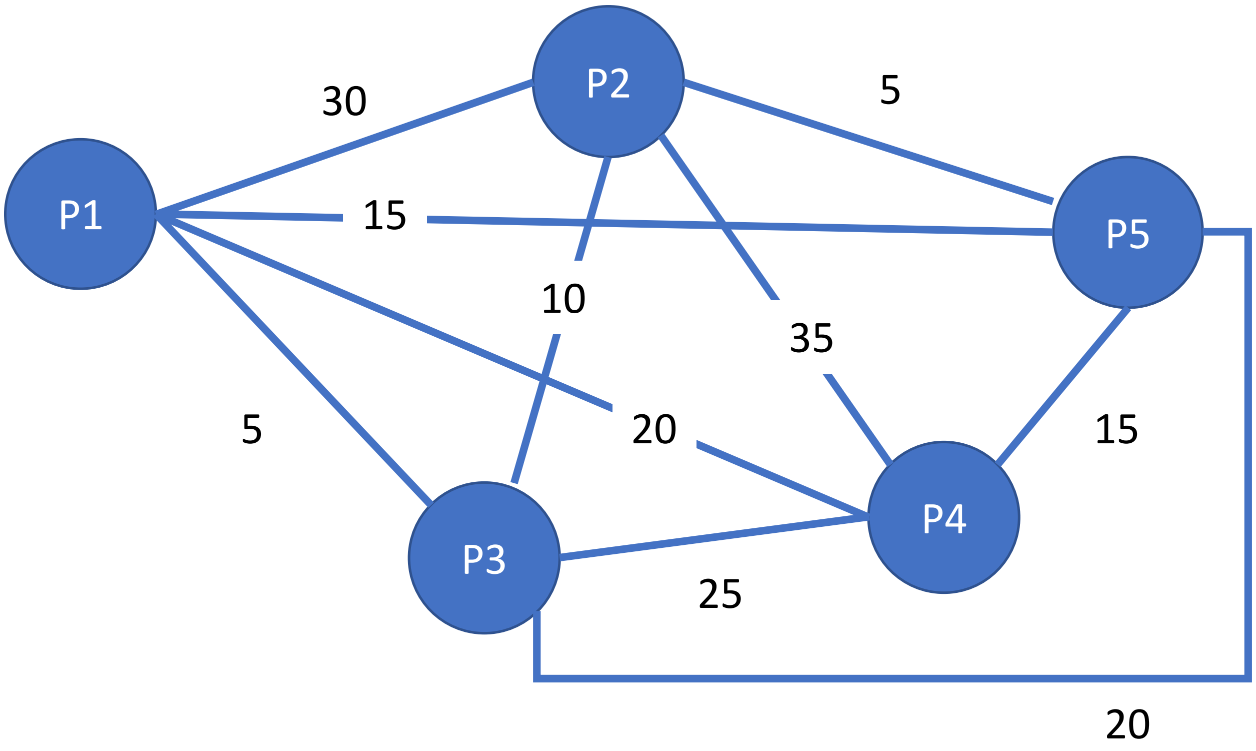
iii. (1 point) Assuming each Promise response for P2's Prepare message takes exactly 2 units, when will P2 send Accept messages?

iv. (1 point) Assuming P2 sends messages at the time stated in the previous parts, which processes will accept P2's proposal?

v. (1 point) What will be the value in P2's proposal?

vi. (3 points) Come up with a scenario where the consensus value changes by switching the arrival time of *one* message. Complete the diagram showing the timing of the promise messages from the acceptors to P2 and the accept messages from P2 back to the acceptors. (For this subpart any messages not shown on the original diagram must take *at least* one time unit, but have no other constraints.)

5. Consider a system of 5 processes, $\{P1, P2, P3, P4, P5\}$ implementing Raft's algorithm for leader election. The one-way delay between each process is specified in the figure below. Assume no processing time. Each of the parts below are independent scenarios.



(a) (1 point) Which server is the most likely to be elected leader in any term? No justification is necessary.

(b) (4 points) Suppose P1 is the leader in term 1. It sends its last heartbeat at time 0 and then fails. Suppose, upon receiving the heartbeat, P2, P3, P4, and P5 set their timeout values to 50, 75, 100, and 150 ms, respectively. Who will each process vote for as the leader in term 2?

(c) (2 points) Suppose now that P2, P3, P4, P5 set their timeout values to 150, 100, 75, and 50 ms, respectively. Who will each process vote for as leader in term 2?

(d) (2 points) Consider the logs of the processes, with each number n denoting an event logged in term n. Order the servers from least up-to-date to most up-to-date:

$P_1 1, 1, 1, 2, 2, 3, 3, 6$
$P_2 1, 1, 1, 2, 2, 3, 3, 3, 4$
$P_3 1, 1, 1, 2, 2, 3, 3, 3, 4, 4$
$P_4 1, 1, 1, 2, 2, 3, 3, 6, 6$
$P_5 1, 1, 1, 2, 2, 3, 3, 3, 4, 5$