Lecture 12: Mutual Exclusion
Why Mutual Exclusion?

- **Bank’s Servers in the Cloud**: Two of your customers make simultaneous deposits of $10,000 into your bank account, each from a separate ATM.
  - Both ATMs read initial amount of $1000 concurrently from the bank’s cloud server
  - Both ATMs add $10,000 to this amount (locally at the ATM)
  - Both write the final amount to the server
  - What’s wrong?
Why Mutual Exclusion?

- **Bank’s Servers in the Cloud**: Two of your customers make simultaneous deposits of $10,000 into your bank account, each from a separate ATM.
  - Both ATMs read initial amount of $1000 concurrently from the bank’s cloud server
  - Both ATMs add $10,000 to this amount (locally at the ATM)
  - Both write the final amount to the server
  - You lost $10,000!
- The ATMs need **mutually exclusive** access to your account entry at the server
  - or, mutually exclusive access to executing the code that modifies the account entry
More Uses of Mutual Exclusion

- Distributed File systems
  - Locking of files and directories
- Accessing objects in a safe and consistent way
  - Ensure at most one server has access to object at any point of time
- Server coordination
  - Work partitioned across servers
  - Servers coordinate using locks
- In industry
  - Chubby is Google’s locking service
  - Many cloud stacks use Apache Zookeeper for coordination among servers
**Problem Statement for Mutual Exclusion**

- *Critical Section* Problem: Piece of code (at all processes) for which we need to ensure there is at most one process executing it at any point of time.

- Each process can call three functions
  - `enter()` to enter the critical section (CS)
  - `AccessResource()` to run the critical section code
  - `exit()` to exit the critical section
ATM1:
enter(S);
// AccessResource()
obtain bank amount;
add in deposit;
update bank amount;
// AccessResource() end
exit(S); // exit

ATM2:
enter(S);
// AccessResource()
obtain bank amount;
add in deposit;
update bank amount;
// AccessResource() end
exit(S); // exit
Approaches to Solve Mutual Exclusion

- Single OS:
  - If all processes are running in one OS on a machine (or VM), then
  - Semaphores, mutexes, condition variables, monitors, etc.
Approaches to Solve Mutual Exclusion (2)

• Distributed system:
  • Processes communicating by passing messages

Need to guarantee 3 properties:
  • **Safety** (essential) – At most one process executes in CS (Critical Section) at any time
  • **Liveness** (essential) – Every request for a CS is granted eventually
  • **Ordering** (desirable) – Requests are granted in the order they were made
Processes Sharing an OS: Semaphores

- Semaphore == an integer that can only be accessed via two special functions
- Semaphore S=1; // Max number of allowed accessors

1. **wait(S)** (or **P(S)** or **down(S)**):

```java
while(1) { // each execution of the while loop is atomic
    if (S > 0) {
        S--;
        break;
    }
}
```

Each while loop execution and S++ are each **atomic** operations – supported via hardware instructions such as compare-and-swap, test-and-set, etc.

2. **signal(S)** (or **V(S)** or **up(s)**):

```
S++; // atomic
```
Our Bank Example Using Semaphores

Semaphore S=1; // shared

ATM1:

wait(S);

// AccessResource()
obtain bank amount;
add in deposit;
update bank amount;
// AccessResource() end
signal(S); // exit

ATM2:

wait(S);

// AccessResource()
obtain bank amount;
add in deposit;
update bank amount;
// AccessResource() end
signal(S); // exit
• In a distributed system, cannot share variables like semaphores
• So how do we support mutual exclusion in a distributed system?
Before solving any problem, specify its System Model:

- Each pair of processes is connected by reliable channels (such as TCP).
- Messages are eventually delivered to recipient, and in FIFO (First In First Out) order.
- Processes do not fail.
  - Fault-tolerant variants exist in literature.
Central Solution

- Elect a central master (or leader)
  - Use one of our election algorithms!
- Master keeps
  - A queue of waiting requests from processes who wish to access the CS
  - A special token which allows its holder to access CS
- Actions of any process in group:
  - enter()
    - Send a request to master
    - Wait for token from master
  - exit()
    - Send back token to master
Central Solution

- Master Actions:
  - On receiving a request from process $P_i$
    - if (master has token)
      - Send token to $P_i$
    - else
      - Add $P_i$ to queue
  - On receiving a token from process $P_i$
    - if (queue is not empty)
      - Dequeue head of queue (say $P_j$), send that process the token
    - else
      - Retain token
**Analysis of Central Algorithm**

- Safety – at most one process in CS
  - Exactly one token
- Liveness – every request for CS granted eventually
  - With $N$ processes in system, queue has at most $N$ processes
  - If each process exits CS eventually and no failures, liveness guaranteed
- FIFO Ordering is guaranteed, in order of requests received at master
Efficient mutual exclusion algorithms use fewer messages, and make processes wait for shorter durations to access resources. Three metrics:

- **Bandwidth**: the total number of messages sent in each `enter` and `exit` operation.
- **Client delay**: the delay incurred by a process at each enter and exit operation (when no other process is in, or waiting)
  
  (We will prefer mostly the enter operation.)
- **Synchronization delay**: the time interval between one process exiting the critical section and the next process entering it (when there is only one process waiting)
Analysis of Central Algorithm

• **Bandwidth**: the total number of messages sent in each *enter* and *exit* operation.
  • 2 messages for enter
  • 1 message for exit

• **Client delay**: the delay incurred by a process at each enter and exit operation (when *no* other process is in, or waiting)
  • 2 message latencies (request + grant)

• **Synchronization delay**: the time interval between one process exiting the critical section and the next process entering it (when there is *only one* process waiting)
  • 2 message latencies (release + grant)
The master is the performance bottleneck and SPoF (single point of failure)
Ring-based Mutual Exclusion

Currently holds token, can access CS

Token: ●
Ring-based Mutual Exclusion

Cannot access CS anymore

Here’s the token!
RING-BASED MUTUAL EXCLUSION

Currently holds token, can access CS

Token: ●
Ring-based Mutual Exclusion

- $N$ Processes organized in a virtual ring
- Each process can send message to its successor in ring
- Exactly 1 token
- `enter()`
  - Wait until you get token
- `exit()` // already have token
  - Pass on token to ring successor
- If receive token, and not currently in `enter()`, just pass on token to ring successor
**Analysis of Ring-based Mutual Exclusion**

- **Safety**
  - Exactly one token
- **Liveness**
  - Token eventually loops around ring and reaches requesting process (no failures)
- **Bandwidth**
  - Per enter(), 1 message by requesting process but up to $N$ messages throughout system
  - 1 message sent per exit()
Analysis of Ring-Based Mutual Exclusion (2)

- Client delay: 0 to $N$ message transmissions after entering enter()
  - Best case: already have token
  - Worst case: just sent token to neighbor
- Synchronization delay between one process’ exit() from the CS and the next process’ enter():
  - Between 1 and $(N-1)$ message transmissions.
  - **Best case**: process in enter() is successor of process in exit()
  - **Worst case**: process in enter() is predecessor of process in exit()
• Client/Synchronization delay to access CS still $O(N)$ in Ring-Based approach.
• Can we make this faster?
Before solving any problem, specify its System Model:

- Each pair of processes is connected by reliable channels (such as TCP).
- Messages are eventually delivered to recipient, and in FIFO (First In First Out) order.
- Processes do not fail.
Ricart-Agrawala's Algorithm

- Classical algorithm from 1981
- Invented by Glenn Ricart (NIH) and Ashok Agrawala (U. Maryland)

- No token
- Uses the notion of causality and multicast
- Has lower waiting time to enter CS than Ring-Based approach
Key Idea: Ricart-Agrawala Algorithm

- enter() at process Pi
  - multicast a request to all processes
    - Request: \(<T, P_i>\), where T = current Lamport timestamp at \(P_i\)
    - Wait until all other processes have responded positively to request
  - Requests are granted in order of causality
- \(<T, P_i>\) is used lexicographically: \(P_i\) in request \(<T, P_i>\) is used to break ties (since Lamport timestamps are not unique for concurrent events)
**Messages in RA Algorithm**

- **enter() at process Pi**
  - set state to **Wanted**
  - multicast "Request" \(<Ti, Pi>\) to all processes, where \(Ti = \) current Lamport timestamp at \(Pi\)
  - wait until **all** processes send back “Reply”
  - change state to **Held** and enter the CS
- **On receipt of a Request \(<Tj, Pj>\) at Pi \((i \neq j)\):**
  - if (state = **Held**) or (state = **Wanted** & \((Ti, i) < (Tj, j)\))
    // lexicographic ordering in \((Tj, Pj)\)
    add request to local queue (of waiting requests)
  - else send “Reply” to \(Pj\)
- **exit() at process Pi**
  - change state to **Released** and “Reply” to **all** queued requests.
Example: Ricart-Agrawala Algorithm

Request message
\( <T, Pi> = <102, 32> \)
Example: Ricart-Agrawala Algorithm

N32 state: Held.
Can now access CS
Example: Ricart-Agrawala Algorithm

N12 state:
Wanted

N12

Request message
<115, 12>

N6

N3

N32

N80

Request message
<110, 80>

N5

N32 state: Held.
Can now access CS

N80 state:
Wanted
**Example: Ricart-Agrawala Algorithm**

N12 state: **Wanted**

N32 state: **Held**. Can now access CS

N80 state: **Wanted**

Request message \(<115, 12>\)

Reply messages

Request message \(<110, 80>\)
Example: Ricart-Agrawala Algorithm

N12 state: Wanted

Request message: <115, 12>

N3 state:

Reply messages

N32 state: Held.
Can now access CS
Queue requests:
<115, 12>, <110, 80>

N6

N80 state: Wanted

Request message: <110, 80>

N32

N5
**Example:** Ricart-Agrawala Algorithm

N80 state: **Wanted**
- Queue requests: <115, 12> (since > (110, 80))

N12 state: **Wanted**
- Request message <115, 12>
- Queue requests: <115, 12>, <110, 80>

N3 state:
- Reply messages
- Request message <110, 80>

N32 state: **Held**.
- Can now access CS
- Queue requests: <115, 12>, <110, 80>

N5 state:
- Request message <110, 80>

N6 state:
- Request message <115, 12>
**Example: Ricart-Agrawala Algorithm**

N12 state: Wanted

Request message: <115, 12>

N32 state: Held.
Can now access CS
Queue requests: <115, 12>, <110, 80>

N80 state: Wanted
Queue requests: <115, 12>
**Example: Ricart-Agrawala Algorithm**

N12 state: **Wanted** (waiting for N80’s reply)

N32 state: **Released**. Multicast Reply to <115, 12>, <110, 80>

N80 state: **Held**. Can now access CS.
Queue requests: <115, 12>

- Request message <110, 80>
- Reply messages
  - N12
  - N3

- Request message <115, 12>
- Reply messages
  - N32
  - N5
  - N6
Analysis: Ricart-Agrawala's Algorithm

• Safety
  • Two processes $P_i$ and $P_j$ cannot both have access to CS
    • If they did, then both would have sent Reply to each other
    • Thus, $(T_i, i) < (T_j, j)$ and $(T_j, j) < (T_i, i)$, which are together not possible
    • What if $(T_i, i) < (T_j, j)$ and $P_i$ replied to $P_j$’s request before it created its own request?
      • Then it seems like both $P_i$ and $P_j$ would approve each others’ requests
      • But then, causality and Lamport timestamps at $P_i$ implies that $T_i > T_j$, which is a contradiction
      • So this situation cannot arise
Analysis: Ricart-Agrawala's Algorithm (2)

- Liveness
  - Worst-case: wait for all other \((N-1)\) processes to send Reply

- Ordering
  - Requests with lower Lamport timestamps are granted earlier
Performance: Ricart-Agrawala's Algorithm

- Bandwidth: $2(N-1)$ messages per enter() operation
  - $N-1$ unicasts for the multicast request + $N-1$ replies
  - $N$ messages if the underlying network supports multicast (1 multicast + $N-1$ unicast replies)
  - $N-1$ unicast messages per exit operation
    - 1 multicast if the underlying network supports multicast
- Client delay: one round-trip time
- Synchronization delay: one message transmission time
Ok, but ...

- Compared to Ring-Based approach, in Ricart-Agrawala approach
  - Client/synchronization delay has now gone down to $O(1)$
  - But bandwidth has gone up to $O(N)$
- Can we get both down?
Maekawa's Algorithm: Key Idea

- Ricart-Agrawala requires replies from *all* processes in group
- Instead, get replies from only *some* processes in group
- But ensure that only process one is given access to CS (Critical Section) at a time
**Maekawa's Voting Sets**

- Each process $P_i$ is associated with a *voting set* $V_i$ (of processes)
- Each process belongs to its own voting set
- *The intersection of any two voting sets must be non-empty*
  - *Same concept as Quorums!*
- Each voting set is of size $K$
- Each process belongs to $M$ other voting sets
- Maekawa showed that $K=M=\sqrt{N}$ works best
- One way of doing this is to put $N$ processes in a $\sqrt{N}$ by $\sqrt{N}$ matrix and for each $P_i$, its voting set $V_i = \text{row containing } P_i + \text{column containing } P_i$. Size of voting set = $2*\sqrt{N}-1$
Example: Voting Sets with N=4

P1’s voting set = V1

V2

p1 p2

V3

p3 p4

V4

p1 | p2
p3 | p4
Maekawa: Key Differences From Ricart-Agrawala

- Each process requests permission from only its voting set members
  - Not from all
- Each process (in a voting set) gives permission to at most one process at a time
  - Not to all
**Actions**

- state = Released, voted = false

- enter() at process Pi:
  - state = Wanted
  - Multicast Request message to all processes in Vi
  - Wait for Reply (vote) messages from all processes in Vi (including vote from self)
  - state = Held

- exit() at process Pi:
  - state = Released
  - Multicast Release to all processes in Vi
**Actions (2)**

- When $P_i$ receives a Request from $P_j$:
  
  ```
  if (state == Held OR voted = true)
      queue Request
  else
      send Reply to $P_j$ and set voted = true
  ```

- When $P_i$ receives a Release from $P_j$:
  
  ```
  if (queue empty)
      voted = false
  else
      dequeue head of queue, say $P_k$
      Send Reply only to $P_k$
      voted = true
  ```
SAFETY

• When a process $P_i$ receives replies from all its voting set $V_i$ members, no other process $P_j$ could have received replies from all its voting set members $V_j$
  • $V_i$ and $V_j$ intersect in at least one process say $P_k$
  • But $P_k$ sends only one Reply (vote) at a time, so it could not have voted for both $P_i$ and $P_j$
**Liveness**

- A process needs to wait for at most $(N-1)$ other processes to finish CS
- But does not guarantee liveness
- Since can have a *deadlock*
- Example: all 4 processes need access
  - P1 is waiting for P3
  - P3 is waiting for P4
  - P4 is waiting for P2
  - P2 is waiting for P1
  - No progress in the system!
- There are deadlock-free versions
Performance

• Bandwidth
  • $2\sqrt{N}$ messages per enter()
  • $\sqrt{N}$ messages per exit()
  • Better than Ricart and Agrawala’s
    $(2*(N-1)$ and $N-1$ messages)
  • $\sqrt{N}$ quite small. $N \sim 1$ million => $\sqrt{N} = 1K$
• Client delay: One round trip time
• Synchronization delay: 2 message transmission times
**Why \( \sqrt{N} \)?**

- Each voting set is of size \( K \)
- Each process belongs to \( M \) other voting sets
- Total number of voting set members (processes may be repeated) = \( KN \)
- But since each process is in \( M \) voting sets
  - \( KN/M = N \Rightarrow K = M \) (1)
- Consider a process \( P_i \)
  - Total number of voting sets = members present in \( P_i \)'s voting set and all their voting sets = \( (M-1)K + 1 \)
  - All processes in group must be in above
  - To minimize the overhead at each process (\( K \)), need each of the above members to be unique, i.e.,
    - \( N = (M-1)K + 1 \)
    - \( N = (K-1)K + 1 \) (due to (1))
    - \( K \sim \sqrt{N} \)
Failures?

- There are fault-tolerant versions of the algorithms we’ve discussed
  - E.g., Maekawa

- One other way to handle failures: Use Paxos-like approaches!
Chubby

- Google’s system for locking
- Used underneath Google’s systems like BigTable, Megastore, etc.
- Not open-sourced but published
- Chubby provides Advisory locks only
  - Doesn’t guarantee mutual exclusion unless every client checks lock before accessing resource

Reference: http://research.google.com/archive/chubby.html
Chubby (2)

- Can use not only for locking but also writing small configuration files
- Relies on Paxos-like (consensus) protocol
- Group of servers with one elected as Master
  - All servers replicate same information
- Clients send read requests to Master, which serves it locally
- Clients send write requests to Master, which sends it to all servers, gets majority (quorum) among servers, and then responds to client
- On master failure, run election protocol
- On replica failure, just replace it and have it catch up
• Mutual exclusion important problem in cloud computing systems
• Classical algorithms
  • Central
  • Ring-based
  • Ricart-Agrawala
  • Maekawa
• Industry systems
  • Chubby: a coordination service
  • Similarly, Apache Zookeeper for coordination
Announcements

• MP2 due this Sunday
• MP2 demos next Monday
  • Signup sheet and demo details on Piazza
• HW2 due next Tuesday
  • Solutions released soon after
  • Graded HW2’s may be returned with/after midterm
**ANNOUNCEMENTS: MIDTERM**

- Midterm Tuesday Oct 13th
- In-class, but there are two classrooms
  - If your last name starts with A-O, go to 1320 DCL
  - Else If your last name starts with P-Z, go to MSEB 119
  - Else you have an awesome last name
- Closed-book, closed-notes.
- Calculators ok.
- No cellphones or other devices, etc.
- Bring pen/pencil
- Multiple choice (short), longer questions (apply + design)
- Sample midterm problems on website
- Also look at HWs, videos, slides, flipped exercises (and textbook)