## Homework 2

CS425/ECE428 Spring 2024
Due: Wednesday, Mar 6 at 11:59 p.m.

| Process ID | Time when "enter" is <br> called (since start of <br> system) | Time spent in critical <br> section after"enter" <br> returns, before calling <br> "exit" |
| :---: | :---: | :---: |
| $P_{1}$ | 10 ms | 50 ms |
| $P_{2}$ | 200 ms | 30 ms |
| $P_{3}$ | 25 ms | 10 ms |
| $P_{4}$ | 5 ms | 20 ms |
| $P_{5}$ | 105 ms | 5 ms |

Table 1: Timings for Q1

1. Consider a distributed system of five processes $\left\{P_{1}, P_{2}, P_{3}, P_{4}, P_{5}\right\}$. Each process needs mutually exclusive access to a critical section. Assume that each process wishes to enter a critical section only once. Table 1 lists the time when each process first makes a blocking call to "enter" the critical section (since the start of the system). It also lists the time each process spends in the critical section after "enter" succeeds, before calling "exit".

For each of the subparts below, assume that the one-way network delay between any two different processes is fixed at 10 ms (i.e. it takes exactly 10 ms for a message to go from $P_{i}$ to $P_{j}$, when $i \neq j$ ). The network delay for any message that a process $P_{i}$ sends to itself is zero. Other than the network delays and the time spent in critical section, assume all other processing takes negligible amount of time.
(a) (5 points) Suppose the system uses the central server algorithm for mutual exclusion, electing $P_{3}$ as the leader. The leader grants requests in the order in which it receives them. When will each process start executing its critical section?
(b) (5 points) Now suppose that the system uses ring-based algorithm for mutual exclusion, with the ring structured as shown below ( P 1 to P 2 to P 3 to P 4 to P 5 to P 1 ).


Figure 1
At time 0 ms (when the system starts up), the token is at $P_{1}$. As specified above, the network delay for passing the token from a given process to its ring successor is 10 ms . When will each process start executing its critical section?
(c) (5 points) Now suppose the processes use the Ricart-Agrawala algorithm for mutual exclusion. Assume all processes start off with a Lamport timestamp of zero, and no other events occur at the processes outside of the events that are part of the algorithm. When will each process start executing its critical section?
2. Consider a system of 6 processes $\{N 1, N 2, N 3, N 4, N 5, N 6\}$ with ids 1 to 6 ( $i$ for $N i$ ). The system uses the ring-based algorithm for leader election, with the six processes arranged in a ring as shown below. The leader election algorithm uses the optimization proposed by Chang \& Roberts to reduce the number of messages passed around the ring when multiple processes initiate the election. The process with highest id must be elected as the leader (which is N6 in this case). It is given that no process fails and no messages are dropped. The time taken to transmit a message from a process to its ring successor is exactly 20 ms , and the processing time at each process is negligible.


If processes N2 and N4 initiate the election simultaneously (at exactly the same time),
(a) (2 points) what is the total number of messages that get exchanged before N2 sets N6 as its elected leader?
(b) (2 points) how long does it take for N 2 to set N 6 as its elected leader after N2 and N4 initiate the election?
3. Consider the following modification of the Bully algorithm: The initiating node (which we assume does not fail) sends an Election message only to the process with the highest id. If it does not get a response after a timeout, it then sends an Election message to the process with the second highest id. If after another timeout it gets no response, it tries the third highest id, and so on. If no higher numbered processes respond, it sends a Coordinator message to all lower-numbered processes.
(a) (1 point) What should a process do when it receives an Election message in order to minimize turnaround time?

For the following parts, consider a distributed system of 7 processes $\left\{P_{1}, P_{2}, \ldots P_{7}\right\} . P_{7}$ has the highest id, followed by $P_{6}$, then $P_{5}$, and so on. The system uses the modified Bully algorithm for leader election (including the solution for 3a). Initially, all 7 processes are alive and $P_{7}$ is the leader. Then $P_{7}$ fails, $P_{3}$ detects this, and initiates the election. $P_{3}$ knows that $P_{7}$ has failed and $P_{6}$ has the highest id among the remaining processes. Assume one-way message transmission time between any two processes is fixed at 20 ms , and timeout is set using the knowledge of this message transmission time.
(b) (1 point) If no other node fails during the election run, how many total messages will be sent by all processes in this election run?
(c) (1 point) If no other node fails during the election run, how long will it take for the election to finish?
(d) (1 point) Now assume that right after $P_{3}$ detects $P_{7}$ 's failure and initiates the election, $P_{6}$ fails. How many total messages will be sent by all processes in this election run?
(e) (1 point) For the above scenario (where $P_{6}$ fails right after $P_{3}$ initiates election upon detecting $P_{7}$ 's failure), how long will it take for the election to finish?
4. Consider a system of six processes $\left[P_{1}, P_{2}, P_{3}, P_{4}, P_{5}, P_{6}\right]$. Each process $P_{i}$ proposes a value $x_{i}$. Let $x_{1}=10, x_{2}=8, x_{3}=5, x_{4}=12, x_{5}=15, x_{6}=6$.
Each process $P_{k}$ must decide on an output variable $y_{k}$ (initialized to undecided), setting it to one of the proposed values $x_{i}$ for $i \in[1,6]$. The safety condition requires that at any point in time, for any two processes $P_{j}$ and $P_{k}$, either $y_{j}$ or $y_{k}$ is undecided, or $y_{j}=y_{k}$ (in other words, the decided value must be same across all processes that have decided).
A consensus algorithm is designed for the above problem that works as follows:

- Each process R-multicasts its proposed value at the same time $t=0 m s$ since start of the system (as per their local clocks).
- As soon as proposed values from all 6 processes are delivered at a process $P_{j}, P_{j}$ sets $y_{j}$ to the maximum of the proposed values it received from the six processes.
- If $y_{j}$ is still undecided at time $(t+$ timeout $), P_{j}$ computes the maximum of the proposed values it has received so far and sets $y_{j}$ to that value.
- Once a process $P_{j}$ decides on $y_{j}$, it does not update $y_{j}$ 's value, and ignores future proposals (if any are received).

Assume that all clocks are perfectly synchronized with zero skew with respect to one-another. The proposed value $x_{i}$ of a process $P_{i}$ gets self-delivered immediately at time $t=0 \mathrm{~ms}$ when $P_{i}$ begins the multicast of $x_{i}$. A message sent from a process to any other process takes exactly $T=10 \mathrm{~ms}$ (and this value is known to all processes). All communication channels are reliable. Processes may fail, but a failed process never restarts.
Suppose the timeout value for the above algorithm is set to 25 ms . Answer the following questions with respect to local time at the processes' clock since the start of the system.
(a) (1 point) Assume no process fails in the system. When will each process decide on a value and what will each of their decided values be?
(b) (1 point) Assume $P_{5}$ fails right after unicasting $x_{5}$ to $P_{2}$ and $P_{6}$ but just before it could initiate the unicast of $x_{5}$ to any of the other processes. When will each of the alive processes decide on a value and what will each of their decided values be?
(c) (2 points) Assume $P_{5}$ fails at right after unicasting $x_{5}$ to $P_{2}$ but just before it could initiate the unicast of $x_{5}$ to any of the other processes. $P_{2}$ fails right after issuing unicast of $x_{5}$ to $P_{6}$ but just before it unicasts it to any other process. When will each of the alive processes decide on a value and what will each of their decided values be?
(d) (2 points) Assume $P_{5}$ fails at right after unicasting $x_{5}$ to $P_{2}$ but just before it could initiate the unicast of $x_{5}$ to any of the other processes. $P_{2}$ fails right after issuing unicast of $x_{5}$ to $P_{6}$ but just before it unicasts it to any other process. Then $P_{6}$ fails right after issuing unicast of $x_{5}$ to $P_{1}$ but just before it unicasts it to any other process. When will each of the alive processes decide on a value and what will each of their decided values be?
(e) (2 points) Assume $P_{4}$ fails right before it could unicast $x_{4}$ to any process. $P_{5}$ fails right after unicasting $x_{5}$ to $P_{2}$ but just before it could initiate the unicast of $x_{5}$ to any of the other processes. $P_{2}$ fails right after issuing unicast of $x_{5}$ to $P_{6}$ but just before it unicasts it to any other process. When will each of the remaining alive processes decide on a value and what will each of their decided values be?
(f) (2 points) What is the smallest value that the timeout should be set to for ensuring safety in this system?
(g) (2 points) Answer Q4c assuming that the timeout is updated to the value in Q4f.
(h) (2 points) Answer Q4d assuming that the timeout is updated to the value in Q4f.
(i) (2 points) Answer Q4e assuming that the timeout is updated to the value in Q4f.

