Midterm Review

PART I

Reliability

Reliability

- Reliability for a giving mission duration t, $R(t)$, is the probability of the system working as specified (i.e., probability of no failures) for a duration that is at least as long as t.
- The most commonly used reliability function is the exponential reliability function:

$$R(t) = e^{-\lambda t}$$

From queueing theory: Probability of zero independent arrivals in t time units (Poisson arrival process)

where λ is the failure rate.

Reliability

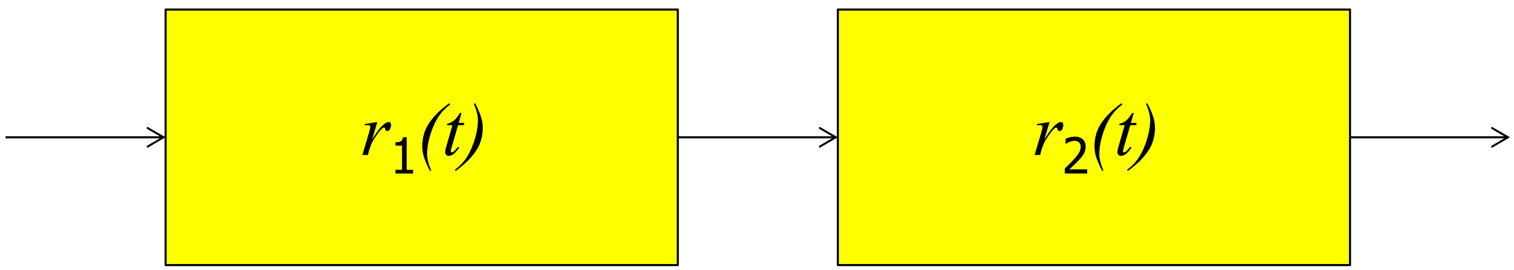
- The most commonly used reliability function is the exponential reliability function:

$$R(t) = e^{-\lambda t}$$

where λ is the failure rate.

- Mean time to failure (MTTF): $1/\lambda$

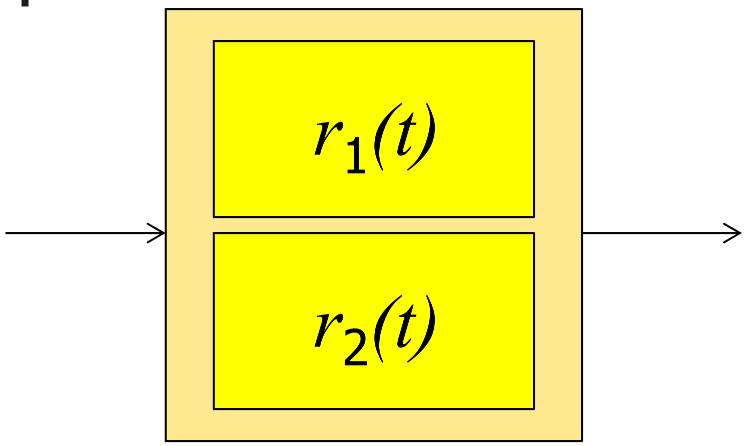
Simple Reliability Modeling



- Total failure rate = $\lambda_1 + \lambda_2$
- Mean time to failure = $1/(\lambda_1 + \lambda_2)$
- Total reliability:

$$R(t) = r_1(t) r_2(t) = e^{-(\lambda_1 + \lambda_2)t}$$

Simple Reliability Modeling

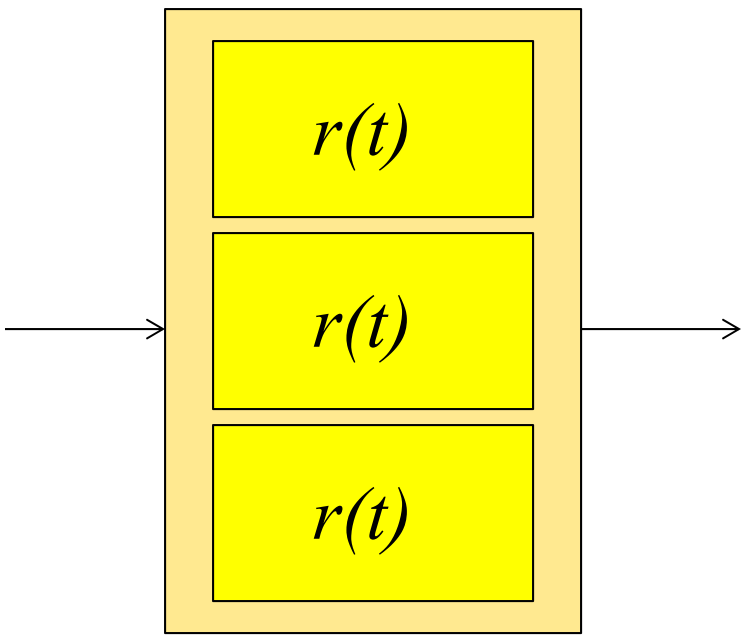


Note: This system needs at least one of the two components to function.

- Total reliability:

$$R(t) = 1 - (1 - r_1(t))(1 - r_2(t))$$

Triple Modular Redundancy



Note: This system needs at least two of the three components to function.

- Total reliability:

$$R(t) = r^3(t) + 3r^2(t)(1 - r(t))$$

Other Implications

$$R(Effort,\ Complexity,\ t) = e^{-kC\ t/E}$$

- Note: splitting the effort greatly reduces reliability.

Well Formed Dependencies

- *Informal intuition:* A reliable component should not *depend* on a less reliable component (it defeats the purpose).
- Design guideline: **Use but do not depend** on less reliable components

Review of Important Theorems

- Total Probability Theorem:

  $P(A) = P(A|C_1)\ P(C_1) + \ldots + P(A|C_n)\ P(C_n)$

  where $C_1, \ldots, C_n$ partition the space of all possibilities

- Bayes Theorem:

  $P(A|B) = P(B|A).\ P(A)/P(B)$

- Other: $P(A,B) = P(A|B)\ P(B)$

Two Sensor Example

- If burglar enters, motion alarm fires 99% of the time and vibration alarm fires 90% of the time. Burglaries occur once a year, motion alarm fires 3 times a year, and vibration alarm fires 10 times a year.
- What are the odds of burglary if both sensors fire?
- P (Burg|A, Vib) = ?
- P (B|A,V) = P(A,V|B) P(B)/P(A,V)

Now what?

OK to say P(A,V|B) = P(A|B)P(V|B)

~~P(A,V) = P(A)P(V)?~~

Remember: If burglar enters, motion alarm fires 99% of the time and vibration alarm fires 90% of the time. Burglaries occur once a year, motion alarm fires 3 times a year, and vibration alarm fires 10 times a year.

Two Sensor Example

- P (Burg|A, Vib) Solution steps:
  - Find the probability of false alarms from:

    $P(A) = P(A|B)\,P(B) + P(A|\bar{B})\,P(\bar{B})$

    $P(V) = P(V|B)\,P(B) + P(V|\bar{B})\,P(\bar{B})$
  - Find the probability of both sensors firing:

    $P(A,V) = P(A,V|B)\,P(B) + P(A,V|\bar{B})\,P(\bar{B})$

    where $P(A,V|B) = P(A|B)P(V|B)$

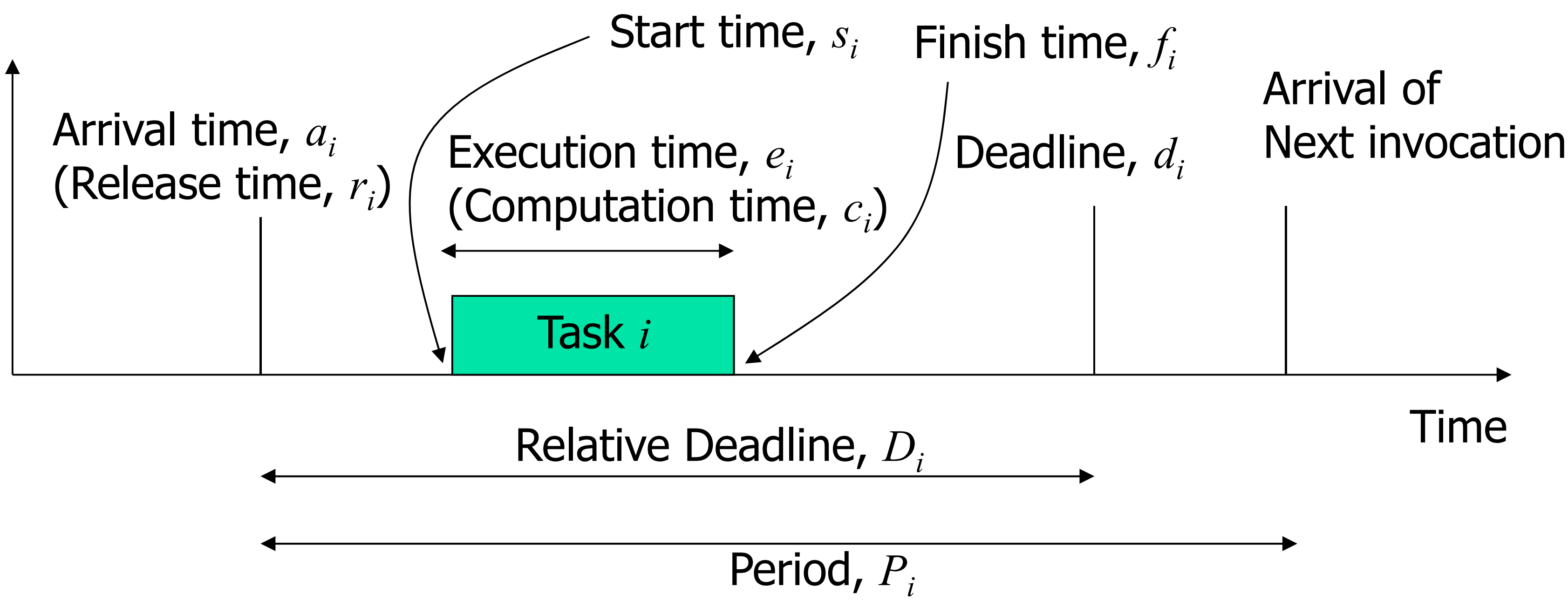
    $P(A,V|\bar{B}) = P(A|\bar{B})P(V|\bar{B})$
  - $P\,(B|A,V) = P(A,V|B)\,P(B)/P(A,V) = 94.62\%$

PART II

Timeliness

Some Terminology

- Tasks, periods, arrival-time, deadline, execution time, etc.



The Schedulability Condition

For n independent periodic tasks with periods equal to deadlines:

The utilization bound of EDF = 1.

The Utilization bound of RM is:

$$U = n\left(2^{1/n} - 1\right)$$

$$n \to \infty \quad U \to \ln 2$$

Practice Question #1

- The probability that a window breaks in a house on any one day is 1/10,000, except when there is a hurricane.
- The probability that a window breaks during a hurricane is 0.3
- The probability that a hurricane passes nearby on any given day is 1/1000
- What are the odds that all 6 windows in Jeff's house break on the same day?

Practice Question #1

- The probability that a window breaks in a house on any one day is 1/10,000, except when there is a hurricane.
- The probability that a window breaks during a hurricane is 0.3
- The probability that a hurricane passes nearby on any given day is 1/1000
- What are the odds that all 6 windows in Jeff's house break on the same day?

- Answer: $1/1000 * (0.3)^6 + 999/1000\ (1/10{,}000)^6$

  $= 1/1000 * (0.3)^6$ (approx.)

Practice Question #2

- The probability of falling debris on planet X is 1/500. The probability that a storage device on a robot breaks when there is falling debris is 0.5. What is the probability that all 4 devices break? (Assume there is no other way for these devices to break.)

Practice Question #2

- The probability of falling debris on planet X is 1/500. The probability that a storage device on a robot breaks when there is falling debris is 0.5. What is the probability that all 4 devices break? (Assume there is no other way for these devices to break.)

- Answer: $1/500\ (0.5)^4$

Observation

- One of the main reasons for failure of large systems is that designers did not properly account for the possibility of correlated failures, and instead viewed them as independent (and hence highly improbably in combination)

Elapsed Time and Reliability

- If the probability of failure within time X is P, what is the probability of failure in time m.X? What is the probability of surviving for time m.X?

Practice Question #3

- The probability of failure on any given day is 1/1000. What is the probability of failure within 5 days?

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- The probability of failure on any given day is 1/1000. What is the probability of failure within 5 days?
- P(Fail) = 1 – P(Survive all 5 days)

  $= 1 - (0.999)^5 = 1 - 0.995 = 0.005$

General Note:

- If the probability of failure within time X is P, what is the probability of failure in time m.X? What is the probability of surviving for time m.X?

- P(surviving time X) = $1 - P$
- P(surviving time mX) = $(1 - P)^m$
- P(failure in time mX) = $1 - (1 - P)^m$

Practice Question #4

- John and Ann do laundry on Sundays with probability 50% each.
  - If their decisions to do laundry are independent, what is the probability that both do laundry on the same Sunday?
  - If their decisions to do laundry are mutually exclusive, what is the probability that both do laundry on the same Sunday?

Practice Question #4

- John and Ann do laundry on Sundays with probability 50% each.
  - If their decisions to do laundry are independent, what is the probability that both do laundry on the same Sunday?
    - P (John, Ann) = P (John) P (Ann) = 0.5 * 0.5 = 0.25
  - If their decisions to do laundry are mutually exclusive, what is the probability that both do laundry on the same Sunday?
    - P (John, Ann)= 0 (for mutual exclusive events)

Note: Independence versus Mutual Exclusion

- For independent events E1, E2, the probability $P(E1, E2) = P(E1)\,P(E2)$
- For mutually exclusive events E1, E2 the probability $P\,(E1, E2) = 0$.

Practice Question #5

- Is this task set schedulable using RM?
  - P1=15, C1=3
  - P2=40, C2=1

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  - P2=40, C2=1

Answer:

- $U = 3/15 + 1/40 < \ln(2)$

  → schedulable using RM!

Practice Question #6

- Is this task set schedulable using EDF?
  - P1=10, C1=3
  - P2=200, C2=14
  - P3=40, C3=11
  - P4=19, C4=6

Practice Question #6

- Is this task set schedulable using EDF?
  - P1=10, C1=3
  - P2=200, C2=14
  - P3=40, C3=11
  - P4=19, C4=6

  - U = 3/10+14/200+11/40+6/19 < 1
    → Schedulable using EDF

Practice Question #7

- Now, assume scheduling is rate monotonic. What is the worst-case response time of each task?
  - $P1=10$, $C1=3$
  - $P2=200$, $C2=14$
  - $P3=40$, $C3=11$
  - $P4=19$, $C4=6$

Practice Question #8

- In the task set below, rate monotonic scheduling is used together with the priority ceiling protocol. What is the worst-case response time of task T2? All times are in seconds.
  - P1=7, C1=3 (includes 1 sec critical section)
  - P2=15, C2=8, D2=11.5