



Exact Schedulability Test

Tarek Abdelzaher



The 4th Credit Project

(Suggested: 1-2 persons per project)

- Option 1: Develop a 30 min survey presentation on an advanced topic of your choice in real-time and embedded computing.
 - Topic name due 10/17.
 - Slides due 11/17.
 - Presentation the week of 12/2
- Example topics:
 - Self-driving cars: the state of the art and future challenges
 - Real-time AI
 - Multicore scheduling – main challenges and results
 - Embedded system security
 - Scheduling Map/Reduce workflows (with emphasis on time support)
 - Participatory and social sensing (crowd-sensing)
 - Software model checking (proving software correctness)
 - IoT market



The 4th Credit Project

(Suggested: 1-2 persons per project)

- Option 2: Implement a real-time or embedded systems service
 - Service name due 10/17.
 - Slides due 11/17.
 - Presentation + Demo the week of 12/2
- Example services:
 - A real-time scheduler for “Intelligence as a Service”
 - Security and diagnostics
 - Disaster response services
 - Social sensing services
 - Your idea here...

Scheduling Taxon

With
Deadline < Period

Periodic Task Scheduling

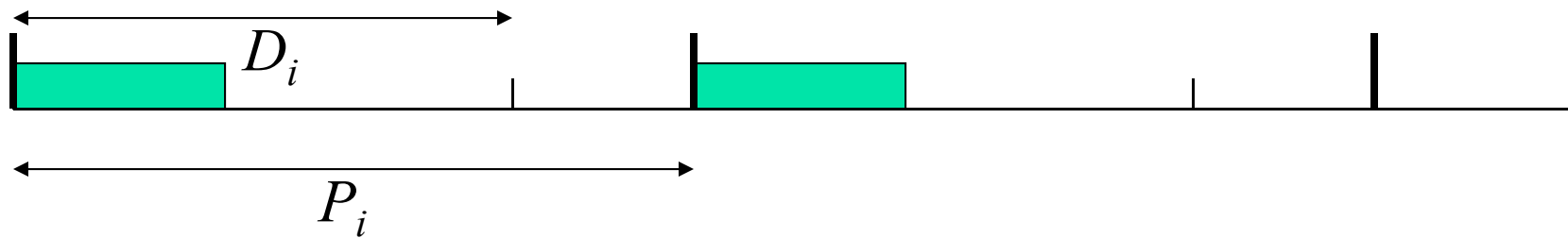
~~Deadline~~

Rate Monotonic

EDF

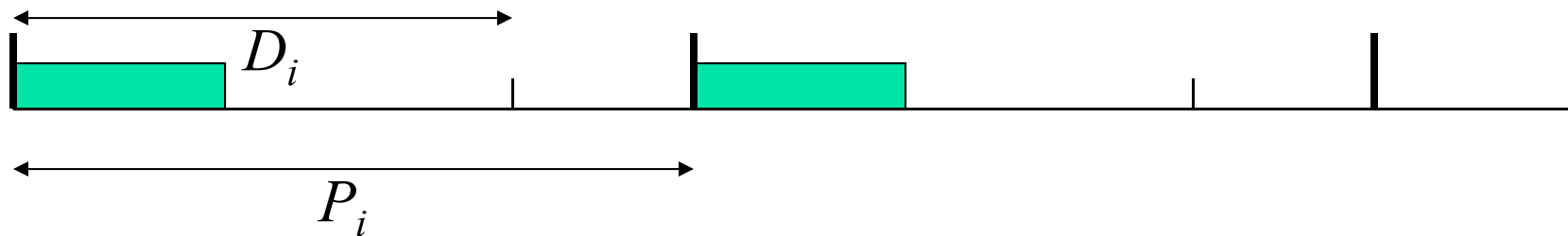
Deadline Monotonic Scheduling

- Consider a set of periodic tasks where each task, i , has a computation time, C_i , a period, P_i , and a relative deadline $D_i < P_i$.



Deadline Monotonic Scheduling

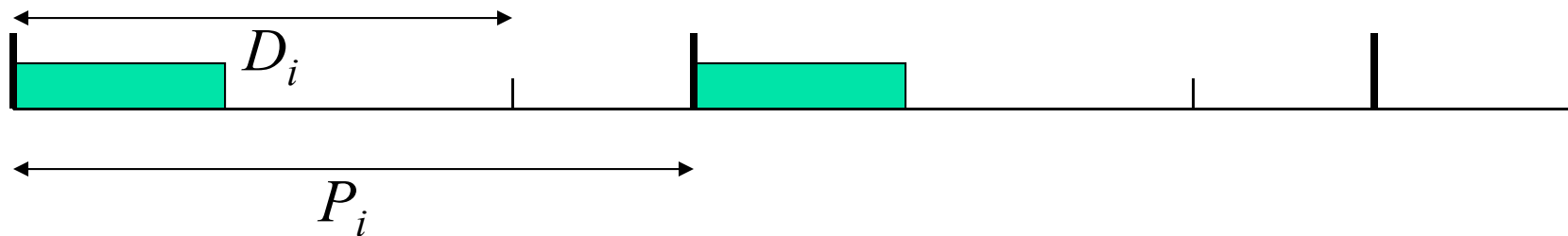
- Consider a set of periodic tasks where each task, i , has a computation time, C_i , a period, P_i , and a relative deadline $D_i < P_i$.



- What is the schedulability condition?

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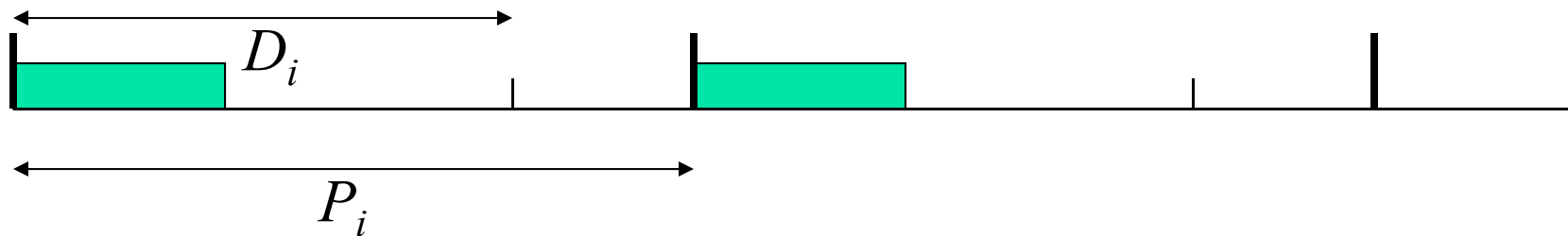


- Schedulability can't be worse than if P_i was reduced to D_i . Thus:

$$\sum_i \frac{C_i}{D_i} \leq n(2^{1/n} - 1)$$

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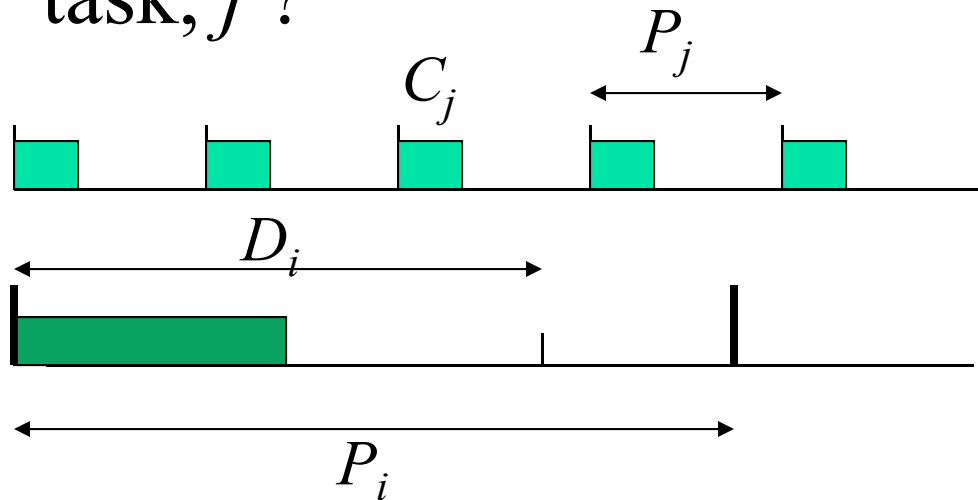
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Problem?

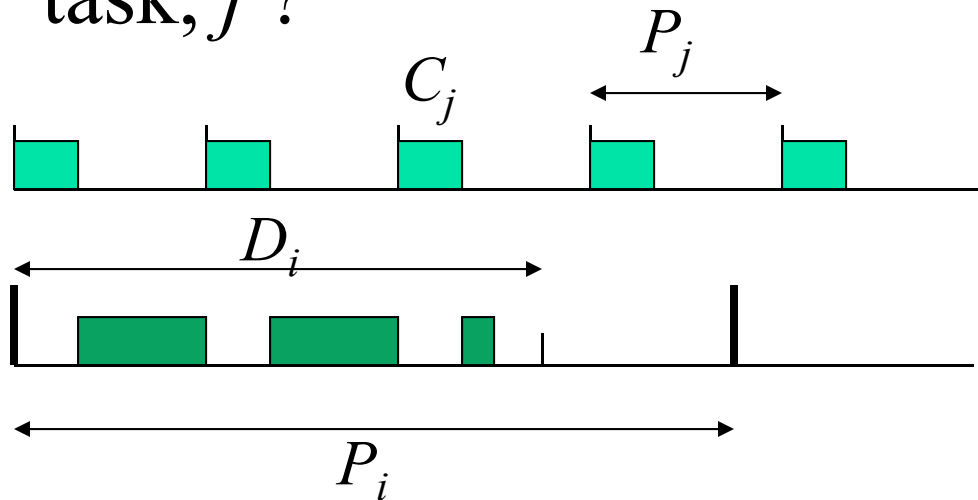
A Better Condition

- Worst case interference from a higher priority task, j ?



A Better Condition

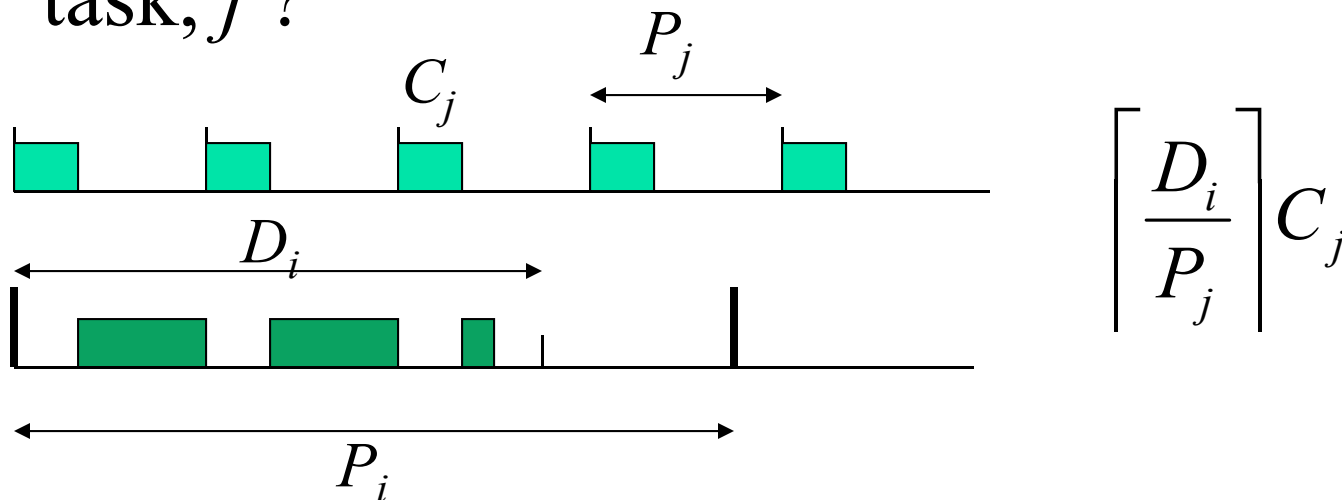
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$$\left\lceil \frac{D_i}{P_j} \right\rceil C_j$$

A Better Condition

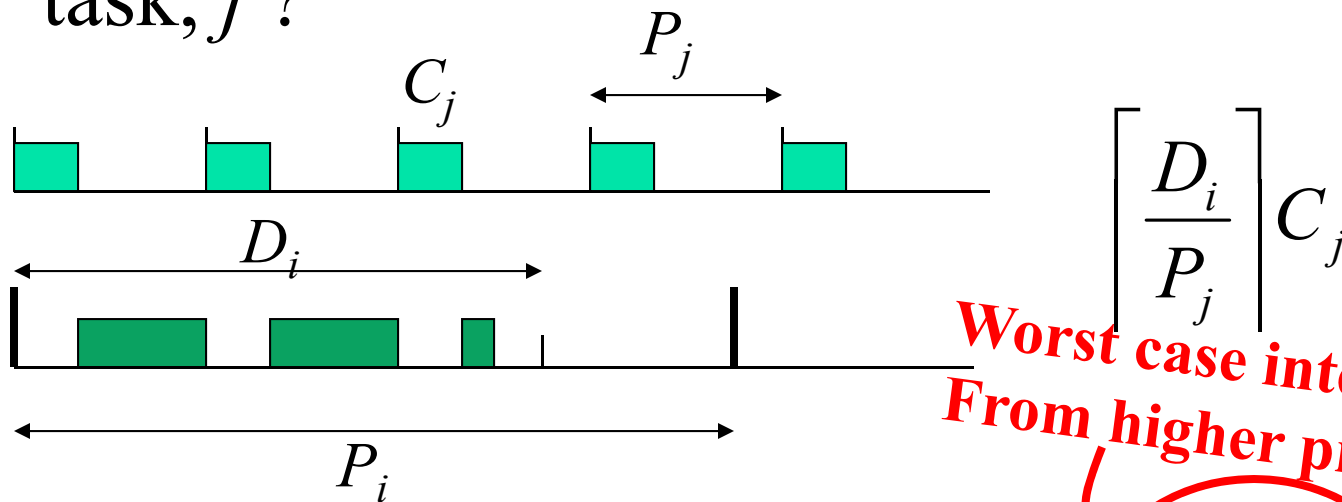
- Worst case interference from a higher priority task, j ?



- Schedulability condition: $C_i + \sum_j \left\lceil \frac{D_i}{P_j} \right\rceil C_j \leq D_i$

A Better Condition

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- Schedulability condition:

$$C_i + \sum_j \left\lfloor \frac{D_i}{P_j} \right\rfloor C_j \leq D_i$$

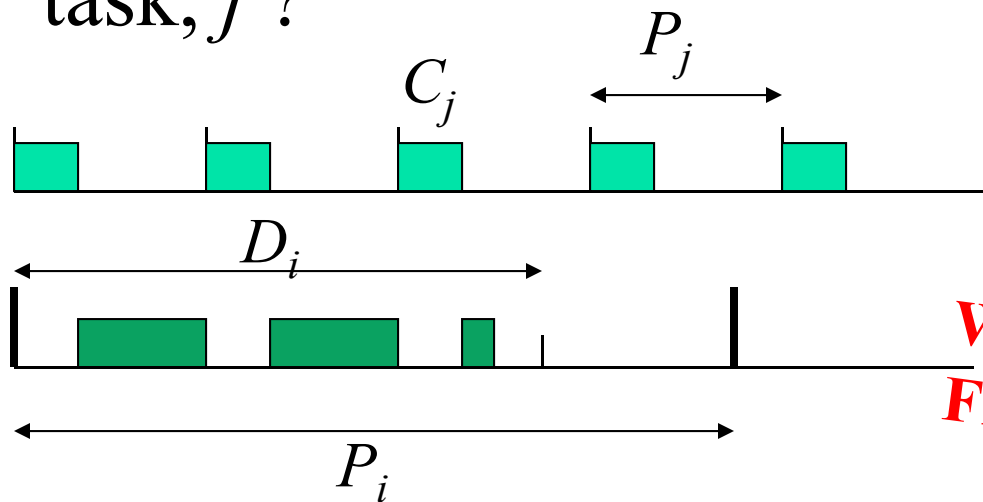
My exec. time

Worst case interference, I ,
From higher priority tasks

My deadline

A Better Condition

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$$\left\lceil \frac{D_i}{P_j} \right\rceil C_j$$

Problem?

**Worst case interference, I ,
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- Schedulability condition:

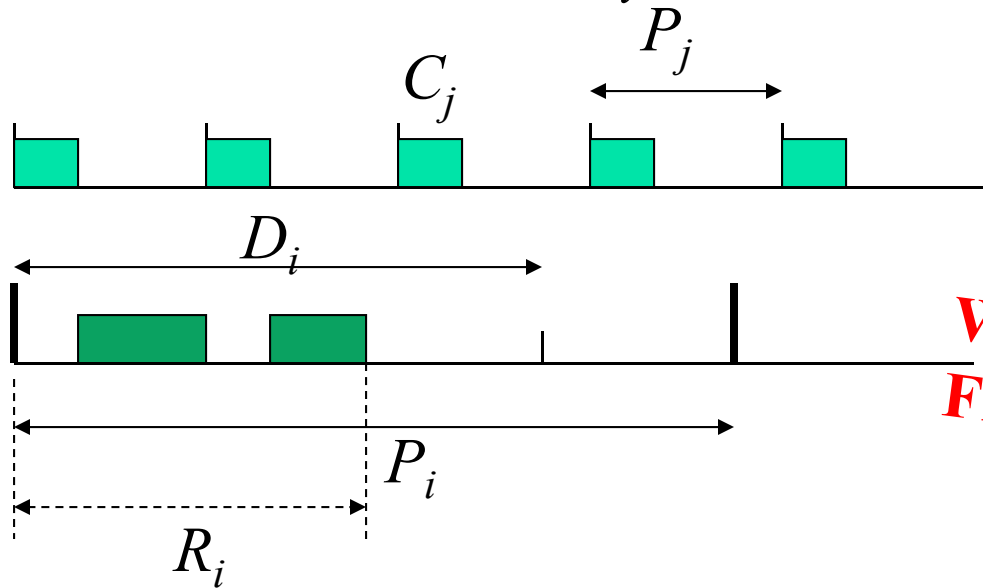
$$C_i + \sum_j \left\lceil \frac{D_i}{P_j} \right\rceil C_j \leq D_i$$

My exec. time (circled around C_i)

My deadline (circled around D_i)

An Exact Condition

- Note: Interference exists only during the response time R_i not the entire D_i

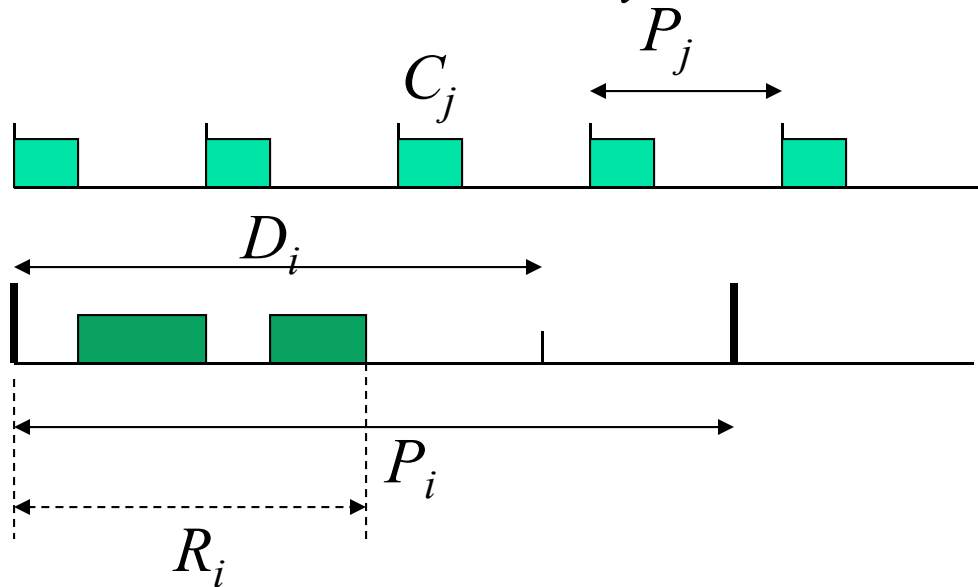


$$I = \sum_j \left[\frac{R_i}{P_j} \right] C_j$$

**Worst case interference, I ,
From higher priority tasks**

An Exact Condition

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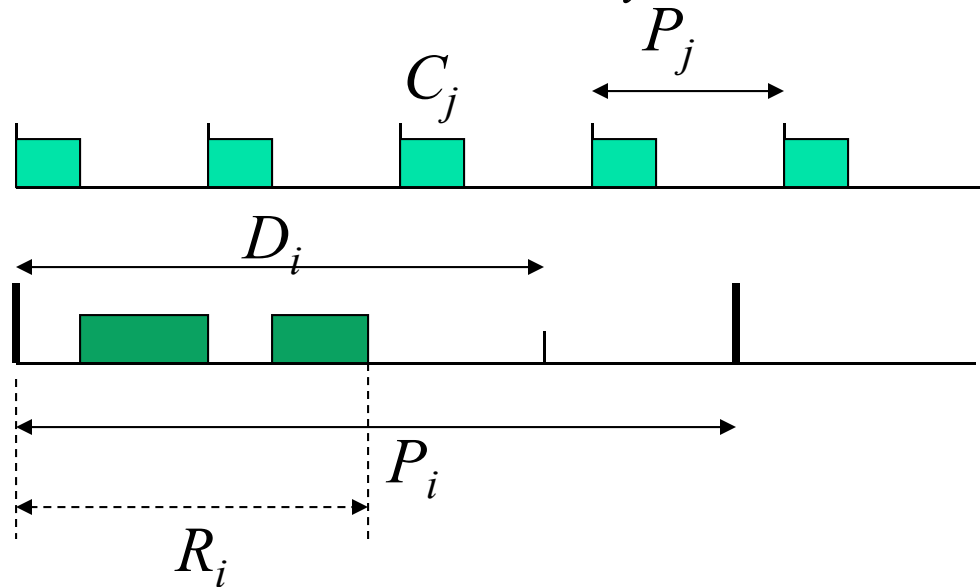
$$I = \sum_j \left[\frac{R_i}{P_j} \right] C_j$$

where

$$R_i = I + C_i$$

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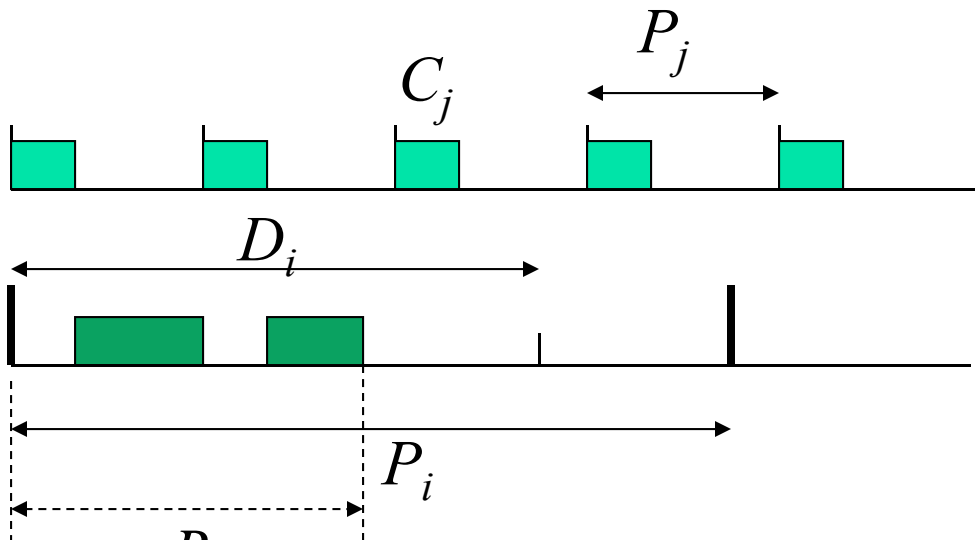
Solve iteratively for the smallest R_i that satisfies both equations



Example

$$I = \sum_j \left[\frac{R_i}{P_j} \right] C_j$$

$$R_i = I + C_i$$



Consider a system of two tasks:

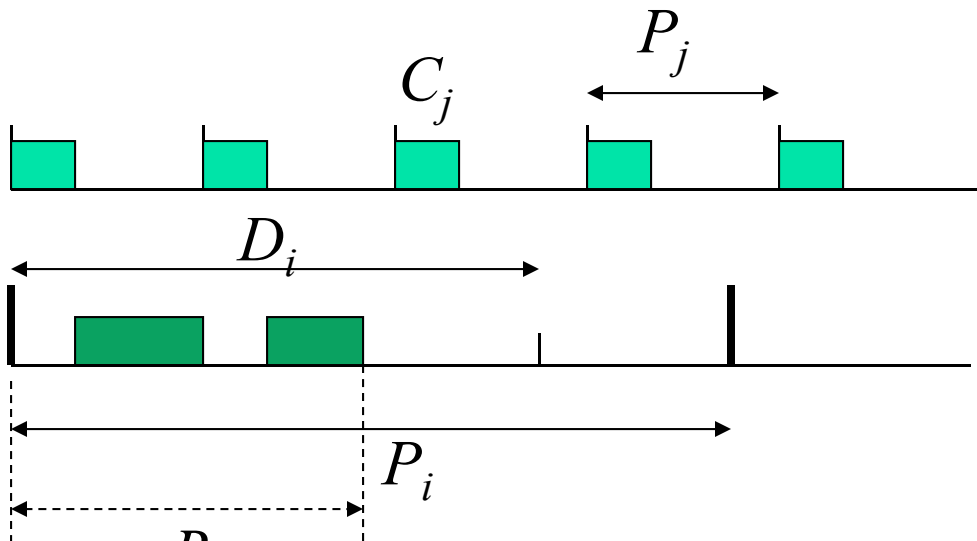
Task 1: $P_1=1.7$, $D_1=0.5$, $C_1=0.5$

Task 2: $P_2=8$, $D_2=3.2$, $C_2=2$

Example

$$I = \sum_j \left[\frac{R_i}{P_j} \right] C_j$$

$$R_i = I + C_i$$



$$I^{(0)} = C_1 = 0.5$$

$$R_2^{(0)} = I^{(0)} + C_2 = 2.5$$

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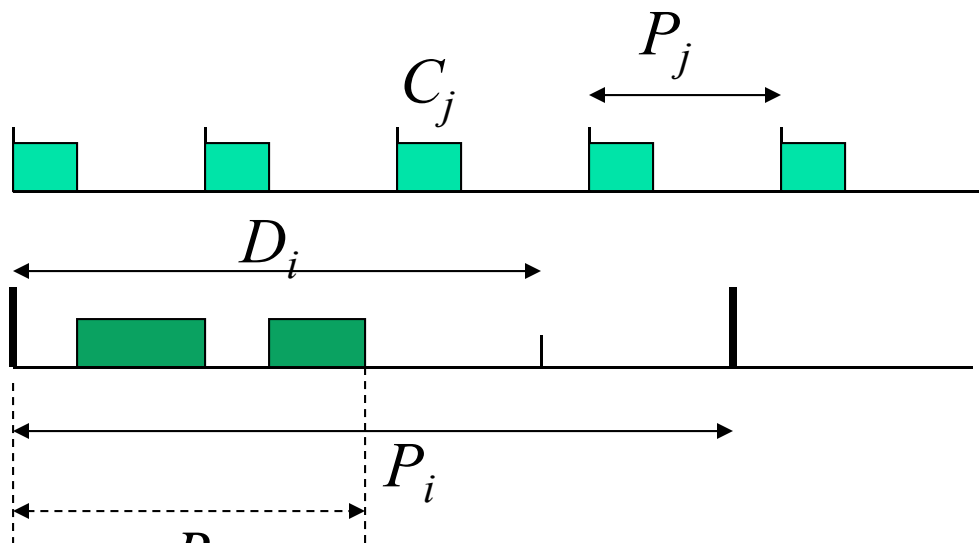
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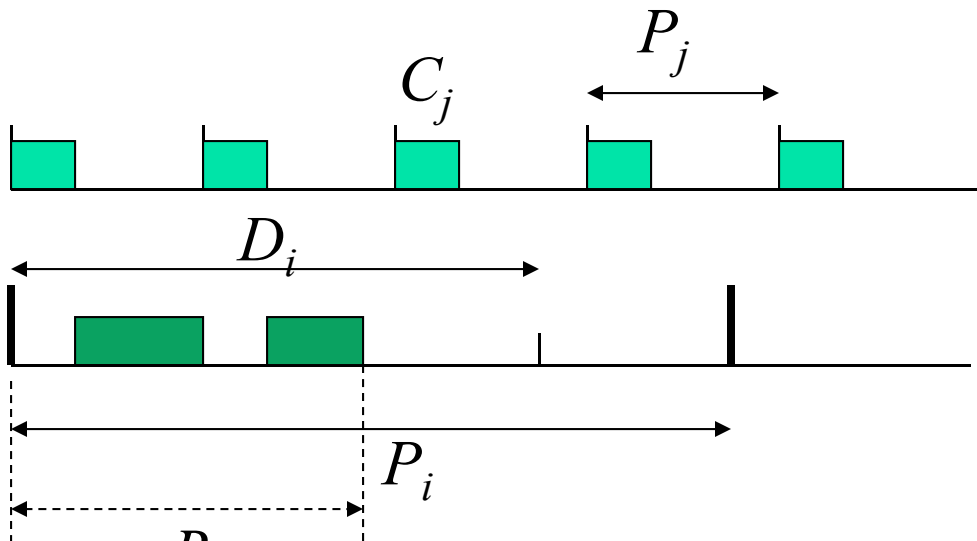
$$I^{(1)} = \left[\frac{R_2^{(0)}}{P_1} \right] C_1 = \left[\frac{2.5}{1.7} \right] 0.5 = 1$$

$$R_2^{(1)} = I^{(1)} + C_2 = 3$$

Example

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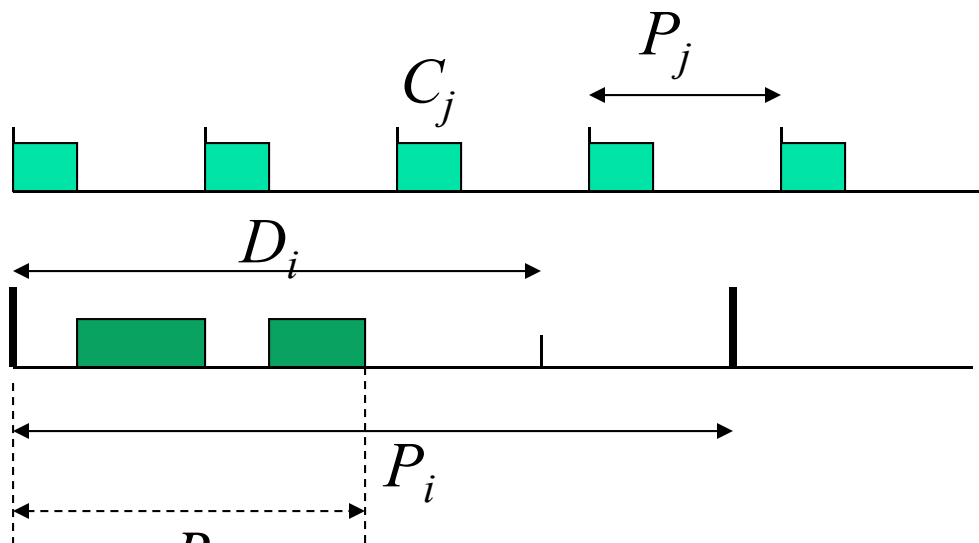
$$I^{(2)} = \left[\frac{R_2^{(1)}}{P_1} \right] C_1 = \left[\frac{3}{1.7} \right] 0.5 = 1$$

$$R_2^{(2)} = I^{(2)} + C_2 = 3$$

Example

$$I = \sum_j \left[\frac{R_i}{P_j} \right] C_j$$

$$R_i = I + C_i$$



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$$R_2^{(2)} = I^{(2)} + C_2 = 3$$

$3 < 3.2 \rightarrow \text{Ok!}$